Wealth Transfers, Indifference Pricing, and XVA Compression Schemes*

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Abstract

Since the 2008–09 financial crisis, banks have introduced a family of XVA metrics to quantify the cost of counterparty risk and of its capital and funding implications: the credit/debt valuation adjustment (CVA and DVA), the costs of funding variation margin (FVA) and initial margin (MVA), and the capital valuation adjustment (KVA).

We revisit from a wealth conservation and wealth transfer perspective at the incremental trade level the cost-of-capital XVA approach developed at the level of the balance sheet of the bank in Albanese and Crépey (2018). Trade incremental XVAs reflect the wealth transfers triggered by the deals due to the incompleteness of counterparty risk. XVA-inclusive trading strategies achieve a given hurdle rate to shareholders in the conservative limit case that no new trades occur.

XVAs represent a switch of paradigm in derivative management, from hedging to balance sheet optimization. This is illustrated by a review of possible applications of the XVA metrics, including a genetic algorithm CVA compression case study on real swap portfolios.

Keywords: Counterparty risk, credit/debt valuation adjustment (CVA and DVA), market incompleteness, wealth transfer, cost of capital, funding valuation adjustment (FVA), margin valuation adjustment (MVA), cost of capital (KVA), funds transfer price (FTP), XVA compression, genetic algorithm.

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1 Introduction

In the aftermath of the financial crisis of 2008–09, regulators launched in a major banking reform aimed at reducing counterparty risk by raising collateralisation and capital requirements and by incentivising central clearing (see Basel Committee on Banking Supervision (2013)). The Basel III banking regulatory framework also set out guidelines for CVA and DVA (credit and debt valuation adjustments), which are valuation metrics for counterparty and own default risk in bilateral markets.

In bilateral as in centrally cleared transactions, collateral nowadays comes in two forms. The variation margin (VM), which is typically re-hypothecable, tracks the (counterparty-risk-free) value of a portfolio. The initial margin (IM) is an additional layer of margin, typically segregated, which is meant as a guarantee against the risk of slippage of a portfolio between default and liquidation. To quantify the respective costs of VM and IM, banks started to price into contingent claims funding and now margin valuation adjustments (FVA and MVA).

On a parallel track, the regulatory framework for the insurance industry has also been reformed, but on the basis of a different set of principles. Insurance claims are largely unhedged and markets are intrinsically incomplete. The cost of capital is reflected into prices. Solvency II focuses on regulating dividend distribution policies in such a way to ensure a sustainable risk remuneration to the shareholders over the lifetime of the portfolio. This is based on the notion of risk margin, which in banking parlance corresponds to a capital valuation adjustment (KVA).

In a related stream of papers, we develop a cost-of-capital XVA approach in incomplete counterparty risk markets. Albanese and Crépey (2018) state the fundamental principles, rooted in the specificities of the balance sheet of a dealer bank. The application of these principles in continuous time leads to a progressive enlargement of filtration setup and nonstandard XVA backward stochastic differential equations (BSDEs) stopped before the bank default time. Crépey, Élie, Sabbagh, and Song (2018) deliver the mathematical tools required for solving such BSDEs and consider further the BSDEs "of the Mc Kean type" that arise when one includes the possibility for the bank to use its capital as a funding source for variation margin. Abbas-Turki, Diallo, and Crépey (2018) deal with the numerical solution of our XVA equations by nested Monte Carlo strategies optimally implemented on graphics processing units (GPUs). Albanese, Caenazzo, and Crépey (2017) apply what precedes to the concrete situation of a bank engaged into bilateral trade portfolios, demonstrating the feasibility of this approach on real banking portfolio involving thousands of counterparties and hundreds of thousands of trades. They also illustrate numerically the importance of the FVA mitigation provided, in the case of uncollateralized portfolios, by the use of capital as a funding source. Armenti and Crépey (2017) and Armenti and Crépey (2018) apply the generic principles of Albanese and Crépey (2018) to the XVA analysis of centrally cleared portfolios: in Armenti and Crépey (2017), this is done under standard regulatory assumptions on the default fund and the funding strategies for initial margins, in order to compare in XVA terms bilateral versus centrally cleared trading networks. Armenti and Crépey (2018), on the other hand, challenge these assumptions in the direction of achieving a greater efficiency (in XVA terms) of centrally cleared trading networks.

The take of the present paper is to show how XVAs represent a switch of paradigm in derivative management, from hedging to balance sheet optimization. This is done by extensively relying on the notion of wealth transfer. This notion is already at the core of Albanese and Crépey (2018), but kind of implicitly and at the level of the whole portfolio of the bank, mainly used a posteriori and for interpretation purposes. By contrast, in this paper, we make it our main tool, exploited directly at the trade incremental level. The rewiring of the theory around the notion of wealth transfer allows re-deriving explicitly and "linearly", with virtually no mathematics, a number of the conceptual relations obtained in Albanese and Crépey (2018).

By the variety of situations that we consider, we try and demonstrate that the wealth transfer view yields a very practical and versatile angle on XVA analysis. The benefit of this angle is to bring in intuition and flexibility. The price for it lies in detail and accuracy, as, beyond the elementary static setup considered as a toy example in Section 4, going deeper into the arguments in order to obtain precise XVA equations unavoidably brings back to all the peculiarities in Albanese and Crépey (2018) and the follow up papers.

Under our cost-of-capital pricing approach, beyond CVA and DVA, the whole XVA castle is rooted on counterparty risk incompleteness. For alternative, replication-based, XVA approaches, see, for instance, Brigo and Pallavicini (2014), Burgard and Kjaer (2011, 2013, 2017), Bichuch, Capponi, and Sturm (2018) (without KVA), or, with a KVA meant as an additional liability like the CVA and the FVA (as opposed to a risk premium in our case), Green, Kenyon, and Dennis (2014). A detailed comparative discussion is provided in Albanese and Crépey (2018, Section 9).

The ***outline*** of the paper goes as follows. Section 2 sets our XVA pricing stage and delivers our main result Theorem 2.1 (which is just another perspective on Albanese and Crépey (2018, Theorem 6.1)). Starting from the limiting case of complete markets, the successive wealth transfers triggered by different forms of counterparty risk incompleteness or trading restrictions are reviewed in Section 3, along with the corresponding XVA implications. Section 4, which is a rewiring of Section B in Albanese and Crépey (2018), yields explicit XVA formulas and illustrates the XVA wealth transfer issue in a one-period static setup. Section 5 illustrates the switch of paradigm that XVAs represent in derivative management, from hedging to balance sheet optimization. Section 6 concludes the main body of the paper by a CVA compression case study on real swap portfolios. Some connections with the Modigliani-Miller theory are discussed in Section A (which develops Section 6.1 in Albanese and Crépey (2018)). Section B provides some details about the evolutionary algorithm used in our CVA compression case study.

The ***main contributions*** of the paper are: From a theoretical point of view, a streamlined presentation of the cost-of-capital XVA approach, rewired around the notion of wealth transfer at the trade incremental level; From a practical point of view, the emphasis on the optimization perspective on the XVA metrics, including a concrete illustration by our CVA compression case study on real swap portfolios.

1.1 Abbreviations

Here is a recapitulative list of the abbreviations introduced in the course of the paper.

BCVA Bilateral CVA.

BDVA Bilateral DVA.

CA Contra-assets valuation.

CCP Central counterparty.

CDS Credit default swap.

CL Contra-liabilities valuation.

CSA Credit support annex.

CVA Credit valuation adjustment (can be unilateral or bilateral).

CVA$^{\mathrm{CL}}$ Contra-liability component of a unilateral CVA.

DFC Default fund contribution.

DVA Debt valuation adjustment (can be unilateral or bilateral).

FDA Funding debt adjustment.

FTP Funds transfer price.

FVA Funding valuation adjustment.

IAS International accounting standard.

IFRS International financial reporting standards.

KVA Capital valuation adjustment.

MDA Margin debt adjustment.

MtM Mark-to-market.

MVA Margin valuation adjustment.

OIS Overnight index swap.

RC Reserve capital.

REPO Repurchase agreement.

RM Risk margin.

SCR Shareholder capital at risk.

UCVA Unilateral CVA.

XVA Generic "X" valuation adjustment.

Also:

BA Value of the derivative portfolio of the bank to the bank as a whole (shareholders and bondholders).

SH Value of the derivative portfolio of the bank to the bank shareholders.

BH Value of the derivative portfolio of the bank to the bank bondholders.

CO Value of the derivative portfolio of the bank to the bank clients (corporate counterparties).

2 XVA Framework

2.1 Agents

Banks play a unique role in the industry as they accept deposits, make loans, and enter into risk transformation contracts with clients. Banks compete with each other to provide their services by offering the best prices to clients. A dealer bank is a price maker which cannot decide on asset selection: trades are proposed by clients and the market maker needs to stand ready to bid for a trade at a suitable price, no matter what the trade is and when it arrives.

When modeling a bank with defaultable debt, we need to consider its shareholders and creditors as separate entities. Specifically, we model a bank as a composite entity split into shareholders and bondholders. Shareholders make investment decisions up until the default of the bank, at which point they are wiped out. Bondholders instead represent the junior creditors of the bank, which have no decision power until the time of default, but are protected by laws such as pari-passu forbidding certain trades that would trigger wealth away from them to shareholders (or to more senior creditors) during the default resolution process of the bank.

Derivative clients (corporate counterparties of the bank) are also individual economic entities. Non-financial firms are characterized by a portfolio of real investments which is separate and in addition to their portfolio of financial contracts. In a reduced form model of the economy where we ignore the real investments portfolios of clients and model only their financial contracts, we lack the information required to decide whether a trade would be optimal to execute or not. Non-financial firms are just viewed as price takers that do not optimize and possibly accept to sustain a loss as a consequence of trading with banks.

The bank also needs an external "funder" to borrow unsecured, as required in last resort by its trading strategy (once all the internal sources of funding, such as received and rehypothecable variation margin, have been exhausted). This funder can be seen as a senior creditor of the bank, which in our setup will be endowed with an exogenous recovery rate.

Last, the bank needs an access to the financial markets, e.g. repo markets, other banks (possibly via CCPs), etc., for setting up a hedge of its portfolio or, more precisely, of its mark-to-market component (as we assume jump-to-default risk hardly hedgeable in practice). We call abstractly the "hedger" of the bank the corresponding agent.

Hence, we consider an economy, with agents, labeled by an index a, coming in five different types:

- Bank shareholders (*sh*), who will only agree upon the bank entering a new trade if appropriately compensated by the client through the entry price of the deal, accounting for the costs of funding and cost of capital in particular;

- Bank *bondholders* (*bh*), who have no saying on trades but are protected by pari-passu type laws;

- Bank clients (or counterparties, *co*), who are price takers and willing to accept a loss in a trade for the sake of receiving (e.g. hedging) benefits that become apparent only once one includes their real investment portfolio, which is not explicitly modeled;

- Bank *funders* (*fu*), who agree to lend cash to the bank unsecured at some risky spread, which can be proxied by the CDS spread of the bank;

- Bank hedgers (*he*), who agree to provide a mark-to-market hedge (fully collateralised back-to-back hedge) of the derivative portfolio of the bank.

As these entities enter into contracts, wealth is transferred among them, as defined and explained in what follows.

2.2 Cash Flows

We assume that, at time 0, agents are already bound by contractual agreements between each other, which obligate them to exchange the related trading cash flows in the future. The cumulative cash flows up to time t received by entity a from all other agents in the economy assuming no new trade is entered (other than the ones initially planned at time 0, even though the latter may include forward starting contracts or dynamic hedging positions) is denoted by a (i.e. we identify agents with the corresponding cash flow processes), premium payments included.

We then assume that at time $t = 0$ a new trade is concluded. We prefix by Δ any trade incremental quantity of interest, e.g. Δa denotes the difference between the cumulative cash flow streams affecting the corresponding agent with and without the new deal.

There are also cash flows affecting our different economic agents, unrelated to the derivative portfolio of the bank. By definition, such cash flows are unchanged upon inclusion of the new deal in the bank portfolio. We assume that none of our economic agents is a monetary authority. Hence, money can neither be created nor destroyed and all relevant entities are included in the model. Under these conditions:

Assumption 2.1 We have that

$$\sum_{agents} \Delta a = 0. \blacksquare \tag{1}$$

Our objective is to assess the incremental impact of the new trade on counterparty risk and on the cost of debt financing and the cost of capital, in such a way that this information can be reflected into entry prices at a level making the shareholders indifferent (at least) to the deal. The analysis can then be repeated at each new trade as frequently as one wishes.

2.3 Valuation Operator

Under a cost-of-capital XVA approach, shareholders decide whether to invest depending on two inputs: a value function and incremental cost of capital. In this part we define the former.

We consider throughout the paper a pricing stochastic basis $(\Omega, \mathbb{G}, \mathbb{Q})$, with model filtration $\mathbb{G} = (\mathcal{G}_t)_{t\in\mathrm{R}_+}$ and pricing measure $\mathbb{Q}$ (the $\mathbb{Q}$ expectation is denoted by $\mathbb{E}$), such that all the processes of interest are $\mathbb{G}$ adapted.

Remark 2.1 In case markets are assumed to be complete, then there is one unique pricing measure $\mathbb{Q}$. In case markets are incomplete, several calibratable risk neutral probabilities can coexist and we need to recognize the related model risk. For this purpose, one can deal with a Bayesian-like prior distribution $\mu(d\mathbb{Q})$ in the space of risk neutral measures. Subjective views of price makers are embedded in the choice of the prior measure μ. In this paper, to keep things simple, we assume that μ is an atomic delta measure, i.e. we simply pick one possible risk neutral measure $\mathbb{Q}$ as it emerges from a calibration exercise and stick to it without including model risk.

