# XVA Analysis From the Balance Sheet

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**Abstract**

XVAs denote various counterparty risk related valuation adjustments that are applied to financial derivatives since the 2007–09 crisis. We root a cost-of-capital XVA strategy in a balance sheet perspective which is key in identifying the economic meaning of the XVA terms. Our approach is first detailed in a static setup that is solved explicitly. It is then plugged in the dynamic and trade incremental context of a real derivative banking portfolio. Our cost-of-capital XVA strategy ensures to bank shareholders a submartingale wealth process corresponding to a target hurdle rate on their capital at risk, consistently between and throughout deals. The model, set on a forward/backward SDE formulation, can be solved efficiently using GPU computing combined with deep learning regression methods in a whole bank balance sheet context. A numerical case study emphasizes the workability and added value of the ensuing pathwise XVA computations.

**Keywords:** Counterparty risk, balance sheet of a bank, market incompleteness, wealth transfer, X-valuation adjustment (XVA), deep learning, quantile regression.

**Mathematics Subject Classification:** 91B25, 91B26, 91B30, 91G20, 91G40, 62G08, 68Q32.

**JEL Classification:** D52, G13, G24, G28, G33, M41.

# 1 Introduction

XVAs, with X as C for credit, D for debt, F for funding, M for margin, or K for capital, are post-2007–09 crisis valuation adjustments for financial derivatives. They deeply affect the derivative pricing task by making it global, nonlinear, and entity dependent. Before these technical implications, the fundamental point is to understand what deserves to be priced and what does not, by rooting the pricing approach in a corresponding collateralization, accounting, and dividend policy of the bank.

Coming after several papers on the valuation of defaultable assets in the 90's, such as Duffie and Huang (1996), Bielecki and Rutkowski (2002, Eq. (14.25) p. 448) yields the formula

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(CVA − DVA) for the firm valuation of bilateral counterparty risk on a swap, assuming risk-free funding. This formula, rediscovered and generalized by others since the 2008–09 financial crisis (cf. e.g. Brigo and Capponi (2010)), is symmetrical, i.e. it is the negative of the analogous quantity considered from the point of view of the counterparty, consistent with the law of one price and the Modigliani and Miller (1958) theorem.

Around 2010, the materiality of the DVA windfall benefit of a bank at its own default time became the topic of intense debates in the quant and academic finance communities. At least, it seemed reasonable to admit that, if the own default risk of the bank was accounted for in the modeling, in the form of a DVA benefit, then the cost of funding (FVA) implication of this risk should be included as well, leading to the modified formula (CVA − DVA + FVA). See for instance Burgard and Kjaer (2011, 2013, 2017), Crépey (2015), Brigo and Pallavicini (2014), or Bichuch, Capponi, and Sturm (2018). See also Bielecki and Rutkowski (2015) for an abstract funding framework (without explicit reference to XVAs), generalizing Piterbarg (2010) to a nonlinear setup.

Then Hull and White (2012) objected that the FVA was only the compensator of another windfall benefit of the bank at its own default, corresponding to the non-reimbursement by the bank of its funding debt. Accounting for the corresponding "DVA2" (akin to the FDA in this paper) brings back to the original firm valuation formula:

$$\mathrm{CVA} - \mathrm{DVA} + \mathrm{FVA} - \mathrm{FDA} = \mathrm{CVA} - \mathrm{DVA},$$

as FVA = FDA.

However, their argument implicitly assumes that the bank can perfectly hedge its own default (cf. Burgard and Kjaer (2013, end of Section 3.1) and see Section 3.5 below). As a bank is an intrinsically leveraged entity, this is not the case in practice. One can mention the related corporate finance notion of debt overhang in Myers (1977), by which a project valuable for the firm as a whole may be rejected by shareholders because the project is mainly valuable to bondholders. But, until recently, such considerations were hardly considered in the field of derivative pricing.

