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Formes de Jacobi et formules de distribution. (French. English summary) [Jacobi forms and distribution formulas]

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Starting from a lattice $L=\mathbb{Z}\omega_1+\mathbb{Z}\omega_2$ in $\mathbb{C}$ and the associated Klein form

$$\mathcal{K}_L(z)=ze^{-|z|^2/2}\prod_{l\in L,l\neq 0}(1-z/l)e^{z/l+(z/l)^2/2},$$

the authors study a Jacobi form in two variables $z,\varphi\in\mathbb{C}$,

$$D_L(z;\varphi)= \exp(2\pi i)(\overline{z}\varphi-\overline{\varphi}z)/(|z_2|^2)\mathrm{Im}(\omega_1/\omega_2)\cdot\mathcal{K}_L(z+\varphi)/(\mathcal{K}_L(z)\mathcal{K}_L(\varphi)).$$

This form is meromorphic in the first variable and not analytic in the second. It has several useful properties already established in previous work, for instance, it is tied to the Weierstrass $\wp$-function via

$$\wp_L(z)-\wp_L(\varphi)=D_L(z,\varphi)D_L(z_1-\varphi).$$

If one has a second lattice $\Lambda\supseteq L$, the authors introduce

$$\mathcal{K}(z;L,\Lambda)=\mathcal{K}_L(z)^{[\Lambda:L]}/\mathcal{K}_\Lambda(z)$$

and prove (Theorem 2.2) as their main result the multiplication distribution formula

$$D_\Lambda(z;\varphi)=\mathcal{K}(z;L,\Lambda)\prod_{\overline{t}\in\Lambda/L}D_L(z;\varphi+t).$$

As a consequence of this result one has another proof of an older distribution formula for elliptic functions obtained by Kubert. Namely (in Theorem 3.1.1), the authors show that each elliptic function can be expressed by products of the D_L's.

The paper closes with two applications which appear as ameliorations of (1) a theorem by Coates, Kubert and Robert concerning Stark units; and (2) a distribution formula for Siegel's φ-function established by Jarvis and Wildeshaus (here the exact root of unity appearing in the formula comes out).

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.