However, we will introduce cost of capital, as a KVA risk-premium entering prices on top of risk-neutral $\mathbb{Q}$ valuation of the cash flows (shareholder cash flows, i.e. pre-bank default cash flows, for alignment of deals to shareholder interest, which drives the trading decisions of the bank as long as it is nondefault). $\blacksquare$

We denote by r a $\mathbb{G}$ progressive OIS rate process, i.e. overnight indexed swap rate, the best market proxy for a risk-free rate as well as the reference rate for the remuneration of cash collateral). Let $\beta = e^{-\int_0^{\cdot} r_t dt}$ be the corresponding risk-neutral discount factor. The representation of valuation by the traders of the bank is encoded into the following:

Definition 2.1 The (time 0) value of a cumulative cash flow stream $\mathcal{P}$ is given by

$$\mathbb{E} \int_{[0,\infty)} \beta_t d\mathcal{P}_t \tag{2}$$

(integral from time 0 included onward, under the convention that all processes are nil before time 0).

In particular, we call mark-to-market MtM of the new deal the value of its contractually promised cash flow stream ρ (i.e. MtM is the value of the deal ignoring the impact of counterparty risk and of its funding and capital implications). For each agent, we denote by the corresponding capitalized acronym $A = \mathrm{SH}, \mathrm{BH}, \mathrm{CO}, \mathrm{FU}, \mathrm{HE}$, the value of a. ∎

Definition 2.2 The wealth transfer triggered by the deal to a given agent is the difference between the values of the corresponding cash flow streams a accounting or not for the new deal, i.e. the value $\Delta\mathrm{A}$ of Δa (by linearity of our valuation operator). ∎

Recalling that the corresponding cash flow streams are premium inclusive (and that the time integration domain includes 0 in (2)):

Assumption 2.2 The risky funding assets and the hedging assets are "fairly" valued, in the sense that $\mathrm{FU} = \mathrm{HE} = 0$. ∎

Hence we no longer report about FU and HE in the sequel.

Lemma 2.1 *We have*

$$\sum_{agents} \Delta\mathrm{A} = 0 \tag{3}$$

and, more specifically,

$$\Delta\mathrm{SH} + \Delta\mathrm{BH} + \Delta\mathrm{CO} = 0. \tag{4}$$

Proof. Definition 2.2 and (1) immediately imply (3), which, under Assumption 2.2, specializes to (4). ∎

Hence, our setup is in line with the Williams (1938) law of conservation of investment value according to which, as a consequence of a financial trade among a number of entities who enter into a contract at time 0 to exchange future cash flows towards each other, the algebraic sum of all wealth transfers at time 0 among all entities involved is zero. But the wealth transfer amount $\Delta\mathrm{A}$ is possibly non-zero, in general, for some entities $a(=sh, bh$, and/or $co)$.

We also introduce $ba = sh + bh$ and $\mathrm{BA} = \mathrm{SH} + \mathrm{BH}$ for the cash flows and value of the derivative portfolio to the bank as a whole. From a balance sheet interpretation point of view that is detailed in Albanese and Crépey (2018, Appendix A), BA and SH correspond to the accounting equity and to the core equity Tier 1 capital of the bank (at least, at the trade incremental level).

2.4 Contra-Assets and Contra-Liabilities

In the case of an investment bank, counterparty risk entails several sources of market incompleteness, or trading restrictions (cf. Sect. A):

- Pari-passu rules, meant to guarantee to the bank creditors the benefit of any residual value within the bank in case of default;
- Bank debt cannot possibly be fully redeemed by the bank shareholders;
- Client default losses cannot be perfectly replicated.

In order to focus on counterparty risk and XVAs, we assume throughout the paper that the market risk of the bank is perfectly hedged by means of perfectly collateralized back-to-back trades. That is, each client deal is replicated, in terms of market risk, by a perfectly collateralized back-to-back trade with another bank. Hence, all remains to be priced is counterparty risk and its capital and funding implications.

More precisely, netting the cash flows of the client portfolio and its hedge results in a set of counterparty risk related cash flows that can be subdivided into counterparty default exposures and funding expenditures, incurred by the bank as long as it is alive, and cash flows received by the bank from its default time onward, when shareholders have already been wiped out by the bondholders, which increase the realized recovery of the latter: See Sect. 4 in an illustrative static setup. Accordingly (see Albanese and Crépey (2018, Definition 3.5) for more technical detail):

Definition 2.3 We call contra-assets and contra-liabilities the synthetic liabilities and assets of the bank that emerge endogenously from its trading, through the impact of counterparty risk on its back-to-back hedged portfolio.

We denote by ca and cl the corresponding cash flow streams, with respective values CA and CL. As the counterparty risk related add-ons are not known yet, this is all "FTP excluded" (cf. Sect. 2.6), i.e. not accounting for the corresponding (to be determined) premium that will be paid by the client to the bank. ∎

Note that, by Definition 2.2, contra-liabilities (such as the DVA) do not benefit to the wealth of shareholders but only to bondholders, because the corresponding cash flows come too late, when shareholders have already been wiped out by the bondholders.

The counterparty related cash flows, i.e. $ca - cl$, are composed of credit and funding cash flows. Under bilateral counterparty risk, there is a credit valuation adjustment (CVA) for each risky counterparty and a debt valuation adjustment (DVA) for default of the bank itself. Each CVA (respectively DVA) is equal to the cost of buying protection against the credit risk of that counterparty (respectively the symmetrical cost seen from the perspective of the counterparty toward the bank). Accordingly, credit cash flows are valued as $(\Delta\mathrm{BDVA} - \Delta\mathrm{BCVA})$. Note that we are dealing with bilateral BCVA and BDVA here, where the CVA and DVA related cash flows between the bank and each given counterparty are only considered until the first occurrence of a default of the two entities (essentially, see Albanese and Crépey (2018, Remark 7.4) for the detail), consistent with the fact that later cash flows will not be paid.

Proposition 2.1 *We have*

$$\mathrm{CA} - \mathrm{CL} = \mathrm{BCVA} - \mathrm{BDVA}. \tag{5}$$

Proof. In view of the above, this is an immediate consequence of $\Delta\mathrm{FU} = 0$ (by Assumption 2.2). ∎

2.5 Cost of Capital

Following the principles of Basel III and Solvency II, we handle (unhedgeable) counterparty risk by means of a combination of a reserve capital account, which is used by the bank to cover systematic losses, and capital at risk to cover exceptional losses.

Shareholders require a dividend premium as compensation for the risk incurred on their capital at risk. The level of compensation required on shareholder capital at risk (SCR) is driven by market considerations. Typically, investors in banks expect a hurdle rate h of about 10% to 12%.

When a bank charges cost of capital to clients, these revenues are accounted for as profits. Unfortunately, prevailing accounting standards for derivative securities are based on the theoretical assumption of market completeness. They do not envision a mechanism to retain these earnings for the purpose of remunerating capital across the entire life of transactions, which can be as long as decades. In complete markets, there is no justification for risk capital. Hence, profits are immediately distributable. A strategy of earning retention beyond the end of the ongoing accounting year (or quarter) is still possible as in all firms, but this would be regarded as purely a business decision, not subject to financial regulation under the Basel III accord.

This leads to an explosive instability characteristic of a Ponzi scheme. For instance, if a bank starts off today by entering a 30-year swap with a client, the bank books a profit. Assuming the trade is perfectly hedged, the profit is distributable at once. But, the following year, the bank still needs capital to absorb the risk of the 29-year swap in the portfolio. If the profits from the trade have already been distributed the previous year, the tentation for the bank to maintain shareholder remuneration levels is to lever up by selling and hedging another swap, booking a new profit and distributing the dividend to shareholders that are now posting capital for both swaps. As long as trading volumes grow exponentially, the scheme self-sustains. When exponential growth stops, the bank return on equity crashes. The great financial crisis of 2008–09 can be analyzed along these lines (cf. the 2008 financial derivative Ponzi scheme displayed in Figure 1): In the aftermath of the crisis, the first casualty was the return on equity for the fixed income business, as profits had already been distributed and market-level hurdle rates could not be sustained by portfolio growth.

Interestingly enough however, in the insurance domain, the Swiss Solvency Test and Solvency II (see Swiss Federal Office of Private Insurance (2006) and Committee of European Insurance and Occupational Pensions Supervisors (2010)), unlike Basel III, do regulate the distribution of retained earnings through a mechanism tied to so called "risk margins" (see Wüthrich and Merz (2013), Eisele and Artzner (2011), or Salzmann

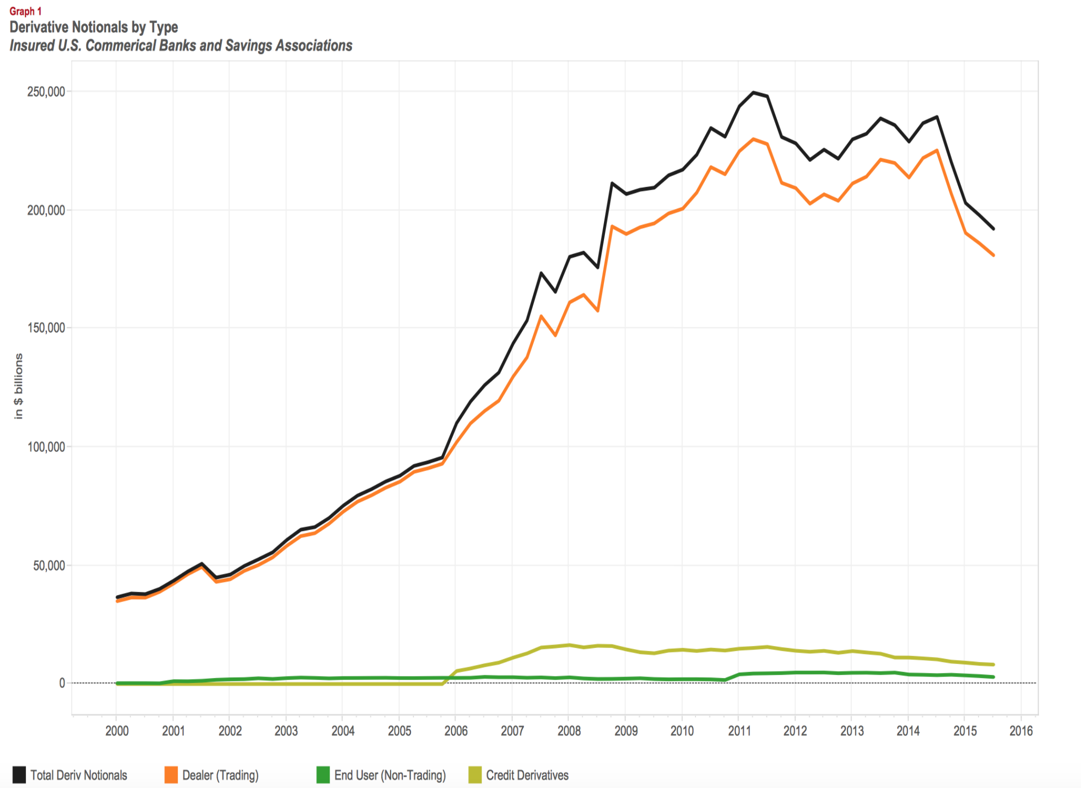

Figure 1: 2008 financial derivatives Ponzi scheme (Source: Office of the comptroller of the currency, Q3 2015 quarterly bank trading revenue report).

and Wüthrich (2010) regarding the risk margin and cost of capital actuarial literature). The accounting standards set out in IFRS 4 Phase II (see International Financial Reporting Standards (2012, 2013)) are also consistent with Solvency II and include a treatment for risk margins that has no analogue in the banking domain.

Under the KVA approach of Albanese and Crépey (2018, Sections 4.2 and 5.3), which provides a continuous-time and banking analog of the above-mentioned insurance framework, earnings are retained into a risk margin (RM) account and distributed gradually and sustainably, at some "hurdle rate" h, to the shareholders. This yields a framework for assessing cost of capital for a bank, passing it on to the bank clients, and distributing it gradually to the bank shareholders through a dividend policy that would be sustainable even in the limit case of a portfolio held on a run-off basis, with no new trades ever entered in the future.

2.6 Funds Transfer Price

A strategy based on capital for unhedged risk and risk margin for dividend distribution allows one to setup an optimal portfolio management framework for a market maker.

Shareholders have decision power in a firm and make investment decisions with the purpose of optimising their own wealth. Accordingly, our broad XVA principle reads as follows:

Assumption 2.3 The bank will commit to an investment decision at time 0 and execute the trade in case ΔSH exceeds the incremental cost of capital or ΔKVA [2] for the trade. ∎

By Definition 2.1, the mark-to-market MtM of the new deal corresponds to its value ignoring counterparty risk and its funding and capital consequences. In line with it:

Assumption 2.4 MtM is the amount required by the bank for setting up the fully collateralized back-to-back market hedge to the deal. ∎

Hence, at inception of a new trade at time 0, the client is asked by the bank to pay the mark-to-market (MtM) of the deal (as the cost of the back-to-back hedge for the bank), plus an add-on, called funds transfer price (FTP), reflecting the incremental counterparty risk of the trade and its capital and funding implications for the bank. Accordingly, Assumption 2.3 is refined as follows.

Definition 2.4 The FTP is computed by the bank in order to achieve

$$\Delta \mathrm{SH} = \Delta \mathrm{KVA}. \blacksquare \tag{6}$$

We emphasize that, since trading cash flows include premium payments (see the beginning of Sect. 2.2), ΔSH contains the FTP (cf. Sect. 2.7 below). Hence, the rationale for Assumption 2.4 is that, should a tentative price be less than MtM plus the FTP that is implicit in (6), then, in line with Assumption 2.3, the bank should refuse the deal at this price, as it would be detrimental to shareholders. We set an equality rather than $\geq$ in (6) in view of the competition between banks, which pushes them to accept from the client "the lowest price admissible to their shareholders": see Albanese and Crépey (2018, Section B.4).

The FTP of a deal can be negative, meaning that the bank should effectively be ready to give some money back to the client (it may do it or not in practice) in order to account for a counteparty(/funding/capital) risk reducing feature of the deal.