The first ones to recast the XVA debate in the perspective of the balance sheet of the bank were Burgard and Kjaer (2011), to explain that an appropriately hedged derivative position has no impact on the dealer's funding costs. Also relying on balance sheet models of a dealer bank, Castagna (2014) and Andersen, Duffie, and Song (2019) end up with conflicting conclusions, namely that the FVA should, respectively should not, be included in the valuation of financial derivatives. Adding the KVA, but in a replication framework, Green, Kenyon, and Dennis (2014) conclude that both the FVA and the KVA should be included as add-ons in entry prices and as liabilities in the balance sheet.

## 1.1 Contents

Our key premise is that counterparty risk entails two distinct but intertwined sources of market incompleteness:

- A bank cannot hedge its own jump-to-default exposure, because this would mean selling protection on its own default, which is nonpractical and, somehow, legally forbidden (see Section 2);
- A bank cannot perfectly hedge counterparty default losses, by lack of sufficiently liquid CDS markets.

We specify the banking XVA metrics that align derivative entry prices to shareholder interest, given this impossibility for a bank to replicate the jump-to-default related cash flows. We develop a cost-of-capital XVA approach consistent with the accounting standards set out in IFRS 4 Phase II (see International Financial Reporting Standards (2013)), inspired from the Swiss solvency test and Solvency II insurance regulatory frameworks (see Swiss Federal Office of Private Insurance (2006) and Committee of European Insurance and Occupational Pensions Supervisors (2010)), which so far has no analogue in the banking domain. Under this approach, the valuation (CL) of the so-called contra-liabilities and the cost of capital (KVA) are sourced from clients at trade inceptions, on top of the (CVA − DVA) complete market valuation of counterparty risk, in order to compensate bank shareholders for wealth transfer and risk on their capital.

The cost of the corresponding collateralization, accounting, and dividend policy is, by contrast with the complete market valuation (CVA − DVA) of counterparty risk,

$$\mathrm{CVA} + \mathrm{FVA} + \mathrm{KVA}, \tag{1}$$

computed unilaterally in a certain sense (even though we do crucially include the default of the bank itself in our modeling), and charged to clients on an incremental run-off basis at every new deal.

All in one, our cost-of-capital XVA strategy makes shareholder equity (i.e. wealth) a submartingale with drift corresponding to a hurdle rate $h$ on shareholder capital at risk, consistently between and throughout deals. Thus we arrive to a sustainable strategy for profits retention, much like in the above-mentioned insurance regulation, but in a consistent continuous-time and banking framework.

Last but not least, our approach can be solved efficiently using GPU computing combined with deep learning regression methods in a whole bank balance sheet context.

## 1.2 Outline and Contributions

Section 2 sets a financial stage where a bank is split across several trading desks and entails different stakeholders. Section 3 develops our cost-of-capital XVA approach in a one-period static setup. Section 4 revisits the approach at the dynamic and trade incremental level. Section 5 is a numerical case study on large, multi-counterparty portfolios of interest rate swaps, based on the continuous-time XVA equations for bilateral trade portfolios recalled in Section A.

The ***main contributions*** of the paper are:

- The one-period static XVA model of Section 3, with explicit formulas for all the quantities at hand, offering a concrete grasp on the related wealth transfer and risk premium issues;

- Proposition 4.1, which establishes the connections between XVAs and the core equity tier 1 capital of the bank, respectively bank shareholder equity (i.e. shareholder wealth);

- Proposition 4.2, which establishes that, under the XVA policy represented by the balance conditions (3) between deals and the counterparty risk add-on (40) throughout deals, bank shareholder equity (i.e. wealth) is a submartingale with drift corresponding to a target hurdle rate $h$ on shareholder capital at risk. This perspective also solves the puzzle according to which, on the one hand, XVA computations are performed on a run-off portfolio basis, while, on the other hand, they are used for computing pricing add-ons to new deals;

- The XVA deep learning simulation / deep learning (quantile) regression computational strategy of Section 4.4;
- The numerical case study of Section 5, which emphasizes the materiality of refined, pathwise XVA computations, as compared to more simplistic XVA approaches.

