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Bayad, Abdelmejid (F-EVRY)

Sommes de Dedekind elliptiques et formes de Jacobi. (French. English, French summaries)
[Elliptic Dedekind sums and Jacobi forms]

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The author produces generalizations to classical Dedekind sums. The new sums arise as products of Jacobi forms summed over a lattice of the form $\{x\tau + y \mid (x, y) \neq (0, 0), 0 \leq x, y \leq p - 1\}$. The Jacobi forms involved are produced as quotients of theta functions associated to the lattice spanned by $\{1, \tau\}$. The main result of this paper is a law of reciprocity for these elliptic Dedekind sums.

Reviewed by *Howard Skogman*

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