2.7 A General Result

As we will see in all the concrete cases reviewed in Sect. 3, a dealer bank only incurs actual (nonnegative) costs before its defaults and actual (nonnegative) benefits from its default time onward, i.e. $bh = cl$, hence BH = CL. A common denominator to all our later formulas (whatever the detailed nature of the wealth transfers involved) is the following (cf. Albanese and Crépey (2018, Proposition 6.1)):

[2] cf. Sect. 3.6.

Theorem 2.1 *Assuming that* $\Delta\mathrm{BH} = \Delta\mathrm{CL}$, *we have*

$$\Delta\mathrm{SH} = \mathrm{FTP} - \Delta\mathrm{CA}. \tag{7}$$

Hence, the investment criterion (6) *is equivalent to*

$$\mathrm{FTP} = \Delta\mathrm{CA} + \Delta\mathrm{KVA} \tag{8}$$

and

$$\begin{aligned} \Delta\mathrm{SH} &= \Delta\mathrm{KVA}, \\ \Delta\mathrm{BH} &= \Delta\mathrm{CL}, \\ \Delta\mathrm{CO} &= -(\Delta\mathrm{CL} + \Delta\mathrm{KVA}). \end{aligned} \tag{9}$$

Proof. Disregarding the FTP, the trade incremental cash flows that affect the bank are $\Delta(ca - cl)$ and he (which includes MtM as the cost of setting up the fully collateralized back-to-back market hedge of ρ, cf. Assumption 2.4). Hence, by Definition 2.3,

$$\Delta\mathrm{BA} = \mathrm{FTP} - (\Delta\mathrm{CA} - \Delta\mathrm{CL}). \tag{10}$$

As a consequence, the assumption that $\Delta\mathrm{BH} = \Delta\mathrm{CL}$ implies

$$\Delta\mathrm{SH} = \mathrm{FTP} - \Delta(\mathrm{CA} - \mathrm{CL}) - \Delta\mathrm{BH} = \mathrm{FTP} - \Delta\mathrm{CA},$$

which is (7). Therefore, the investment criterion (6) is rewritten as (8). The last identity in (9) readily follows from the two first ones and from (4). ∎

As a concluding remark to this section, let us now assume, for the sake of the argument (cf. Albanese and Crépey (2018, Section 5.3)), that the bank, all other things being equal, would be able to hedge (or monetize) its own jump-to-default exposure through a further deal corresponding to the delivery by the bank of a cash flow stream cl in exchange of a premium fee “fairly” priced as its valuation CL. Then, switching from the FTP given by (8) to a modified

$$\mathrm{FTP} = \Delta\mathrm{CA} - \Delta\mathrm{CL} + \Delta\mathrm{KVA} = \Delta\mathrm{BCVA} - \Delta\mathrm{BDVA} + \Delta\mathrm{KVA}, \tag{11}$$

but accounting for the extra income to shareholders provided by the premium fees of the hedge in each portfolio including and excluding the new deal, all shareholder cash flows are now exactly the same as before (in each portfolio including and excluding the new deal), but the bondholders are entirely wiped out by the hedge. Hence we have, instead of (9),

$$\begin{aligned} \Delta\mathrm{SH} &= \Delta\mathrm{KVA}, \\ \Delta\mathrm{BH} &= 0, \\ \Delta\mathrm{CO} &= -\Delta\mathrm{KVA}. \end{aligned} \tag{12}$$

This shows that, under the trading strategy including the hedge, our investment criterion (6) becomes equivalent to the modified pricing rule (11), which supersedes (8), and the corresponding wealth transfers are given by (12).

If, in addition, the trading loss of the bank shareholders was also hedged out, then there would be no more risk for the bank and therefore no capital at risk required, hence the KVA would vanish and we would be left with the "complete market formulas"

$$\mathrm{FTP} = \Delta\mathrm{BCVA} - \Delta\mathrm{BDVA} \text{ and } \Delta\mathrm{SH} = \Delta\mathrm{BH} = \Delta\mathrm{CO} = 0$$

(cf. Sect. 3.1). However, in practice, a bank cannot hedge its own default nor replicate its counterparty default exposure, hence we remain with the outputs of Theorem 2.1. The exact specification of the different terms in (8)–(9) depends on the trading setup, as our next sections illustrate.

3 Wealth Transfers Triggered by Market Incompleteness

In this section we provide more detailed (but still conceptual) formulas for the FTP and the different wealth transfers involved, under more and more realistic assumptions regarding the trading restrictions and market incompleteness faced by a dealer bank. See the papers commented upon in Sect. 1 (or see Sect. 4 in a static static setup) for definite XVA formulas or equations in various concrete trading setups.

3.1 The Limiting Case of Complete Markets

In the special case of complete markets (without trading restrictions, in particular), wealth transfers between bank shareholders and bank creditors can occur but are irrelevant to investment decisions and they have no impact on prices, i.e. there is no point to distinguish shareholders from bondholders. The (complete market Modigliani-Miller form of) justification of this statement (cf. Sect. 2.7 and A) is that, in case

$$\Delta\mathrm{BA} = \Delta\mathrm{SH} + \Delta\mathrm{BH} \geq 0$$

but "it seems that" $\Delta\mathrm{SH} < 0$, shareholders would still be able to increase their wealth (canceling out the wealth transfer from them to bondholders triggered by the deal) by buying the firm debt prior to executing the trade, making $\Delta\mathrm{SH}$ nonnegative (and $\Delta\mathrm{BH}$ diminished by the same amount, in line with the conservation law (3)). Moreover, in complete markets, there is no justification for capital at risk, so that there is no cost of capital either. Hence, the criterion (6) for an investment decision is rewritten as

$$\Delta\mathrm{BA} = 0. \tag{13}$$

In complete markets without trading restrictions, funding comes for free because unsecured derivatives can be REPOed, i.e. posted as a guarantee against the corresponding funding debt, which therefore is free of credit risk. That is, $fu = 0$.

Hence, consistent with (10) and (5) (but with even $fu = 0$ here, instead of only $\mathrm{FU} = 0$ in general), we obtain

$$\Delta\mathrm{BA} = \Delta\mathrm{BDVA} - \Delta\mathrm{BCVA} + \mathrm{FTP}. \tag{14}$$

An application of the investment criterion (13) to (14) yields

$$\mathrm{FTP} = \Delta\mathrm{BCVA} - \Delta\mathrm{BDVA} \tag{15}$$

(and we have $\Delta\mathrm{SH} = \Delta\mathrm{BH} = \Delta\mathrm{CO} = 0$), which flows into the reserve capital account maintained by the bank for dealing with counterparty risk in expectation (and no capital at risk, hence no risk margin, are required).

As discussed in Sect. 2.7 and Sect. A, the (complete market form of Modigliani-Miller) argument whereby shareholders buy hedge the default of the bank is crucial in relation to the pricing rule (15). If shareholders cannot do so, then this pricing rule triggers wealth transfers from shareholders to bondholders by the amount

$$\Delta\mathrm{SH} = -\Delta\mathrm{BDVA}, \qquad \Delta\mathrm{BH} = \Delta\mathrm{BDVA}. \tag{16}$$

The reason for this wealth transfer is that gains conditional to the default of the bank represent cash flows which are received by the bank bondholders at time of default. They do not benefit the shareholders, which at that point in time are wiped out, unless precisely these gains can be monetized before the default through hedging by the bank (see the computations in Sect. 2.7,).

Accounting standards for derivative securities such as IAS 39 followed by IFRS 9 have been designed around a concept of "fair valuation", which implicitly depends on the complete market hypothesis (cf. Sect. 2.5). Under this understanding the valuation of a bilateral contract is "fair" if both parties agree on the valuation independently: This is the case if there exists a replication strategy that precisely reproduces the cash flow stream of a given derivative contract.

In complete markets, there is no distinction between price maker and price taker. There is also no distinction between entry price and exit price of a derivative contract. The fair valuation of a derivative asset is the price at which the asset can be sold. All buyers would value the derivative at the exact same level, as any deviation from the cost of replication would lead to an arbitrage opportunity. In particular, fair valuations are independent of endowments and any other entity specific information.

The most glaring omission in a complete market model for a bank is a justification for equity capital as a loss absorbing buffer. Capital may still be justified as required to finance business operations. A bank is justified to charge fees for services rendered, but these fees should not depend on the risk profile of the trades. They should only be proportional to the operational workload (in the sense of a volume based fee) of the bank. Once the fees are received, a portion is allocated to cover operational costs and the remainder is released into the dividend stream.

3.2 DVA Wealth Transfer Triggered by Shareholders Not Being Able to Redeem Bank Debt

As explained at different levels in Sect. 2.7, 3.1 and A, if shareholders were able to freely trade bank debt, the interest of shareholders would be aligned with the interests of the firm as a whole (shareholders and bondholders altogether). However, in reality, trading restrictions prevent shareholders from effectively offsetting wealth transfers to bondholders by buying bank debt. Hence these wealth transfers can only compensated by clients in the form of suitable valuation adjustments at deal inception. The debt valuation adjustment (DVA) illustrates well this phenomenon. As a bank enters a new trade, they pass on to clients the credit valuation adjustment (CVA) as a compensation against the counterparty default risk. The DVA is the CVA the counterparty assigns to the bank. If valuations were fair, a bank should symmetrically recognise also a DVA benefit to clients. However, since the DVA can be monetised by the bank only by defaulting, managers are reluctant to recognise a DVA benefit to clients. If they did, they would effectively trigger a wealth transfer as in (16) from shareholders to bondholders that they would not be able to hedge. In order to ensure $\Delta\mathrm{SH} = 0$ (we are not considering KVA yet) in spite of this, a bank should charge, instead of (15),

$$\mathrm{FTP} = \Delta\mathrm{BCVA}, \tag{17}$$

so that the wealth transfer to bondholders becomes borne by the client of the deal instead of the bank shareholders. This yields, instead of (16),

$$\begin{aligned} &\Delta\mathrm{SH} = 0, \\ &\Delta\mathrm{BH} = \Delta\mathrm{BDVA}, \\ &\Delta\mathrm{CO} = -\Delta\mathrm{BH}. \end{aligned} \tag{18}$$

Conceivable DVA hedges would involve the bank selling credit protection on itself, an impossible trade, or violations of the pari passu rule on debt seniority. Following up to these considerations, regulators have started to de-recognize the DVA as a contributor to core equity tier I capital, the metric that roughly represents the value of the bank to the shareholders (see Basel Committee on Banking Supervision (2012)).

3.3 $\mathrm{CVA}^{\mathrm{CL}}$ Wealth Transfer Triggered by Shareholders Bankruptcy Costs

The Basel Committee on Banking Supervision (2012) went even further and decided that banks should compute a unilateral CVA, which we denote by UCVA. This can be interpreted as saying that shareholders face a bankruptcy cost, equal to UCVA at default time, which goes to benefit bank bondholders. This bankruptcy cost corresponds to the transfer to bondholders of the residual amount on the reserve capital account of the bank in case the latter defaults. Accounting for this feature, upon entering a new trade, if entry prices are struck at the indifference level for shareholders (still ignoring capital and its KVA implication, here and until Sect. 3.6), we obtain $\mathrm{FTP} = \Delta\mathrm{UCVA}$ and

$$\begin{aligned} &\Delta\mathrm{SH} = 0, \\ &\Delta\mathrm{BH} = \Delta\mathrm{BDVA} + \Delta\mathrm{CVA}^{\mathrm{CL}}, \\ &\Delta\mathrm{CO} = -\Delta\mathrm{BH}. \end{aligned} \tag{19}$$

Here, by definition, $\mathrm{CVA}^{\mathrm{CL}}$ is the difference between the unilateral UCVA and BCVA, i.e. the valuation of the counterparty default losses occurring beyond the default of the bank itself. The acronym CL in $\mathrm{CVA}^{\mathrm{CL}}$ stands for contra-liability (see Definition 2.3), because $\mathrm{CVA}^{\mathrm{CL}}$ indeed corresponds to a "contra-liability component" of the UCVA.

3.4 FVA Wealth Transfer Triggered by the Impossibility to REPO Derivatives

A related form of wealth transfer occurs in the case of costs of funding for variation margin. The acquisition of assets funded with unsecured debt triggers a wealth transfer from shareholders to bondholders. This happens because shareholders sustain a cost of carry for unsecured debt while bondholders benefit out of having a claim on the asset in case the bank defaults.

Remark 3.1 In this regard, REPO contracts are a more efficient method for funding asset acquisitions since shareholders sustain far lower funding rates (close to risk-free rates); on the flip side however, bondholders do not have a claim on an asset passed as collateral in a REPO transaction. ∎

Applying Williams (1938)'s wealth conservation principle to unsecured debt valuation (cf. Sect. 2.3 and Albanese and Crépey (2018, Section 4.1)), the cost of carry of debt to shareholders equals the gain to bondholders induced by the non reimbursement by the bank of the totality of its funding debt to the funder if it defaults. In Albanese and Andersen (2014, 2015) and Albanese and Crépey (2018), the wealth transferred from shareholders to bondholders by the cost of unsecured debt is called funding valuation adjustment (FVA), while the wealth received by bondholders through the accordingly increased recovery rate is denoted with FDA. Wealth conservation implies that FDA = FVA (akin to FU = 0 in Assumption 2.2). So, in accounts, the fair valuation of the derivative portfolio of the bank (i.e. of the counterparty risk related cash flows, under our back-to-back hedge assumption of market risk) should also contain a term FDA equal to FVA in absolute value, but contributing to the bank fair valuation with an opposite sign, i.e. appearing as an asset, of the contra-liability kind, in the bank balance sheet (see Albanese and Crépey (2018, Section A and Figure 2) for the detailed balance sheet perspective on the XVA metrics). This way, the accounting equity of the bank, i.e. the wealth of the bank as a whole, does not depend on the funding spreads or funding policies of the bank, a result in the line of Williams (1938)'s law, which holds independently of whether markets are complete or not (cf. our general identity (5)).