From a broader point of view, this paper reflects a shift of paradigm regarding the pricing and risk management of financial derivatives, from hedging to balance sheet optimization, as quantified by relevant XVA metrics. In particular (compare with the end of Section 1.1), our approach implies that the FVA (and also the MVA, see Remark 2.1) should be included as an add-on in entry prices and as a liability in the balance sheet; the KVA should be included as an add-on in entry prices, but not as a liability in the balance sheet.

From a computational point of view, this paper opens the way to second generation XVA GPU implementation. The first generation consisted of nested Monte Carlo implemented by explicit CUDA programming on GPUs (see Albanese, Caenazzo, and Crépey (2017), Abbas-Turki, Diallo, and Crépey (2018)). The second generation takes advantage of GPUs leveraging via pre-coded CUDA/AAD deep learning packages that are used for the XVA embedded regression and quantile regression task. An economic capital based KVA approach is then not only conceptually more satisfying, but also much simpler to implement, than a regulatory capital based KVA approach.

# 2 Balance Sheet and Capital Structure Model of the Bank

We consider a dealer bank, which is a market maker, involved into bilateral derivative portfolios with clients. For simplicity, we only consider European derivatives.

The bank has two (main) kinds of stakeholders, ***shareholders*** and ***bondholders***. The shareholders have the control of the bank and are solely responsible for investment decisions before bank default. The bondholders represent the junior creditors of the bank, who have no decision power until bank default, but are protected by laws, of the pari-passu type, forbidding trades that would trigger value away from them to shareholders during the default resolution process of the bank. The bank also has senior creditors, represented in our framework by an ***external funder***, who can lend unsecured to the bank and is assumed to enjoy an exogenously given recovery rate in case of default of the bank.

We consider three kinds of business units within the bank (see Figure 1 for the corresponding picture of the bank balance sheet, also referring to Table 1 after it for a list of the main financial acronyms used in the paper): the ***CA desks***, i.e. the CVA desk and the FVA desk (or Treasury) of a bank, in charge of contra-assets, i.e. of counterparty risk and its funding implications for the bank; the ***clean desks***, who focus on the market risk of the contracts in their respective business lines; the ***management*** of the bank, in charge of the dividend release policy of the bank.

Collateral means cash or liquid assets that are posted to guarantee a netted set of transactions against defaults. It comes in two forms: variation margin, which is re-hypotecable, i.e. fungible across netting sets, and initial margin, which is segregated. We assume cash only collateral. Posted collateral is supposed to be remunerated at the risk-free rate (assumed to exist, with overnight index swap rates as a best market proxy).

**Remark 2.1** To alleviate the notation, in this conceptual section of the paper, we only consider an FVA as the global cost of raising collateral for the bank, as opposed to a distinction, in the industry and in later sections in the paper, between an FVA, in the strict sense of the cost of raising variation margin, and an MVA for the cost of raising initial margin. ∎

***Amounts on dedicated cash accounts of the bank:***

| | | |
|---|---|---|
| **CM** | Clean margin | Definition 2.1 and Assumption 2.1 |
| **RC** | Reserve capital | Definition 2.1 and Assumption 2.1 |
| **RM** | Risk margin | Definition 2.1 and Assumption 2.1 |
| **UC** | Uninvested capital | Definition 2.1 and Assumption 2.1 |

***Valuations:***

| | | |
|---|---|---|
| **CA** | Contra-assets valuation | (2), (15), and (45) |
| **CL** | Contra-liabilities valuation | Definition 2.1 and (17), (32), and (40) |
| **CVA** | Credit valuation adjustment | (16), (15), (46), and (54)–(55) |
| **DVA** | Debt valuation adjustment | (17) and (16) |
| **FDA** | Funding debt adjustment | (17) and (22) |
| **FV** | Firm valuation of counterparty risk | (20) and (22) |
| **FVA** | Funding valuation adjustment | Remark 2.1, (16), (15), and (46) |
| **KVA** | Capital valuation adjustment | (3), (25), and (50) |
| **MtM** | Mark-to-market | (3) and (14) |
| **MVA** | Margin valuation adjustment | Remark 2.1, (30), (46), and (56) |
| **XVA** | Generic "X" valuation adjustment | First paragraph |

