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Sommes elliptiques multiples d'Apostol-Dedekind-Zagier. (French. English, French summaries) [Multiple elliptic Apostol-Dedekind-Zagier sums]

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Starting with the classical Klein function $\mathcal{K}_L$ of a lattice $L=\mathbb{Z}\tau+\mathbb{Z}\tau\in\mathfrak{H}$, the author defines a Jacobi form $D_\tau(z,\varphi)z$, $\varphi\in\mathbb{C}$, which is meromorphic in the first variable but not analytic in the second. D_τ is strongly tied to the Weierstrass $\wp$-function, as one has (among other functional relations coming from older work by the author) the equation

$$D_\tau(z,\varphi)D_\tau(z,-\varphi)=\wp_L(z)-\wp_L(\varphi).$$

For $O_L=\{x\in L,\ xL\subset L\},\ m\in\mathbb{N}$, and coprime elements $p,a_0,\dots,a_n\in O_L\setminus O_L^\times$ the author defines elliptic Dedekind sums $d(p;a_1,\dots,a_n;m,\varphi,z,\tau)$ and multiple elliptic Apostol-Dedekind-Zagier sums $S_k(p;a_1,\dots,a_n;m,\varphi,z,\tau)$ built from sums, products and (for S_k) derivations of D_τ's with appropriate arguments.

The main result of the note is reciprocity laws expressing sums of the d's (resp. S_k's) like

$$\sum_{l=0}^{m} S_k(a_l;\ a_0,\dots,\check{a}_e,\dots,a_n;m,\varphi,\tau)$$

for $k\in\mathbb{N},\ a_0+\cdots+a_n+m\equiv 0 \mod dO_L$ and φ a division point $\neq O$ in $\mathbb{C}/L$ as elliptic analogues to classical results by Zagier from 1973 and Apostol from 1950.

Reviewed by *Rolf Berndt*

References

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.