However, the resulting notion of fair valuation of the portfolio (or wealth of the bank as a whole) is only relevant in a complete market. If markets are incomplete in the sense that shareholders are forbidden to buy bank bonds, then one needs to refocus on shareholder interest. As the FVA subtracts from shareholder value (because the corresponding cash flows are pre-bank default), investment decisions do depend on funding strategies. That is, in order for an investment to be acceptable, the bank needs to ensure that the change in FVA is passed on to the client. The same amount is then also transferred as a net benefit to bank bondholders.

As a result, FTP and wealth transfers for indifference entry prices in the case of unsecured funding of variation margin are given by FTP $= \Delta$UCVA $+ \Delta$FVA and

$$\begin{aligned} &\Delta \mathrm{SH} = 0, \\ &\Delta \mathrm{BH} = \Delta \mathrm{BDVA} + \Delta \mathrm{CVA}^{\mathrm{CL}} + \Delta \mathrm{FVA}, \\ &\Delta \mathrm{CO} = -\Delta \mathrm{BH}. \end{aligned} \tag{20}$$

3.5 MVA Wealth Transfer Triggered by Different Funding Policies for Initial Margins

Initial margin (IM) offers a fourth example of wealth transfer. When the IM posted by the bank is funded using debt unsecured to the funder, an additional wealth transfer ΔMVA from shareholders to bondholders is triggered (unless the bank could hedge its own default, see Sect. 2.7 and A). Hence, at indifference, we have

$$\mathrm{FTP} = \Delta \mathrm{UCVA} + \Delta \mathrm{FVA} + \Delta \mathrm{MVA} \tag{21}$$

and

$$\begin{aligned} &\Delta \mathrm{SH} = 0, \\ &\Delta \mathrm{BH} = \Delta \mathrm{BDVA} + \Delta \mathrm{CVA}^{\mathrm{CL}} + \Delta \mathrm{FVA} + \Delta \mathrm{MVA}, \\ &\Delta \mathrm{CO} = -\Delta \mathrm{BH}. \end{aligned} \tag{22}$$

It is debatable whether or not unsecured collateral funding strategies are unavoidable. As a rule, wealth transfers, entry prices for clients, and investment decisions by banks depend on the collateral strategies which are enacted. For instance, in case initial margin posting would be delegated to a non-banking specialist lender without funding costs and recovering the portion of IM unused to cover losses if the bank defaults, then, since the IM is sized as a large quantile of the return distribution over the margin period, the corresponding MVA is bound to be much smaller than the one resulting from unsecured borrowing at the bank CDS spread. See Albanese et al. (2017, Section 4.3) and Armenti and Crépey (2018, Section 4.2) for details.

Remark 3.2 Strategies to achieve secured funding for variation margin are discussed in Albanese, Brigo, and Oertel (2013) and Albanese et al. (2015). However, such funding schemes are much more difficult to implement for VM because VM is far larger and more volatile than IM. ∎

3.6 KVA Wealth Transfer Triggered by the Cost of Capital Which is Required by the Impossibility of Hedging out Counterparty Default Losses

The formulas (21)–(22) do not account for the cost of the capital earmarked to absorb exceptional losses (beyond the expected losses already accounted for by reserve capital). In our framework we include this cost as a capital valuation adjustment (KVA), which is dealt with separately as a risk premium, flowing into a risk margin account distinct from the reserve capital account. Specifically, we define our KVA as the cost of remunerating shareholders at some constant hurdle rate $h > 0$ for their capital at risk. The hurdle rate is the instantaneous remuneration rate of one unit of shareholder capital at risk, which can be interpreted as a risk aversion parameter of the shareholders (cf. Albanese and Crépey (2018, Section B.4)). This corresponds to the Solvency II notion of risk margin.

Accounting further for cost of capital, indifference in the sense of (6) for shareholders corresponds to

$$\mathrm{FTP} = \Delta\mathrm{UCVA} + \Delta\mathrm{FVA} + \Delta\mathrm{MVA} + \Delta\mathrm{KVA} \tag{23}$$

and

$$\Delta\mathrm{SH} = \Delta\mathrm{KVA}, \tag{24}$$

$$\Delta\mathrm{BH} = \Delta\mathrm{BDVA} + \Delta\mathrm{CVA}^{\mathrm{CL}} + \Delta\mathrm{FVA} + \Delta\mathrm{MVA}, \tag{25}$$

$$\Delta\mathrm{CO} = -\Delta\mathrm{BH} - \Delta\mathrm{KVA}. \tag{26}$$

If the deal occurs, the sum of the first three incremental amounts in (23), i.e. $\Delta\mathrm{UCVA} + \Delta\mathrm{FVA} + \Delta\mathrm{MVA}$, accrues to reserve capital, while the last term $\Delta\mathrm{KVA}$ accrues to the risk margin account.

We emphasize that the FTP formula (23) makes the price of the deal both entity-dependent (via, for instance, the CDS funding spread of the bank, which is a major input to the FVA) and portfolio-dependent (via the trade incremental feature of the FTP), far away from the law of one price, the complete market notion of fair valuation, and the (complete market form of) the Modigliani-Miller invariance principle.

4 XVA Formulas and Wealth Transfers in a Static Setup

In this section, which is a rewiring around the notion of wealth transfer of Section B in Albanese and Crépey (2018), we illustrate the XVA wealth transfer issues in an elementary static one-year setup, with r set equal to 0.

Assume that at time 0 a bank, with no prior endowment and equity E corresponding to its initial wealth, enters a derivative position with a client. We drop the $\Delta\cdot$ notation in this section, where every quantity of interest prior to the deal is simply 0, so that all our price and XVA notation refers to the new deal at time 0.

The bank and its client are default prone with zero recovery and no collateralization (no variation or initial margins). We denote by J and J_1 the survival indicators of the bank and its client at time 1 (both being assumed alive at time 0), with default probability of the bank $\mathbb{Q}(J = 0) = \gamma$ and no joint default for simplicity, i.e $\mathbb{Q}(J = J_1 = 0) = 0$. The bank wants to charge to its client an add-on (or obtain from its client a rebate, depending on whether the bank is seller or buyer), denoted by CA, accounting for its expected counterparty default losses and funding expenditures, as well as a KVA risk premium.

The all-inclusive XVA add-on to the entry price for the deal, which we call funds transfer price (FTP), follows as

$$\mathrm{FTP} = \underbrace{\mathrm{CA}}_{\text{Expected costs}} + \underbrace{\mathrm{KVA}}_{\text{Risk premium}} . \tag{27}$$

4.1 Cash Flows

The counterparty risk related cash flows affecting the bank before its default are its counterparty default losses

$$\mathcal{C}^{\circ} = (1 - J_1)\rho^{+}$$

and its funding expenditures $\mathcal{F}^{\circ}$. Accounting for the to-be-determined add-on CA (the KVA paid by the client at time 0 is immediately transferred by the bank management to the shareholders), the bank needs to borrow $(\mathrm{MtM} - \mathrm{CA})^{+}$ unsecured or invest $(\mathrm{MtM} - \mathrm{CA})^{-}$ risk-free, depending on the sign of $(\mathrm{MtM} - \mathrm{CA})$, in order to pay $(\mathrm{MtM} - \mathrm{CA})$ to the client. In accordance with Assumption 2.2, unsecured borrowing is "fairly" priced as its valuation (cf. Definition 2.1) $\gamma \times$ the amount borrowed by the bank, so that the funding expenditures of the bank amount to

$$\mathcal{F}^{\circ} = \gamma(\mathrm{MtM} - \mathrm{CA})^{+} \tag{28}$$

(deterministically in this one-period setup). We assume further that a fully collateralized back-to-back market hedge is set up by the bank in the form of a deal with a third party, with no entrance cost and a payoff to the bank of $-(\rho - \mathrm{MtM}) = -he$ at time 1, irrespective of the default status of the bank and the third party at time 1.

Collecting all cash flows, the result of the bank over the year is (cf. Albanese and Crépey (2018, Section B.2) for a detailed calculation)

$$ba = -(1 - J_1)\rho^{+} - \gamma(\mathrm{MtM} - \mathrm{CA})^{+} + (1 - J)\big(\rho^{-} + (\mathrm{MtM} - \mathrm{CA})^{+}\big) + \mathrm{FTP}. \tag{29}$$

Introducing further

$$\mathcal{C}^{\bullet} = (1 - J)\rho^{-},\ \mathcal{F}^{\bullet} = (1 - J)(\mathrm{MtM} - \mathrm{CA})^{+}, \tag{30}$$

we thus have

$$\begin{aligned} ba &= \underbrace{\mathrm{FTP} - \overbrace{(\mathcal{C}^{\circ} + \mathcal{F}^{\circ})}^{ca}}_{sh} + \underbrace{\mathcal{C}^{\bullet} + \mathcal{F}^{\bullet}}_{bh=cl} \\ &= -\underbrace{\big(\mathcal{C}^{\circ} - \mathcal{C}^{\bullet} - \mathrm{FTP}\big)}_{co} - \underbrace{\mathcal{F}^{\circ} - \mathcal{F}^{\bullet}}_{fu}, \end{aligned} \tag{31}$$

where the identification of the different terms follows from their financial interpretation.

4.2 Static XVA Formulas

In a static one-period setup, there are no unilateral versus first-to-default issues and no bank accounts involved (as no rebalancing of the trading strategies at intermediate time points is necessary), hence no $\mathrm{CVA}^{\mathrm{CL}}$ issue either (cf. Sect. 3.3). Moreover we assume no collateralization, hence there is nothing related with initial margin. As a consequence, contraliabilities in this context reduce to the DVA and the FDA. Accordingly, the FTP and wealth transfer formulas (8) and (9) reduce to

$$\begin{aligned} &\mathrm{FTP} = \mathrm{CA} + \mathrm{KVA} = \mathrm{CVA} - \mathrm{DVA} + \mathrm{CL} + \mathrm{KVA}, \\ &\mathrm{SH} = \mathrm{KVA}, \mathrm{BH} = \mathrm{CL}, \mathrm{CO} = -\mathrm{CL} - \mathrm{KVA}, \end{aligned} \tag{32}$$

where

$$\mathrm{CA} = \underbrace{\mathbb{E}\mathcal{C}^{\circ}}_{\mathrm{CVA}} + \underbrace{\mathbb{E}\mathcal{F}^{\circ}}_{\mathrm{FVA}}. \tag{33}$$

Hence, using also (28),

$$\mathrm{CVA} = \mathbb{E}\big[(1 - J_1)\rho^{+}\big], \ \mathrm{FVA} = \frac{\gamma}{1+\gamma}(\mathrm{MtM} - \mathrm{CVA})^{+}, \tag{34}$$

and

$$\mathrm{CL} = \underbrace{\mathbb{E}[(1 - J)\rho^{-}]}_{\mathrm{DVA}} + \underbrace{\mathbb{E}[(1 - J)(\mathrm{MtM} - \mathrm{CA})^{+}]}_{\mathrm{FDA}=\gamma(\mathrm{MtM}-\mathrm{CA})^{+}=\mathrm{FVA}}. \tag{35}$$

As for the KVA, it is meant to remunerate the shareholders at some hurdle rate h (e.g. 10%) for the risk on their capital, i.e.

$$\mathrm{KVA} = h\mathrm{E}. \tag{36}$$

Moreover, as the bank shareholders are in effect CVA and FVA traders in this setup (where market risk is hedged out and there is no MVA), we may add that it would be natural to size E by some risk measure (such as value at risk or, better, expected shortfall, at some "sufficiently high" confidence level, e.g. 97.5%) of the trading loss(-and-profit) of the shareholders given as (cf. (31))

$$L = \mathcal{C}^{\circ} + \mathcal{F}^{\circ} - \mathrm{CA} = -sh + \mathrm{KVA} \tag{37}$$

(i.e. the pure trading loss not accounting for the KVA risk premium). Hence, in the static setup, our hurdle rate h is nothing but the return on equity (ROE).

Note moreover that

$$\mathrm{CA} - \mathrm{CL} = \mathrm{CVA} + \mathrm{FVA} - (\mathrm{DVA} + \mathrm{FDA}) = \mathrm{CVA} - \mathrm{DVA}, \tag{38}$$

as FVA = FDA.

5 Derivative Management: From Hedging to XVA Compression

The great financial crisis of 2008–09 emphasized the incompleteness of counterparty risk. XVAs represent the ensuing switch of paradigm in derivative management, from hedging to balance sheet optimization. In this section we illustrate this evolution by a discussion of two potential applications of the XVA metrics in the optimization mode (beyond the basic use of computing them and charging them into prices for some target hurdle rate $h > 0$ that would be set by the management of the bank).

Of course, for any potential application of the XVA metrics to be practical, one needs efficient XVA calculators in the first place: XVAs involve heavy computations at the portfolio level, which yet need sufficient accuracy so that trade incremental numbers are not in the numerical noise of the machinery. In practice, banks mostly rely on exposure-based XVA computational approaches, based on time 0 XVAs reformulated as integrals of market expected exposures against suitable kernels, such as CDS curves (see e.g. (42)–(43) in the case of the CVA). This is somehow enhanced by the regulation, which requires banks to compute their mark-to-future cubes[3] already for the determination of their credit limits (through potential future exposures, i.e. maximum expected credit exposures over specified periods of time calculated at some quantile levels). Exposure-based approaches are also convenient for computing the XVA sensitivities that are required for XVA hedging purposes[4] (see Green and Kenyon (2014), Huge and Savine (2017), or Antonov, Issakov, McClelland, and Mechkov (2018)). The most advanced (but also quite demanding) implementations are nested Monte Carlo strategies optimized with GPUs (see Albanese, Bellaj, Gimonet, and Pietronero (2011) and Abbas-Turki et al. (2018)).