***Also:***

| | | |
|---|---|---|
| **CR** | Capital at risk | (48) |
| **CET1** | Core equity tier I capital | (2) and (37) |
| **EC** | Economic capital | Definitions 3.2 and A.1 |
| **FTP** | Funds transfer price | (40) |
| **SHC** | Shareholder capital (or equity) | (2) and (38) |
| **SCR** | Shareholder capital at risk | Assumption 2.1 and (24) |

Table 1: Main financial acronyms and place where they are introduced conceptually and/or specified mathematically in the paper, as relevant.

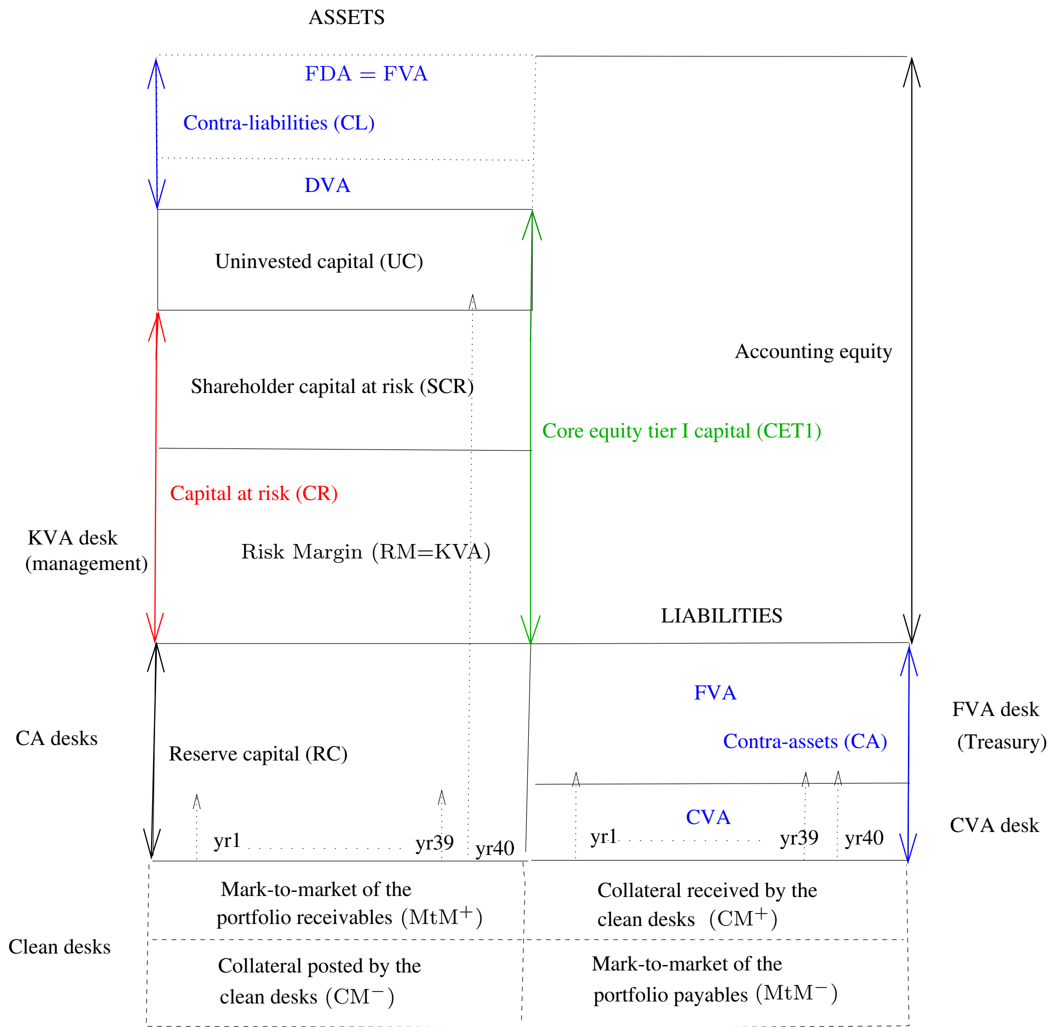


Figure 1: Balance sheet of a dealer bank. Contra-liability valuation (CL) at the top is shown in dotted boxes because it is only value to the bondholders (see Section 3.5). Mark-to-market valuation (MtM) of the client portfolio by the clean desks, as well as the corresponding collateral (clean margin CM), are shown in dashed boxes at the bottom. Their role will essentially vanish in our setup, where we assume a perfect clean hedge by the bank. The arrows in the left column represent trading losses of the CA desks in "normal years 1 to 39" and in an "exceptional year 40" with full depletion (i.e. refill via UC, under Assumption 2.1.ii) of RC, RM, and SCR. The numberings yr1 to yr40 are fictitious yearly scenarios in line with a 97.5% expected shortfall of the one-year-ahead trading losses of the bank that we use for defining its economic capital. The arrows in the right column symbolize the average depreciation in time of contra-assets between deals. The collateral between the bank and the clients is not shown to alleviate the picture.

The CA desks guarantee the trading of the clean desks against client defaults, through a ***clean margin account***, which can be seen as (re-hypotecable) collateral exchanged between the CA desks and the clean desks. The corresponding clean margin amount (CM) also plays the role of the funding debt of the clean desks put at their disposal at a risk-free cost by the Treasury of the bank. This is at least the case when CM $> 0$ (clean desks clean margin receivers). In the case when CM $< 0$ (clean desks clean margin posters), ($-$CM) corresponds to excess cash generated by the trading of the clean desks, usable by the Treasury for its other funding purposes. See the bottom, dashed boxes in Figure 1.

