Report on the habiltiation of
Monsieur BAYAD Abdelmejid

by Werner Bley

The scientific work of A. Bayad is centered around the classical theory of elliptic and modular functions. In particular, he studies Jacobi forms in two variables and uses their analytic properties to derive results in arithmetic.

Let $L \subset \mathbb{C}$ denote a complex lattice, $z \in \mathbb{C}$, and write $K_L(z)$ for the Klein function. The function $K_L(z)$ is a normalization (due to Klein) of the Weierstrass σ-function and has been studied by many authors. Its analytic properties are well known. Let $z, \varphi \in \mathbb{C} \setminus L$ and define

$$D_L(z;\varphi) := e^{\pi i E_L(z;\varphi)} \frac{K_L(z+\varphi)}{K_L(z)K_L(\varphi)}.$$

Here $E_L(z;\varphi) = ad - bc$, if $z = a\omega_2 + b\omega_1, \varphi = c\omega_2 + d\omega_1, a, b, c, d \in \mathbb{R}$, with any $\mathbb{Z}$-basis $\{\omega_1, \omega_2\}$ of L such that $\operatorname{Im}(\frac{\omega_1}{\omega_2}) > 0$. It is this Jacobi form $D_L(z;\varphi)$ which plays a central role in Bayad's work.

In the following I will briefly summarize Bayad's habilitation thesis, to which I refer by [Th]. All other references are those from the reference list of [Th].

As the work of Bayad clearly shows, the function $D_L(z;\varphi)$ has many remarkable properties which have interesting applications in different areas of mathematics. In paragraph 2 of [Th] Bayad gives an exaustive list of these properties. Among these the following two are in my opinion the most important. To state them we let $\Lambda \subseteq \mathbb{C}$ be another complex lattice with $L \subseteq \Lambda$, $[\Lambda : L] = l$.

(i) Additive distribution law

$$\sum_{t \in \Lambda/L} D_L(lz;\varphi+t) = D_\Lambda(z;\varphi).$$

(ii) Multiplicative distribution law

$$K(\varphi; L, \Lambda) \prod_{t \in \Lambda/L} D_L(z+t;\varphi) e^{-2\pi i E_L(t;\varphi)} = D_\Lambda(z;\varphi).$$

Here

$$K(\varphi; L, \Lambda) := \frac{K_L(z)^{[\Lambda:L]}}{K_\Lambda(z)}.$$

Paragraph 3 of [Th] is dedicated to a first application, an analogue of Gauss' quadratic reciprocity law for a quadratic imaginary field K. For $\alpha, \beta \in \mathcal{O}_K$ with $(\alpha, 2) = (\beta, 2) = (\alpha, \beta) = 1$ one defines a quadratic residue symbol $\left(\frac{\alpha}{\beta}\right)_2 \in \{\pm 1\}$ which should be considered as the elliptic analogue of the Legendre symbol. In [8] Bayad proves an explicit reciprocity law using properties of the Weierstrass $\wp$-function. In Appendix 2 of [Th] he gives yet another proof of his result, this