5.1 Capital/Collateral Optimisation of Inter-Dealer Trades

In this section, we consider a bilateral derivative market with banks and clients. Each bank is modelled as having an exogenously specified CDS curve which also implies funding costs. Each bank also uses a certain hurdle rate for passing cost of capital to clients. We assume that clients hold a fixed portfolio of derivative trades with one or more of the banks and are indifferent to trades between dealers. We want to identify capital/collateral optimisation inter-dealer trades that would be mutually beneficial[5] to the shareholders of two dealer counterparties and find a way to achieve a "Pareto optimal equilibrium", in the sense that there exists no additional mutually beneficial inter-dealer trade.

Considering a tentative inter-dealer trade, with contractually promised cash flows ρ, between a tentative "seller bank" a (meant to "pay" ρ) and a tentative "buyer bank" b

[3]Prices of spanning instruments in future time points and scenarios, from which expected exposure profiles are easily deduced.

[4]Hedging of spread risks, as jump-to-default risk can hardly be hedged.

[5]In the sense of Assumption 2.3.

(meant to "receive" ρ, i.e. "pay" $-\rho$) , we denote the corresponding FTPs as (cf. (23)):

$$\begin{aligned}
\mathrm{FTP}_a(\rho) &= \Delta\mathrm{UCVA}_a(\rho) + \Delta\mathrm{FVA}_a(\rho) + \Delta\mathrm{MVA}_a(\rho) + \Delta\mathrm{KVA}_a(\rho) \\
\mathrm{FTP}_b(-\rho) &= \Delta\mathrm{UCVA}_b(-\rho) + \Delta\mathrm{FVA}_b(-\rho) + \Delta\mathrm{MVA}_b(-\rho) + \Delta\mathrm{KVA}_b(-\rho).
\end{aligned}$$

Note that all incremental XVA terms are entity specific as they depend on the endowment of each dealer, i.e. of their current portfolio. The FVA and the MVA also depend on the entity specific funding spread. Finally, the KVA depends additionally on the target hurdle rate set by management.

Consistent with Assumption 2.3, bank a would be happy to sell the contract at any price $\geq \mathrm{MtM} + \mathrm{FTP}_a(\rho)$, whereas bank a would be happy to buy it at any price $\leq \mathrm{MtM} - \mathrm{FTP}_b(-\rho)$. Hence The sales of the contract ρ from a to b may be a win-win provided

$$\mathrm{FTP}_a(\rho) + \mathrm{FTP}_b(-\rho) < 0. \tag{39}$$

However, allocated shareholder capital sets a constraint on trading. As a consequence of the new trade, shareholder capital at risk (cf. Sect. 2.5) of each bank $i = a, b$ changes by the amount $\Delta\mathrm{SCR}_i$. Hence the sales can only occur if

$$\mathrm{SCR}_i + \Delta\mathrm{SCR}_i \leq \mathrm{SHC}_i, \text{ for } i = a, b, \tag{40}$$

where SHC_i is the shareholder capital (at risk or uninvested) for bank i.

In conclusion:

Proposition 5.1 *The transaction whereby bank a agrees on receiving the additional future cash-flows ρ from bank b (contractually promised cash-flows, ignoring counterparty risk and its capital and funding consequences) can be a win-win for both parties if and only if* (39) *holds, subject to the constraint* (40). *In this case, if the transaction occurs at the intermediate price*

$$\begin{aligned}
\mathrm{MtM} &+ \mathrm{FTP}_a(\rho) - \frac{1}{2}\big(\mathrm{FTP}_a(\rho) + \mathrm{FTP}_b(-\rho)\big) \\
&= \mathrm{MtM} - \mathrm{FTP}_b(-\rho) + \frac{1}{2}\big(\mathrm{FTP}_a(\rho) + \mathrm{FTP}_b(-\rho)\big)
\end{aligned}$$

paid by bank a to bank b, then the shareholders of both banks a and b mutually benefit of a positive net wealth transfer equal to

$$\Delta w = -\frac{1}{2}\big(\mathrm{FTP}_a(\rho) + \mathrm{FTP}_b(-\rho)\big) > 0. \blacksquare \tag{41}$$

5.2 Optimal Liquidation of the CCP Portfolio of a Defaulted Clearing Member

Another potential application of the XVA metrics is for dealing with default or distress resolutions. Specifically we consider the problem of the liquidation of the CCP derivative portfolio of a defaulted clearing member, dubbed "defaulted portfolio" for brevity henceforth. Here CCP stands for a central counterparty (also called clearing house, see Armenti and Crépey (2017, 2018) and European Parliament (2012) for references). A CCP nets the contracts of each of its clearing members (typically broker arms of major banks) with all the other members and collects variation and initial margins in the same spirit as for bilateral trades, but at the netted portfolio level for each member. In addition, a CCP deals with extreme and systemic risk through an additional layer of protection, called default fund, contributed by and pooled among the clearing members. We denote by μ_i the (nonnegative) default fund refill allocation weights, which determine how much each surviving clearing member must contribute to the refill of the default fund in case the latter has been eroded by the default of a given member, e.g. μ_i proportional to the current default fund contributions (DFC) of the surviving members. We denote by $L^{\star}$ the liquidation gap (or negative of the residu, if negative) of the CCP accounting for everything else than the liquidation properly said, i.e. for the margins and default fund contribution of the defaulted member, for the volatility and liquidity slippage of the defaulted portfolio during the cure period separating the default from the liquidation, and for the loss (or gain, counted negatively) of the CCP on its hedge (if any) of the defaulted portfolio over the cure period. We denote by MtM the (possibly volatility and liquidity impacted) mark-to-market[6] of the defaulted portfolio at the liquidation time where it is reallocated between the surviving members.

In a first stage, we assume that the defaulted portfolio is proposed as an indivisible package to the surviving members, which are just left with the freedom of proposing a price for global novation, i.e. replacing the defaulted member in all its future contractual obligations related to the defaulted portfolio.

One may then consider the following reallocation and pricing scheme. The CCP computes the FTPs of each surviving member corresponding to every other member receiving the portfolio of the defaulted member, i.e. their respective incremental XVA costs in each of these alternative scenarios.

The scenario giving rise to the lowest aggregated FTP, say member 1 receiver of the defaulted portfolio, is implemented. Accordingly, member 1 recovers the portfolio of the defaulted member and an MtM amount of cash from the CCP, whereas each surviving member (1 itself included) receives its corresponding FTP from the CCP, so that everybody is indifferent to the reallocation in MtM and XVA terms. The ensuing overall liquidation loss (or negative of the gain, if negative) is $L = L^{\star} + \mathrm{MtM} + \mathrm{FTP}_1$, where FTP_1 denotes the aggregated FTPs of the surviving members corresponding to this reallocation of the portfolio to member 1. In addition:

- If $L > 0$, meaning erosion by L of the default fund, then each surviving member (1 included) pays $L \times \mu_i$ to the CCP as refill to the default fund;
- If $L \leq 0$, then the CCP uses the residu $(-L)$ for, prioritarily, reimbursing any non-consumed DFC and IM (in this order) of the defaulted member to its liquidator and, if there remains a surplus after that, distributing it to the surviving clearing members proportional to their μ_i.

[6]Counted, sign-wise, as a debt to the other clearing members.

Let us now assume that the defaulted portfolio would instead be rewired with a surviving member corresponding to an aggregated FTP greater than FTP_1, say member 2, along with the required amounts of cash making each survivor indifferent to the reallocation in MtM and XVA terms. Then one must replace L by $L' = L^\star + \mathrm{MtM} + \mathrm{FTP}_2$ in the bullet points above, where FTP_2 is the aggregated FTP of all clearing members when member 2 receives the defaulted portfolio. As $\mathrm{FTP}_2 > \mathrm{FTP}_1$ by definition, therefore $L' > L$, implying that everybody would end up worse off than in the first alternative.

Last, we assume that, instead of being reallocated as an indivisible package, the defaulted portfolio is divided by the CCP into sub-packages reallocated one just after the other to possibly different clearing members. Iterating the above procedure, we denote by MtM^l the mark-to-markets and by FTP^l the recursively cheapest aggregated FTPs of the successive lots l (cheapest over consideration of the different scenarios regarding which survivor receives the lot l). The overall liquidation loss (or negative of the gain, if negative) becomes

$$L'' = L^\star + \sum_l (\mathrm{MtM}^l + \mathrm{FTP}^l) = L^\star + \mathrm{MtM} + \sum_l \mathrm{FTP}^l \leq L,$$

because of the multiple embedded optimizations (so that, in particular, $\sum_l \mathrm{FTP}^l \leq \mathrm{FTP}_1$). Hence, everybody would now end up better off than in the first alternative.

5.3 XVA Compression Cycles

In the context of Sect. 5.1, a solution to reach Pareto optimal equilibrium with no additional mutually beneficial inter-dealer trades would be to run iterative bidding cycles to discover trades that could be profitable to two banks. By running virtual bidding cycles, we can also answer other questions such as how to find a Pareto optimal reallocation of the portfolio of one given bank in case this portfolio is being liquidated (cf. Sect. 5.2). Another related application could be how to perform a partial liquidation in case a bank is in regulatory administration and the objective is to restore compliance with regulatory capital requirements.

Such systematic, market-wide bidding cycles would be very useful to optimise bilateral derivative markets by releasing costs for funding and capital into the dividend stream at participating broker dealers. A practical implementation problem, however, is that bilateral portfolios are held confidentially by each dealer and cannot be disclosed as an open bidding cycle would require: In order for a bidding cycle to be realistic and implementable in real life situations, it has to be designed in such a way that trade data at each bank is handled securely and not revealed to the other participants except when a mutually beneficial trade is identified and there is consensus on both sides to discover it.

Likewise, in the default resolution setup of Sect. 5.2, one should be careful that, in the course of the process, the CCP does not disclose to members any information, direct or indirect (i.e. via XVAs or incremental XVAs) relative to the portfolio of the other members.

A possible solution to this portfolio confidentiality problem could be as follows. Suppose that an XVA calculator is present within the firewall of each bank. This XVA calculator would be separate from the internal XVA calculator but hopefully will not deviate too much from it. Internal calculators, to avoid model risk, would be seeded with precisely the same calibrated models and guaranteed to produce identical results when loaded with identical portfolios under identical conditions and market data.

Suppose also that these calculators can share confidentially information with each other corresponding to each trade or set of trades which would be a candidate for novation. In this case, the existence of win-win trades would be discovered by the calculators without trade information itself being revealed.

Once the existence of a win-win trade such as (39)–(40) is detected by the XVA calculator, the two parties would then have to take the initiative to communicate with each other to exploit the opportunity if they so choose. Namely, the hypothetical seller would know which trade (or set of trades) would be worthwhile selling and to which peer. The hypothetical seller will be notified and it will be up to her to decide whether to start a conversation with the peer. In particular, before proceeding the seller has to verify that the calculator result on entry prices is acceptably accurate. If the seller then opens a communication channel with the peer, she will disclose the nature of the trades in question and ask the peer to verify a price on his own internal systems. If both internal systems agree that there is a trade opportunity, then the trade takes place.

In the future such largely automated XVA compression cycles could favorably replace the current XVA compression procedures that monopolize hundreds of quants twelve hours in a row in major tier 1 banks[7].

6 CVA Compression Case Study[8]

As stated by the Basel Committee on Banking Supervision (2015), major counterparty credit losses on OTC derivative portfolios in 2008 arose from XVA accounting losses (increase of contra-asset valuation) rather than from actual default on their counterparties. For instance, a bank incurs a CVA loss when the market perceives a deterioration of the credit risk of a client. This has motivated the creation of XVA desks for dealing with these risks.

One of the tasks of XVA desks consists in optimizing XVA metrics through clearing/offsetting trades. The optimization serves two purposes : reducing costs originated from immobilized regulatory capitals such as MVA; Proposing (given an initial portfolio) trade opportunities to an existing counterparty for benefiting from an FTP cut. The second point, specialized to the CVA between the bank and an existing client of the bank, motivates the present section.

[7]Source: David Bachelier, panel discussion Capital & margin optimisation, Quantminds International 2018 conference, Lisbon, 16 May 2018.

[8]The authors would like to thank Hugo Lebrun, Rémi Ligoureau, and Guillaume Macey (HSBC France), for their assistance in the conduct of this study.

Remark 6.1 We focus on CVA compression for simplicity in our case study, being understood that the algorithmic and numerical methodologies developed below are generic. This approach could and should be extended to further XVA metrics (see e.g. Kondratyev and Giorgidze (2017), who use genetic algorithms for determining an optimal trade-off between MVA compression and transaction costs), and in fact to the FTP as a whole, as soon as these are available with sufficient speed, for computation at the portfolio level, and accuracy, for numerical significance of the results at the incremental trade level. ∎

Compressing the CVA amounts to finding the trade that has the most opposite counterparty risk exposure to an initial portfolio, under incremental market risk constraints so that the new trade does not penalize the bank in terms of trading limits.

The complexity of the problem stems from the non-linearity in the optimization due, in particular, to the hybrid nature of the state space: An incremental trade is described by a combination of continuous and discrete parameters. This rules out convex optimization schemes and leads instead to the use of genetic algorithms. Hereafter we show how a genetic algorithm with penalization can efficiently find a CVA offsetting trade, while limiting the impact on the market exposure profile in order to respect the separation of mandates between the XVA desk, in charge of managing counterparty risk, and the other, dubbed "clean", trading desks of the bank, in charge of their respective mark-to-market.

6.1 Incremental CVA

The CVA (credit valuation adjustment) is the cost of the potential default from a counterparty at some random time τ. During the lifetime of its contracts $[0, T]$, the bank is exposed to a potential loss called expected positive exposure (EPE) :

$$\mathrm{EPE}(t) = \mathbb{E}\mathrm{MtM}_t^+,\ 0 \le t \le T. \tag{42}$$

In case of actual default, the counterparty refunds the corresponding exposure of the bank at certain rate R called recovery rate. The haircut $(1 - R)$, called loss given default, is lost for the bank. This loss is a cash flow from the bank to the counterparty at default time. The (time 0) UCVA of the bank can then be seen as its expected discounted loss, i.e. (cf. Crépey, Bielecki, and Brigo (2014, Chapter 3) or Green (2015))

$$\mathrm{UCVA} = (1 - R)\mathbb{E}[\beta_\tau \mathbb{1}_{\tau<T}\mathrm{MtM}_\tau^+] = \int_0^T \beta_s^{(\gamma)}\mathrm{EPE}(s)\lambda(s)ds, \tag{43}$$

where β is the risk-neutral discount factor, $\beta^{(\gamma)}$ is the discount factor adjusted for the default of the client (corresponding to a discount rate shifted by the default intensity γ), and $\lambda(\cdot)$ is the CDS curve of the counterparty (assuming for simplicity a deterministic β and independence between credit and exposure). For conciness in the sequel we just write CVA for the UCVA in (43).