In addition, the CA desks value the contra-assets (future counterparty default losses and funding expenditures), charge them to the clients at deal inception, deposit the corresponding payments in a ***reserve capital account***, and then are exposed to the corresponding payoffs. As time proceeds, contra-assets realize and are covered by the CA desks with the reserve capital account.

On top of reserve capital, the so-called risk margin is sourced by the management of the bank from the clients at deal inception, deposited into a ***risk margin account***, and then gradually released as KVA payments into the shareholder dividend stream.

Another account contains the ***shareholder capital at risk*** earmarked by the bank to deal with exceptional trading losses (beyond the expected losses that are already accounted for by reserve capital).

Last, there is one more bank account with shareholder ***uninvested capital***.

All cash accounts are remunerated at the risk-free rate.

**Definition 2.1** We write CM, RC, RM, SCR, and UC for the respective (risk-free discounted) amounts on the clean margin, reserve capital, risk margin, shareholder capital at risk, and uninvested capital accounts of the bank. We also define

$$\mathrm{SHC} = \mathrm{SCR} + \mathrm{UC}\,, \quad \mathrm{CET1} = \mathrm{RM} + \mathrm{SCR} + \mathrm{UC}. \blacksquare \tag{2}$$

From a financial interpretation point of view, before bank default, SHC corresponds to shareholder capital (or equity), i.e. ***shareholder wealth***; CET1 is the ***core equity tier I capital*** of the bank, representing the financial strength of the bank when assessed from a regulatory, structural solvency point of view, i.e. the sum between shareholder capital and the risk margin (which is also loss-absorbing), but excluding the value CL of the so-called contra-liabilities (see Figure 1). Indeed, the latter is only value to the bondholders (cf. Section 3.5), hence only accounting equity.

