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Valuation q-adique et relation de distribution additive pour certaines fonctions q-périodiques. (French. French summary) [q-adic valuation and additive distribution relation for certain q-periodic functions]
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Let K be a local field, let $\overline{K}$ be its algebraic closure and let q be an element of K^* with strictly positive valuation. The author takes advantage of the well-known fact that most of the classical q-expansions of modular or elliptic functions converge p-adically. Inspired by the recent paper by the author and G. Robert ["Une relation de distribution additive satisfaite par une famille de fonctions elliptiques", Bull. Soc. Math. France, to appear] he proves local analogues of the results of that article.

Given natural numbers p, l such that $(p, l) = 1$ and a primitive p-torsion point $\psi \in \overline{K}^*/q^{\mathbf{Z}}$, a "local Siegel function" associated to the subgroup $q^{\mathbf{Z}}\psi^{\mathbf{Z}}$ and a "local discriminant function" are defined in a quite natural way. In addition, the q-valuations of these functions, evaluated at certain torsion points, are computed.

Let $\varphi \in E[p] \setminus \langle\psi\rangle$ be another p-torsion point of $\overline{K}^*/q^{\mathbf{Z}}$. Using the local version of the Abel-Jacobi theorem the author constructs a q-periodic function $D(\cdot, \varphi, \langle\psi\rangle)$ with given divisor $\sum_{\rho\in\langle\psi\rangle}((\varphi\rho) - (\rho))$ and also provides an expression of D in terms of the Siegel functions and the discriminant.

The main result of the paper is then an additive distribution law which, for two l-torsion points $\alpha, \gamma \in \overline{K}^*/q^{\mathbf{Z}}$, relates a certain sum of the functions $D(\cdot, \varphi, \langle\psi\rangle)$ with $D(\cdot, z_0, \langle\alpha\psi\rangle)$, where z_0 is an lp-torsion point depending on φ and γ. This distribution law may be considered as a local analogue of a resolvent formula studied by S.-P. Chan [J. Reine Angew. Math. **375/376** (1987), 67–82; MR0882292 (88j:11077)].

In the appendix the author uses his results of the previous sections to simplify some of the local computations of his previous paper with the reviewer and P. Cassou-Noguès [J. Algebra **179** (1996), no. 1, 145–190; MR1367846 (96k:11131)], where certain products of these functions (or their complex analogues) are used to construct quadratic Stickelberger elements.

Reviewed by *Werner Bley*

References

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.