Remark 6.2 Under a credit support agreement (CSA), MtM should be replaced by $(\mathrm{MtM}-\mathcal{C})$ in (42)–(43), where $\mathcal{C}$ is the collateral posted by the counterparty. Obviously collateral can mitigate the EPE and the CVA considerably. In the data of our case study there is no CSA, i.e. $\mathcal{C} = 0$. ∎

Non-linearity of MtM^+ with respect to the portfolio payoff components imposes CVA calculations at the counterparty portfolio (netting set) level. Hence, trade incremental CVA computations require two portfolio-wide calculations: one without the new trade and another one including it. Of course it is possible to store the paths simulated for the initial portfolio and reuse them each time we want to compute a new trade incremental XVA. In any case each trade incremental CVA computation requires the forward simulation of the mark-to-market process of the new deal.

6.2 Optimization Problem

We want to select a trade that minimizes the CVA variation given an initial portfolio. In the context of our case study we limit ourselves to monocurrency swaps (with value converted into euros). Negative CVA variations are desired as they signify a counterparty risk reduction. We impose the additional swap to be at par so that it can be entered at no cost, which is equally desirable from the bank and from the client perspective. The counterparty is identical for the initial portfolio and the additional swap as credit risk budget is often assigned at counterparty level.

The variable parameters for our optimization are: the new swap notional (bounded and positive), its maturity (within a list of dates), the direction of the position (payer or receiver, encoded as a binary parameter), and the deal currency (within an enumerated list). These variables define a discrete solution space $\mathcal{A}$.

Formally, we obtain the following fitness minimization problem:

$$\underset{x\in\mathcal{A}}{\mathrm{minimize}} \quad f(x) = \Delta\mathrm{CVA}(x) + \alpha|\mathrm{DV01}(x)|, \tag{44}$$

where x embodies the new deal (swap), $\Delta\mathrm{CVA}(x)$ is its incremental CVA, $\mathrm{DV01}(x)$ its DV01 [9], and α is a trade-off parameter. By so doing, we hope to decrease the counterparty risk without impacting the market risk and the trading loss and profit of the bank. In other words, we want to find a "right way risk" swap that offsets our initial portfolio cash flows. This swap can be flagged to the client as a product with reduced additional costs.

We address the optimization problem (44) by a genetic minimization algorithm: The state space $\mathcal{A}$ is viewed as a space of "chromosomes" x, the "genes" (components) of which evolve randomly along the iterations of the algorithm as detailed in Sect. B. These algorithms offer no theoretical guarantee of convergence but they are often found the most efficient approach in practice for dealing with such hybrid (partly continuous,

[9] The DV01 of an interest rate derivative is the variation of its mark-to-market when we shift the yield curve of one basis point.

partly discrete/combinatorial, hence without clearly defined gradient and Hessian), nonconvex, and high-dimensional optimization problems.

We present CVA compression results on two anonymized real data sets (the underlying interest rate and FX models are proprietary and cannot be disclosed in the paper). First we take a portfolio composed of payer swaps and we do not penalize our objective function. The point is to check that the algorithm will select a trade opposite to the portfolio exposure. Then, to respect the above-mentioned segregation of mandates between the XVA and the mark-to-market (such as interest rates, FX, etc.) desks of the bank, we activate the DV01 penalization on the same initial payer swap portfolio. Last, we launch the penalized optimization on an initial portfolio with a more complex risk profile, to see if the algorithm succeeds in converging in the case of a more complex, "nonconcave" exposure, vanishing at some future maturities.

In all our experiments, the initial portfolio is composed of monocurrency euro swaps. The fixed rate of all swaps is at par. There is no collateral to mitigate the exposure and all swaps in the initial portfolio are in euros.

We set the genetic algorithm population size (number of fitness evaluations per iteration) to 100 individuals and we limit the number of iterations to 5. This setting amounts to evaluating the fitness function on 600 different individuals. The space of solutions (or chromosomes) is discretized as follows:

- Notional: 10^5 to 8×10^6 with step 10^5,
- Maturity: 1 to 20 years with step 1 year, 30 years and 50 years,
- Currency: EUR, GBP, USD JPY,
- Position: pay/receive.

Thus our admissible space contains 14080 points, which implies that our algorithm explores around 4.2% of the state space.

Concerning the genetic algorithm hyperparameters (see Sect. B), the mutation rate is set to 20% and the crossover rate to 50%. In the genetic algorithms literature the crossover rate is often close to one, but for problems with few genes (i.e. components of x, or parameters, only four in our case), it is recommended to select a smaller value.

6.3 Payer Portfolio Without Penalization

First, we consider a portfolio only composed of payer swaps. The expected exposure (EE) $\mathbb{E}\mathrm{MtM}_t$ and the expected positive and negative exposures (EPE and ENE) $\mathbb{E}\mathrm{MtM}_t^{\pm}$ are shown as functions of time t in Figure 2, which illustrates the asymmetric risk profile of the portfolio. The point is to verify that the algorithm will select a receiver swap with a maturity comparable to those of the swap of the initial portfolio.

Table 1 reports after each iteration the three best solutions (from top to bottom) ever found since the beginning of the algorithm (in terms of the overall fitness criterion (44)). A positive incremental CVA means that we increase the counterparty risk of the bank. The initial portfolio CVA amounts to 34929€. We also report the |DV01|s of

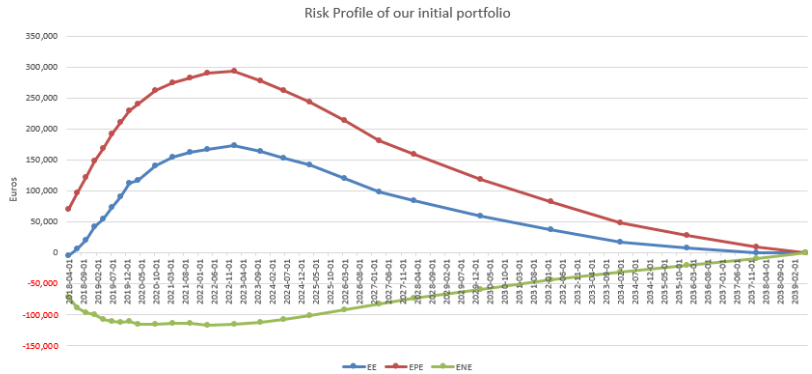

Figure 2: Risk profile of the portfolio (payer portfolio without penalization)

the enriched portfolios in order to be able to assess the impact of the penalization in our next experiment.

Iter.	Mat. (yrs)	Not. (K€)	Rate (%)	Curr.	Pos.	ΔCVA (€)	$\frac{-\Delta CVA}{CVA}$ (in %)	|DV01|(€)
0	10	4800000	1.6471	GBP	Receive	-8019	23.0	4484
	10	4700000	1.6471	GBP	Receive	-7948	22.8	4390
	10	4600000	1.6471	GBP	Receive	-7872	22.5	4297
1	17	5600000	1.4623	EUR	Receive	-17249	49.4	8648
	12	5400000	1.7036	GBP	Receive	-9163	26.2	5957
	16	3900000	0.6377	JPY	Receive	-8760	25.1	6137
2	14	6600000	1.3416	EUR	Receive	-21680	62.1	8626
	17	5100000	1.4623	EUR	Receive	-19729	56.5	7875
	17	5600000	1.4623	EUR	Receive	-17249	49.4	8648
3	14	6600000	1.3416	EUR	Receive	-21680	62.1	8626
	17	5100000	1.4623	EUR	Receive	-19729	56.5	7875
	17	5600000	1.4623	EUR	Receive	-17249	49.4	8648
4	17	3300000	1.4623	EUR	Receive	-27300	78.2	5096
	12	6100000	1.2203	EUR	Receive	-25382	72.7	6959
	11	5600000	1.147	EUR	Receive	-23009	65.9	5908
5	17	3300000	1.4623	EUR	Receive	-27300	78.2	5096
	12	6100000	1.2203	EUR	Receive	-25382	72.7	6959
	12	5100000	1.2203	EUR	Receive	-25264	72.3	5818

Table 1: Evolution of optimal solutions after each iteration (payer portfolio without penalization).

A stabilization of the algorithm is observed after 4 iterations, on a solution (incremental trade) leading to a CVA gain of about 27300€, i.e. about 78% of the initial portfolio CVA. The maturity and the notional are the two most sensitive genes in the optimization. The maturity of the swap is chosen by the algorithm so as to reduce the exposure peak (as opposed to the exposure profile as a whole): The decrease of the exposure on the first 8 years of the portfolio is visible in terms of EPE profile on figure

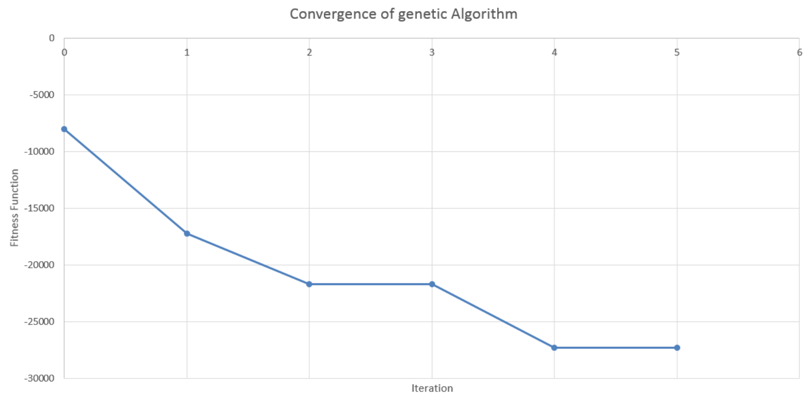

Figure 3: Fitness value as a function of iteration number (payer portfolio without penalization).

4 and of CVA profile[10] on Figure 5.

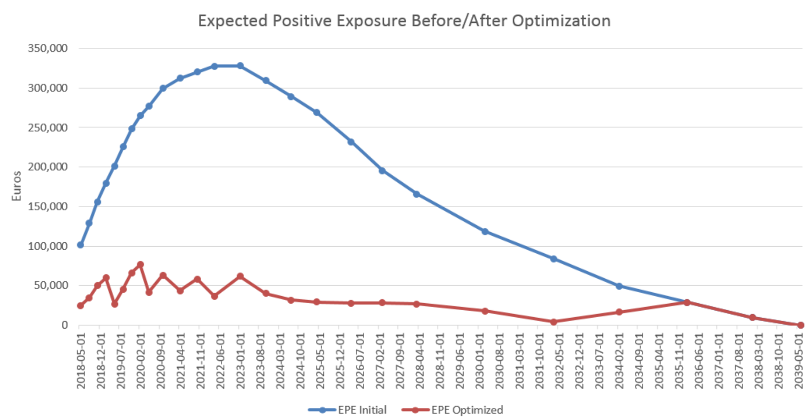

Figure 4: Risk profile of the portfolio before and after optimization (payer portfolio without penalization).

6.4 Payer Portfolio With Penalization

We keep the same initial portfolio but we now penalize our objective function with the |DV01| of the incremental swap. Our regularization parameter α (see (44)) is set to one.

The gains in CVA are of the same order of magnitude as those without penalization (92% of the CVA gain without penalization), for 20% of |DV01| less than before. The second and third best solutions also achieve a great CVA gain, while diminishing the

[10]Term structure obtained by integrating the EPE profile against the CDS curve of the counterparty from time 0 to an increasing upper bound $t \leq T$ (cf. (43)).

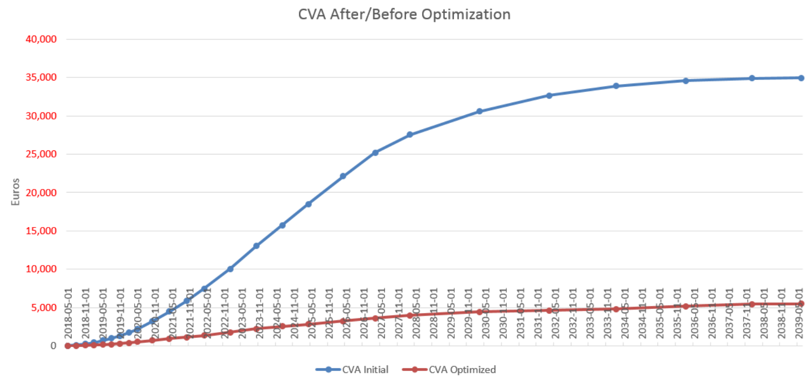

Figure 5: CVA curve before and after optimization (payer portfolio without penalization).