**Remark 2.2** The purpose of our capital structure model of the bank is not to model the default of the bank, like in a Merton (1974) model, as the point of negative equity (CET1 $< 0$). In the case of a bank, such a default model would be unrealistic. For instance, at the time of its collapse in April 2008, Bear Stearns had billions of capital. In fact, the legal definition of default is an unpaid coupon or cash flow, which is a liquidity (as opposed to solvency) issue. We will actually model the default of the bank as a totally unpredictable event at some exogenous time $\tau$ calibrated to the credit default swap (CDS) curve referencing the bank. Indeed we view the latter as the most reliable and informative credit data regarding anticipations of markets participants about future recapitalization, government intervention, bail-in, and other bank failure resolution policies.

Instead, the aim of our capital structure model is to put in a balance sheet perspective the contra-assets and contra-liabilities of a dealer bank, items which are not present in the Merton model and play a key role in our XVA analysis. $\blacksquare$

In line with the Volcker rule banning proprietary trading for a bank, we assume a perfect market hedge of the client portfolio by the clean desks of the bank, in a sense to be specified below in the respective static and continuous-time setups. By contrast, as jump-to-default exposures (own jump-to-default exposure, in particular) cannot be hedged by the bank (cf. Section 1.1), we conservatively assume no XVA hedge.

We work on a measurable space $(\Omega, \mathfrak{A})$ endowed with a calibrated risk-neutral pricing measure $\mathbb{Q}^*$, with related expectation denoted by $\mathbb{E}^*$. Note that, as common in the context of XVA computations, the historical probability measure required for capital at risk computations is taken equal to $\mathbb{Q}^*$ (the discrepancy between the two is left to model risk). Moreover, for notational simplicity, we use the risk-free asset as a numéraire.

## 2.1 Run-Off Portfolio

Until Section 4.2, we consider the case of a portfolio held on a run-off basis, i.e. set up at time 0 and such that no new unplanned trades enter the portfolio in the future.

The trading cash flows of the bank (cumulative cash flow streams starting from 0 at time 0) then consist of

- the contractually promised cash flows $\mathcal{P}$ from clients,
- counterparty credit cash flows $\mathcal{C}$ to clients (i.e., because of counterparty risk, the effective cash flows from clients are $\mathcal{P} - \mathcal{C}$),
- risky funding cash flows $\mathcal{F}$ to the external funder, and
- hedging cash flows $\mathcal{H}$ of the clean desks to financial hedging markets (note that all cash flow differentials can be positive or negative).

See Section 3.1 and (43)–(44) for concrete specifications in respective one-period and continuous-time setups.

**Assumption 2.1** i. ***(Self-financing condition)*** RC + RM + SCR + UC − CM evolves like the received trading cash flows $\mathcal{P} - \mathcal{C} - \mathcal{F} - \mathcal{H}$.

ii. ***(Mark-to-model)*** The amounts on all the accounts but UC are marked-to-model (hence the last, residual amount, UC, plays the role of an adjustment variable). Specifically, we assume that the following ***shareholder balance conditions*** hold at all times:

$$\mathrm{CM} = \mathrm{MtM}\,,\ \ \mathrm{RC} = \mathrm{CA}\,,\ \ \mathrm{RM} = \mathrm{KVA}, \tag{3}$$

for theoretical target levels MtM, CA, and KVA to be specified in later sections of the paper (which will also determine the theoretical target level for SCR).

iii. ***(Agents)*** The initial amounts $\mathrm{MtM}_0$, $\mathrm{CA}_0$, and $\mathrm{KVA}_0$ are provided by the clients at portfolio inception time 0. Resets between time 0 and the bank default time $\tau$ (excluded) are on bank shareholders. At the (positive) bank default time $\tau$, the property of the residual amount on the reserve capital and risk margin accounts is transferred from the shareholders to the bondholders of the bank. ∎

Under a cost-of-capital XVA approach, we define valuation so as to make shareholder trading losses (that include marked-to-model liability fluctuations) centered, then we add a KVA risk premium in order to ensure to bank shareholders some positive hurdle rate $h$ on their capital at risk.

