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MR1478032 (98h:11138) 11R27 (11R29)
Bayad, Abdelmejid (F-EVRY-LAP); **Robert, Gilles** (F-GREN-F)
Amélioration d'une congruence pour certains éléments de Stickelberger quadratiques. (French. English, French summaries) [Improvement of a congruence for certain quadratic Stickelberger elements]
Bull. Soc. Math. France **125** (1997), *no. 2,* 249–267.

Let $E/\mathbf{C}$ be an elliptic curve and let $\Omega\subseteq\mathbf{C}$ denote its period lattice. Let $\langle\psi\rangle\subseteq E[p]$ be a cyclic subgroup of order $p>1$ of the group of p-torsion points of $E(\mathbf{C})$. Finally, let $\varphi\in E[p]\setminus\langle\psi\rangle$.

The authors study Ω-elliptic functions $D_\Omega(z,\varphi,\langle\psi\rangle)$ associated to the divisor $\sum_{\rho\in\langle\psi\rangle}(\varphi+\rho)-(\rho)$, which exist by the theorem of Abel-Jacobi and are determined up to a multiplicative constant.

These functions appeared quite frequently in the recent literature, mainly used for the construction of certain Lagrange resolvents with properties analogous to those of the classical Gauss sums. On the one hand the factorization of these resolvents led to Galois module structure results for rings of integers, and on the other hand it served for the construction of "quadratic" Stickelberger elements.

Unfortunately, almost every author chooses his own normalization for $D_\Omega(z,\varphi,\langle\psi\rangle)$, which makes it hard to compare the results. Bayad and Robert now introduce the very natural normalization $\lim_{z\to 0}D_\Omega(z,\varphi,\langle\psi\rangle)=1$, which leads, after suitable specializations, to a beautiful additive distribution law (compare this also to similar formulas in papers of S.-P. Chan [J. Reine Angew. Math. **375/376** (1987), 67–82; MR0882292 (88j:11077)] and R. Schertz [J. Number Theory **39** (1991), no. 3, 285–326; MR1133558 (92j:11130)]).

Based on this the present authors improve on the results concerning quadratic Stickelberger elements of A. Bayad, W. Bley and P. Cassou-Noguès [J. Algebra **179** (1996), no. 1, 145–190; MR1367846 (96k:11131)], which also served as a starting point for their investigations.

Reviewed by *Werner Bley*

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