Iter.	Mat. (yrs)	Not. (K€)	Rate (%)	Curr.	Pos.	ΔCVA (€)	$\frac{-\Delta\text{CVA}}{\text{CVA}}$ (in %)	$\lvert$DV01$\rvert$(€)
0	10	4500000	1.6471	GBP	Receive	-7790	22.3	4218
	10	4600000	1.6471	GBP	Receive	-7871	22.5	4311
	10	4700000	1.6471	GBP	Receive	-7947	22.8	4405
1	17	5600000	1.4731	EUR	Receive	-16892	48.4	8706
	10	4500000	1.6471	GBP	Receive	-7790	22.3	4217
	10	4600000	1.6471	GBP	Receive	-7871	22.5	4311
2	14	6600000	1.3336	EUR	Receive	-21888	62.7	8654
	17	5600000	1.4731	EUR	Receive	-16892	48.4	8706
	17	6100000	1.4531	EUR	Receive	-15038	43.1	9466
3	14	6600000	1.3336	EUR	Receive	-21888	62.7	8654
	17	5600000	1.4731	EUR	Receive	-16892	48.4	8706
	9	4500000	0.9584	EUR	Receive	-10454	29.9	3945
4	10	6600000	1.3336	EUR	Receive	-21888	62.7	8654
	11	6600000	1.3999	EUR	Receive	-18825	53.9	9207
	17	5600000	1.4731	EUR	Receive	-16892	48.4	8706
5	11	2900000	1.3811	EUR	Receive	-25059	71.7	4039
	18	1500000	1.48	EUR	Receive	-18258	52.3	2442
	17	1500000	1.4531	EUR	Receive	-16553	47.4	2327

Table 2: Evolution of optimal solutions after each iteration (payer portfolio with penalization).

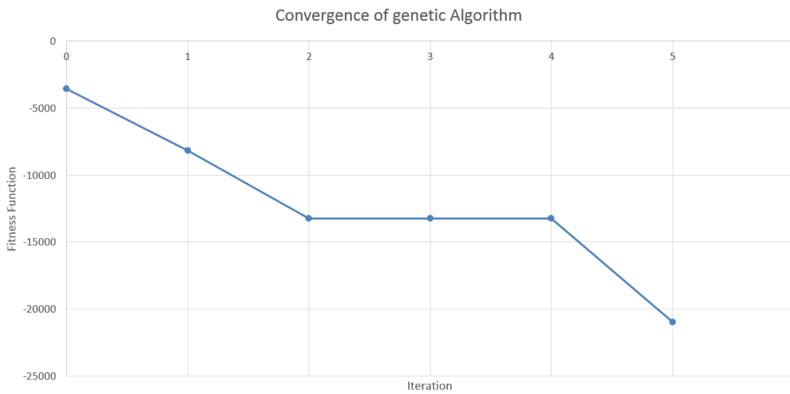

Figure 6: Fitness as a function of iteration number (payer portfolio with penalization).

|DV01| by a factor three with respect to the nonpenalized case. In the context of a payer portfolio, |DV01| control and CVA gain are two antagonistic targets. This may explain why the algorithm seems to struggle in finding a stable solution (the last iteration still decreases the fitness significantly). Proposed solutions choose either to shrink the maturity or to shorten the notional for building a less aggressive strategy.

During the execution, the algorithm first optimizes the CVA and then (in iteration 5) reduces the |DV01|. This is due to difference of order of magnitude between ΔCVA and |DV01| (recalling $\alpha = 1$): incremental ΔCVA is more important, hence the algorithm only takes care of the penalization once ΔCVA has been compressed.

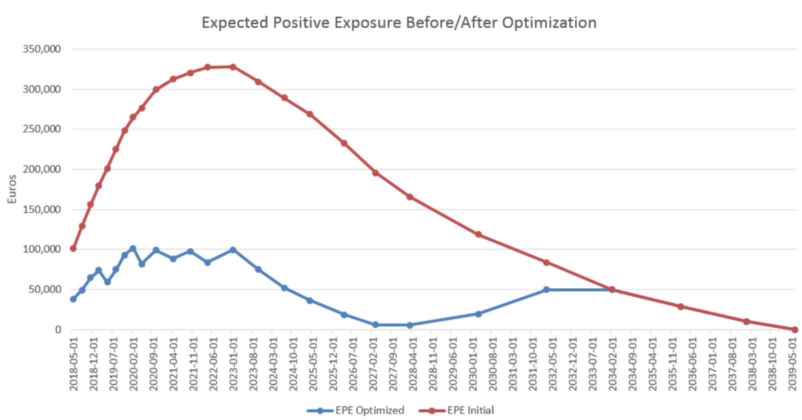

Figure 7: Risk profile of portfolio before and after optimization (payer portfolio with penalization).

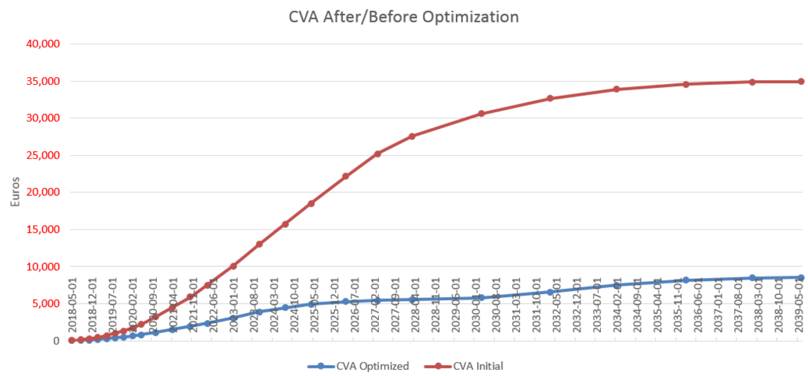

Figure 8: CVA curve before and after optimization (payer portfolio with penalization).

6.5 Hybrid Portfolio With Penalization

We challenge our genetic algorithm with a more balanced initial portfolio, as shown in Figure 9 (to be compared with Figure 2). The initial CVA is now 6410€. We set our regularization parameter α (see (44)) to 0.3 (as opposed to 1 in the previous case, in view of the lower initial CVA of the portfolio).

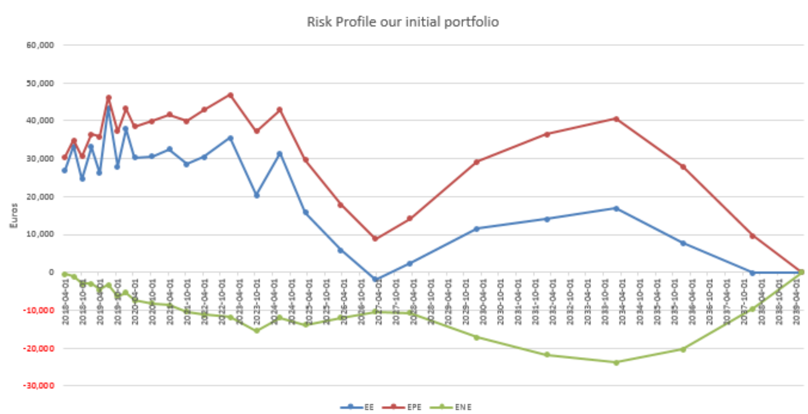

Figure 9: Risk profile of the portfolio (hybrid portfolio with penalization).

The stabilization of the algorithm occurs after three iterations, showing that, for the hybrid portfolio, |DV01| penalization and ΔCVA play less antagonistic roles. This is obtained by a relatively small notional and a maturity limited to 9 years, versus 11 years in the previous case of a payer portfolio with penalization. Figure 11 explains the choices operated by the algorithm: As we restrict our incremental strategy to one swap, the algorithm limits the EPE until the first positive peak before 2026. A better strategy outside our solution space would be to add a second swap with entry date in 2028 and end date in 2037.

Iter.	Mat. (yrs)	Not. (K€)	Rate (%)	Curr.	Pos.	ΔCVA (€)	$\frac{-\Delta CVA}{CVA}$ (in %)	|DV01|(€)
0	1	6000000	0.025	JPY	Receive	14	-0.2	609
	1	6100000	0.025	JPY	Receive	14	-0.2	619
	1	6300000	0.025	JPY	Receive	14	-0.2	640
1	8	1500000	0.8565	EUR	Receive	-1905	29.7	1177
	6	2300000	0.586	EUR	Receive	-1166	18.2	1370
	9	700000	1.608	GBP	Receive	-820	12.8	595
2	8	1500000	0.8565	EUR	Receive	-1905	29.7	1177
	6	2300000	0.586	EUR	Receive	-1166	18.2	1370
	9	700000	1.608	GBP	Receive	-82	12.8	595
3	9	1900000	0.9584	EUR	Receive	-2284	35.6	1665
	8	1500000	0.8565	EUR	Receive	-1905	29.7	1177
	7	2700000	0.7225	EUR	Receive	-1628	25.4	1865
4	9	1900000	0.9584	EUR	Receive	-2284	35.6	1665
	8	1500000	0.8565	EUR	Receive	-1905	29.7	1177
	7	2700000	0.7225	EUR	Receive	-1628	25.4	1865
5	9	1900000	0.9584	EUR	Receive	-2284	35.6	1665
	8	1500000	0.8565	EUR	Receive	-1905	29.7	1177
	9	2500000	0.9584	EUR	Receive	-1942	30.3	2192

Table 3: Evolution of optimal solutions after each iteration (hybrid portfolio with penalization).

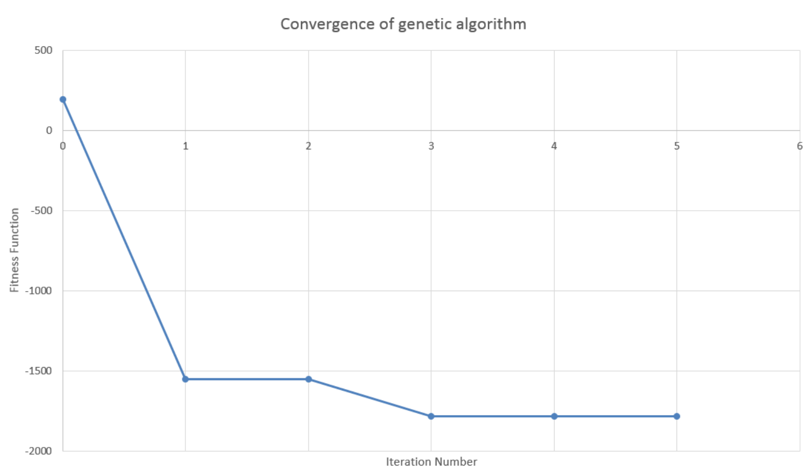

Figure 10: Fitness value as a function of iteration number (hybrid portfolio with penalization)

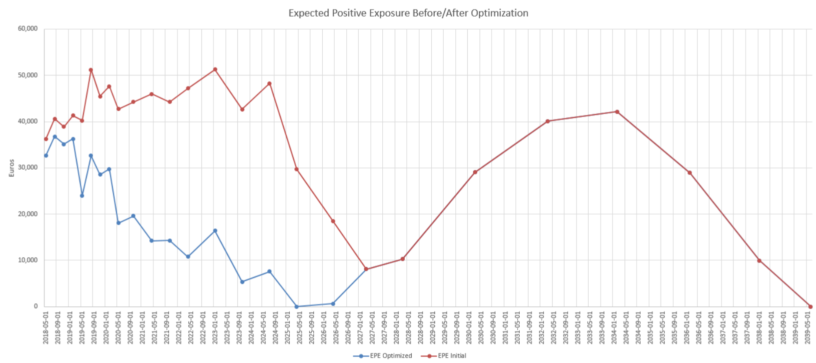

Figure 11: Risk profile of portfolio before and after optimization (hybrid portfolio with penalization).

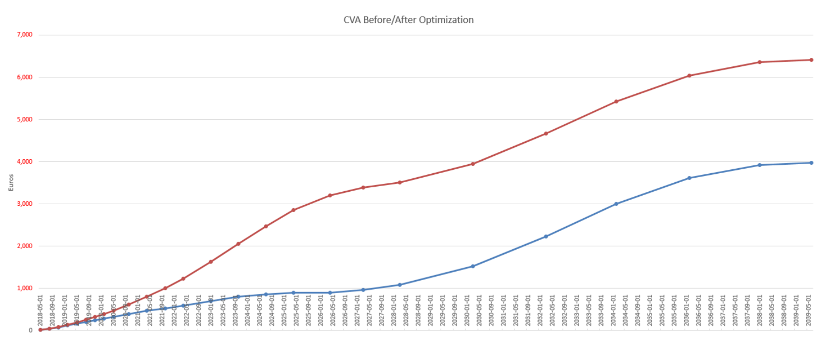

Figure 12: CVA curve before and after optimization (hybrid portfolio with penalization).

6.6 Perspectives

This case study is only a first step toward more complex optimizations.

The solution space would need to be extended for a finest control of the incremental exposure. One could thus enlarge the solution space with crosscurrency swaps and cleared swaps. Strategies involving several additional trades could be implemented. The penalization could be refined with a focus on forward marked to market (our current |DV01| penalization only controls spot market risk, which may lead to undesirable solutions).

A Connections with the Modigliani-Miller Theory

The Modigliani-Miller celebrated invariance result is in fact not one but several related propositions, developed in a series of papers going back to the seminal Modigliani and Miller (1958) paper. These propositions are different facets of the broad statement that the funding and capital structure policies of a firm are irrelevant to the profitability of its investment decisions. Ssee e.g. Baron (1976), Miller (1988), and Villamil (2008) for various discussions and surveys. We emphasize that we do not need or use such result (or any negative form of it) in our paper, but there are interesting connections to it, which we develop in this section.

A.1 Modigliani-Miller Irrelevance, No Arbitrage, and Completeness

Modigliani-Miller (MM) irrelevance, as we put it for brevity hereafter, was initially understood by its authors as a pure arbitrage result. They even saw this understanding as their main contribution with respect to various precedents, notably Williams (1938)'s law of conservation of investment value (see Sect. 2.3). So, quoting the footnote page 271 of Modigliani and Miller (1958) :

> "See, for example, J. B. Williams [21, esp. pp. 72-73]; David Durand [3]; and W. A. Morton [15]. None of these writers describe in any detail the mechanism which is supposed to keep the average cost of capital constant under changes in capital structure. They seem, however, to be visualizing the equilibrating mechanism in terms of switches by investors between stocks and bonds as the yields of each get out of line with their 'riskiness.' This is an argument quite different from the pure arbitrage mechanism underlying our proof, and the difference is crucial."

