Développements limités usuels avec $x_0 = 0$ (Maclaurin) :

$$\begin{aligned}
\log(1+x) &= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots + (-1)^{n+1}\frac{x^n}{n} + o(x^n) \\
&= \sum_{k=1}^{n} (-1)^{k+1}\frac{x^k}{k} + o(x^n) \\
\log(1-x) &= -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \ldots - \frac{x^n}{n} + o(x^n) \\
&= \sum_{k=1}^{n} \frac{x^k}{k} + o(x^n) \\
e^x &= 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots + \frac{x^n}{n!} + o(x^n) \\
&= \sum_{k=1}^{n} \frac{x^k}{k!} + o(x^n) \\
\sin x &= x - \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+2}) \\
&= \sum_{k=1}^{n} (-1)^k \frac{x^{2k+1}}{(2k+1)!} + o(x^{2n+2}) \\
\cos x &= 1 - \frac{x^2}{2} + \frac{x^4}{4!} + \ldots + (-1)^n \frac{x^{2n}}{(2n)!} + o(x^{2n+1}) \\
&= \sum_{k=1}^{n} (-1)^k \frac{x^{2k}}{(2k)!} + o(x^{2n+1}) \\
\tan x &= x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + o(x^7) \\
\arcsin x &= x + \frac{x^3}{6} + \frac{3x^5}{40} + \frac{5x^7}{112} + \ldots + \frac{(2n)!}{4^n (n!)^2 (2n+1)} x^{2n+1} + o(x^{2n+2}) \\
&= \sum_{k=0}^{n} \frac{(2k)!}{4^k (k!)^2 (2k+1)} x^{2k+1} + o(x^{2n+2}) \\
\arccos x &= \frac{\pi}{2} - \arcsin x \\
&= \frac{\pi}{2} - \sum_{k=0}^{n} \frac{(2k)!}{4^k (k!)^2 (2k+1)} x^{2k+1} + o(x^{2n+2}) \\
\arctan x &= x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots + (-1)^n \frac{x^{2n+1}}{2n+1} + o(x^{2n+2}) \\
&= \sum_{k=0}^{n} (-1)^k \frac{x^{2k+1}}{2k+1} + o(x^{2n+2}) \\
\sinh x &= x + \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots + \frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+2})
\end{aligned}$$

$$
\begin{aligned}
&= \sum_{k=0}^{n} \frac{x^{2k+1}}{(2k+1)!} + o(x^{2n+2}) \\
\cosh x &= 1 + \frac{x^2}{2} + \frac{x^4}{4!} + \ldots + \frac{x^{2n}}{(2n)!} + o(x^{2n+1}) \\
&= \sum_{k=0}^{n} \frac{x^{2k}}{(2k)!} + o(x^{2n+1}) \\
\tanh x &= x - \frac{x^3}{3} + \frac{2x^5}{15} - \frac{17x^7}{315} + o(x^7) \\
(1+x)^\alpha &= 1 + \alpha x + \frac{\alpha(\alpha-1)}{2}x^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{3!}x^3 + \ldots \\
&\quad \ldots + \frac{\alpha(\alpha-1)(\alpha-2)\ldots(\alpha-(n-1))}{n!}x^n + o(x^n) \quad \text{avec } \alpha \in \mathbb{R} \\
&= \sum_{k=0}^{n} \binom{\alpha}{n} x^k + o(x^n) \quad \text{si } \alpha \in \mathbb{N}
\end{aligned}
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avec les cas particuliers

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\begin{aligned}
\sqrt{1+x} &= 1 + \frac{x}{2} - \frac{x^2}{2^3} + \frac{x^3}{2^4} + \ldots + (-1)^{n-1}\frac{3\cdot 5\cdot 7\cdot \ldots \cdot (2n-3)}{n!2^n}x^n + o(x^n) \\
\sqrt{1-x} &= 1 - \frac{x}{2} - \frac{x^2}{2^3} - \frac{x^3}{2^4} - \ldots - \frac{3\cdot 5\cdot 7\cdot \ldots \cdot (2n-3)}{n!2^n}x^n + o(x^n) \\
\frac{1}{\sqrt{1+x}} &= 1 - \frac{x}{2} + \frac{3x^2}{2^3} - \frac{5x^3}{2^4} + \ldots + (-1)^n\frac{3\cdot 5\cdot 7\cdot \ldots \cdot (2n-1)}{n!2^n}x^n + o(x^n) \\
\frac{1}{1+x} &= 1 - x + x^2 - x^3 + \ldots + (-1)^n x^n + o(x^n) \\
&= \sum_{k=0}^{n} (-1)^k x^k + o(x^n) \\
\frac{1}{1-x} &= 1 + x + x^2 + x^3 + \ldots + x^n + o(x^n) \\
&= \sum_{k=0}^{n} x^k + o(x^n)
\end{aligned}
$$