But, thirty years later, judging by the footnote page 99 in Miller (1988), the view of Miller on their result had evolved:

> "For other, and in some respects, more general proofs of our capital structure proposition, see among others, Stiglitz (1974) for a general equilibrium proof showing that individual wealth and consumption opportunities are unaffected by capital structures; See Hirshleifer (1965) and (1966) for a

> state preference, complete-markets proof; Duffie and Shafer (1986) for extensions to some cases of incomplete markets"

Non-arbitrage and completeness are intersecting but non-inclusive notions. Hence, implicitly, in Miller's own view, MM invariance does not hold in general in incomplete markets (even assuming no arbitrage opportunities). As a matter of fact, we can read page 197 of Gottardi (1995):

> "When there are derivative securities and markets are incomplete the financial decisions of the firm have generally real effects"

and page 9 of Duffie and Sharer (1986):

> "As to the effect of financial policy on shareholders, we point out that, generically, shareholders find the span of incomplete markets a binding constraint. This yields the obvious conclusion that shareholders are not indifferent to the financial policy of the firm if it can change the span of markets (which is typically the case in incomplete markets). We provide a trivial example of the impact of financial innovation by the firm. De-Marzo (1986) has gone beyond this and such earlier work as Stiglitz (1974), however, in showing that shareholders are indifferent to the trading of existing securities by firms. Anything the firm can do by trading securities, agents can undo by trading securities on their own account. Indeed, any change of security trading strategy by the firm can be accomodated within a new equilibrium that preserves consumption allocations. Hellwig (1981) distinguishes situations in which this is not the case, such as limited short sales."

Regarding MM irrelevance or not in incomplete markets (including some of the references that appear in the above quotations and other less closely related ones): Baron (1976), Milne (1975), Hagen (1976), and Hellwig (1981) deal with the impact of the default riskiness of the firm; Miller (1995) and Balling (2015) discuss the special case of banks, notably from the angle of the bias introduced by government repayment guarantees for bank demand deposits; Cline (2015) tests empirically MM irrelevance for banks, concluding to MM offsets of the order of half what they should be if MM irrelevance would fully hold.

A.2 The XVA Case

A bit like with limited short sales in Hellwig (1981), a (seemingly overlooked) situation where shareholders may "find the span of incomplete markets a binding constraint" is when market completion or, at least, the kind of completion that would be required for MM invariance to hold, is legally forbidden. This may seem a narrow situation but it is precisely the XVA case, which is also at the crossing between market incompleteness and the presence of derivatives pointed out as the MM 'non irrelevance case' in Gottardi (1995). The contra-assets and contra-liabilities that emerge endogenously from the impact of counterparty risk on the derivative portfolio of a bank (cf. Definition 2.3) cannot be "undone" by shareholders, because jump-to-default risk cannot be replicated by a bank: This is practically impossible in the case of contra-assets, for lack of available or sufficiently liquid hedging instruments (such as CDS contracts with rapidly varying notional on corporate names that would be required for replicating CVA exposures at client defaults); It is even more problematic in the case of contra-liabilities, because a bank cannot sell CDS protection on itself (this is forbidden by law) and it has a limited ability in buying back its own debt (as, despite the few somehow provocative statements in Miller (1995), a bank is an intrinsically leveraged entity).

As a consequence, MM irrelevance is expected to break down in the XVA setup. In fact, as seen in the main body in the paper, cost of funding and cost of capital are material to banks and need be reflected in entry prices for ensuring shareholder indifference to the trades.

More precisely, the XVA setup is a case where a firm's valuation is invariant to funding strategies and, still, investment decisions are not. The point here is a bit subtle. Saying that the value of a company is independent of financing strategies does not imply that investment decisions do not depend on financing strategies. There two numbers we can look at: the value of the equity E and the sum of the value of equity and debt, $\mathrm{E} + \mathrm{D}$. Equity holders will naturally seek to optimize E and will accept an investment opportunity if $\Delta\mathrm{E}$ is positive. Williams' law implies that equity plus debt, $\mathrm{E} + \mathrm{D}$, stays invariant under a certain financial transaction. But this does not imply in general that shareholders are indifferent to the transaction: Shareholders are indifferent if $\Delta\mathrm{E} = 0$, not if $\Delta(\mathrm{E} + \mathrm{D}) = 0$. To go from Williams' wealth conservation law to MM irrelevance, we have to assume complete markets or, at least, the availability of certain trades to shareholders. Namely, assuming shareholders can and do change financing strategy, then, even if we start with $\Delta\mathrm{E} < 0$ for a given transaction (but $\Delta(\mathrm{E} + \mathrm{D}) = 0$), we may conclude that equity shareholders are actually indifferent as there exists a change in financing strategy for which $\Delta\mathrm{E} = 0$. However, in the XVA case, the bank cannot freely buy back its own debt, so such a change is not possible and only Williams' wealth conservation law remains, whereas MM irrelevance breaks down: See Sect. 4 for illustration in a pedagogical static setup.

B Optimization by Genetic Algorithm

Genetic algorithms belong to the class of evolutionary algorithms relying on the principles of survival of the strongest and genetic mutation.

At each iteration, the fitness[11] is computed for each solution (also named chromosome) of an initial population[12]. The values returned by the objective function are used for selecting chromosomes from the population. Among numerous selection methods, we can quote fitness proportionate selection, ranking proportionate selection (convenient as it requires no scaling for the fitness values), or tournament selection (selection

[11] The value returned by the objective function of the problem.

[12] A set of chromosomes.

of the best among randomly drawn chromosomes). The common intention is to build a probability law that samples in priority individuals with best fitness values. For a comparison of classical selection methods, we mention the work of Blickle and Thiele (1995).

The mutation stage is intended to maintain some diversity inside the population in order to avoid local minima trap. It randomly changes one gene[13] of one chromosome drawn with a probability p called mutation rate.

Selection and mutation play opposite roles, because a focus on fitness leads to a quicker convergence toward a local minima. Conversely, a too heavily mutated population results into a slow random research.

In addition, a crossover operator plays the role of a reproduction inside the algorithm. A distribution (often the same as the one used for selection) is chosen for selecting chromosomes from a population of the previous iteration and for building pairs of chromosomes through crossover recombination. The principle of crossover is to build two children chromosomes from parents chromosomes. Children share gene values of their parents but a gene value from one parent cannot be inherited by both children. A crossover mask decides for each gene in which parent a child can copy the gene version. One of the most popular crossover mask is the following:

Example B.1 (Single point crossover) *Let (p_1, p_2) be a pair of chromosomes chosen as parents and let (c_1, c_2) denote the children. We assume that each chromosome has four genes A, B, C, D, that p_1 has gene versions $\{A_1, B_1, C_1, D_1\}$ and p_2 has gene versions $\{A_2, B_2, C_2, D_2\}$. For a single point crossover we draw uniformly an integer i such the first i genes for child 1 are inherited from p_1 and the remaining genes are transferred from p_2 to c_1. If we draw $i = 2$, then c_1 has gene versions $\{A_1, B_1, C_2, D_2\}$ and c_2 has gene values $\{A_2, B_2, C_1, D_1\}$.* ∎

The role of crossover operator is paradoxical as it can be seen as a combination of mutations (it increases genetic diversity) whilst promoting chromosomes with highest fitnesses. For more details on different versions of mutation and crossover operators, the reader is referred to Goldberg et al. (1989).

The execution behavior of any genetic algorithm is mostly determined by the choice of the selection operator, the number of solutions affected by a mutation and the number of chromosomes affected by a crossover. We denote by r_m the percentage of individuals in a population affected by a mutation (also named mutation rate) and by r_c the percentage of individuals concerned by crossover recombination (also named crossover rate).

See Algorithm 1 and Figure 13 for the algorithm in pseudo-code and skeleton forms.

B.1 Customized Algorithm

In the theoretical literature on genetic algorithms, a (tentative) solution is represented as a bit string. In practice, however, string bits representation of parameters do not

[13] A gene is a component of our solution. In our case, it can be the notional or the counterparty.

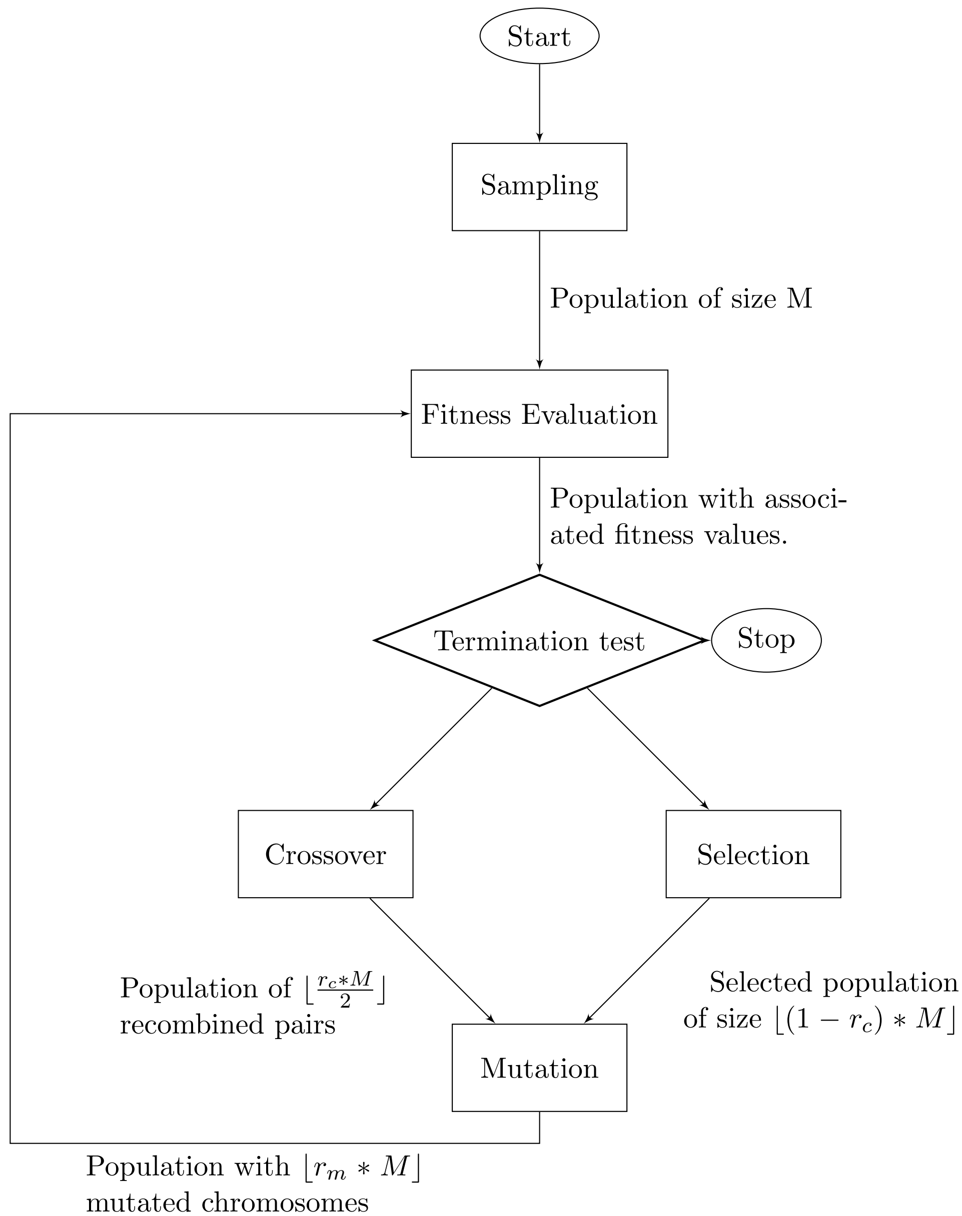

Figure 13: Skeleton for genetic algorithm with rank proportionate selection.

Data: An initial population P_{init} of size M and the associated fitness values for each chromosome, a crossover rate r_c, a mutation rate r_m.

initialization;

while *Stopping condition is unsatisfied.* **do**

  Save in $P_{selected}$ $\lfloor (1 - r_c) * M \rfloor$ chromosomes selected with rank proportionate method from P_{init} ;

  Save in $P_{crossover}$ $\lfloor r_c * M \rfloor$ chromosomes selected with rank proportionate method from P_{init} ;

  Recombine (uniformly without replacement) $\lfloor \frac{r_c * M}{2} \rfloor$ pairs from $P_{crossover}$;

  Merge $P_{crossover}$ and $P_{selected}$ in $P_{mutated}$;

  Mutate randomly $\lfloor r_m * M \rfloor$ in $P_{mutated}$;

  for *Each chromosome c in* $P_{mutated}$ **do**

    Compute the fitness value of c;

  end

end

Result: A new population and the associated fitness values.

Algorithm 1: Pseudo-Code for genetic algorithm with rank proportionate selection.

give enough control on the mutation distribution. Namely, in string bits representation, mutations affect all bits uniformly, whereas we might want to mutate some parameters more frequently (e.g. to try "more" notional values for our swap, a feature found important in our numerics). Hence, we rather model our solution as a variable string, a choice also made in Kondratyev and Giorgidze (2017).

We choose rank proportionate selection to avoid any problem of data scaling. More precisely, if we have a population $\mathcal{P} = \{1, ..., N\}$ of N solutions and the associated fitnesses $(f_i)_{i \in \mathcal{P}}$, then the probability to select chromosome i is

$$p_i = \frac{2R(f_i)}{N(N+1)},$$

where R is a rank function that returns one for the lowest value (in the context of a minimization problem).

Concerning the crossover operator, we use a uniform crossover mask through which the choice of gene inherited from one parent or another is drawn with a uniform probability.

The mutation operator selects uniformly a gene allele (value) to modify. Namely, the different alleles of a gene have the same probability to be drawn as new value for this gene; However, the probability to mutate a gene is proportional to the number of allele he can take. For instance, in our experiments where the notional can take 80 different values, the currency 4 values, the position 2 possible values, and the maturity 22 values, when a chromosome is selected for mutation, the probability to mutate each of its genes is equal to $\frac{80}{108}$ for the notional , $\frac{4}{108}$ for the currency, $\frac{2}{108}$ for the position, and $\frac{22}{108}$ for the maturity.

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