In what follows, this is developed, first, in a static setup, which can be solved explicitly, and then, in a dynamic and trade incremental setup, as suitable for dealing with a real derivative banking portfolio.

# 3 XVA Analysis in a Static Setup

In this section, we apply the cost-of-capital XVA approach to an uncollateralized portfolio made of a single deal, $\mathcal{P}$ (random variable promised to the bank), between a client and a bank, without prior endowment, in an elementary one-period (one year) setup. All the trading cash flows $\mathcal{P}$, $\mathcal{C}$, $\mathcal{F}$, and $\mathcal{H}$ are then random variables (instead of processes in a multi-period setup). Risky funding assets are assumed fairly priced by the market, i.e. $\mathbb{E}^*\mathcal{F} = 0$.

The bank and client are both default prone with zero recovery to each other. The bank also has zero recovery to its external funder. We denote by $J$ and $J_1$ the survival indicators (random variables) of the bank and client at time 1, with default probability of the bank $\mathbb{Q}^*(J = 0) = \gamma$.

Since prices and XVAs only matter at time 0 in a one-period setup, we identify all the XVA processes, as well as the mark-to-market (valuation by the clean desks) MtM of the deal, with their values at time 0.

For any random variable $Y$, we define

$$Y^\circ = JY \text{ and } Y^\bullet = -(1-J)Y\,, \quad \text{hence } Y = Y^\circ - Y^\bullet. \tag{4}$$

Let $\mathbb{E}$ denote the expectation with respect to the bank survival measure, say $\mathbb{Q}$, associated with $\mathbb{Q}^*$, i.e., for any random variable $\mathcal{Y}$,

$$\mathbb{E}\mathcal{Y} = (1-\gamma)^{-1}\mathbb{E}^*(\mathcal{Y}^\circ) \tag{5}$$

(which is also equal to $\mathbb{E}\mathcal{Y}^\circ$) . The notion of bank survival measure was introduced in greater generality by Schönbucher (2004). In the present static setup, (5) is nothing but the $\mathbb{Q}^*$ expectation of $\mathcal{Y}$ conditional on the survival of the bank (note that, whenever $\mathcal{Y}$ is independent from $J$, the right-hand-side in (5) coincides with $\mathbb{E}^*\mathcal{Y}$).

**Lemma 3.1** *For any random variable $\mathcal{Y}$ and constant $Y$, we have*

$$Y = \mathbb{E}^*(\mathcal{Y}^\circ + (1-J)Y) \Longleftrightarrow Y = \mathbb{E}\mathcal{Y}. \tag{6}$$

***Proof***. Indeed,

$$Y = \mathbb{E}^*(J\mathcal{Y} + (1-J)Y) \Longleftrightarrow \mathbb{E}^*(J(\mathcal{Y}-Y)) = 0 \Longleftrightarrow \mathbb{E}(\mathcal{Y}-Y) = 0 \Longleftrightarrow Y = \mathbb{E}\mathcal{Y}, \tag{7}$$

where the equivalence in the middle is justified by (5). ∎

**Remark 3.1** We will account for the mark-to-market of the deal and for the free funding source provided by reserve capital RC = CA (cf. (3)). But, for simplicity in a first stage, we will ignore the possibility of using capital at risk for funding purposes. This possibility will be introduced in Section 3.4. ∎

## 3.1 Cash Flows

**Lemma 3.2** *Given the (to be specified)* MtM *and* CA *amounts (cf. Assumption 2.1.ii), the credit and funding cash flows $\mathcal{C}$ and $\mathcal{F}$ of the bank and its trading loss (and profit) $L$ are such that*

$$\begin{aligned}
&\mathcal{C}^\circ = J(1-J_1)\mathcal{P}^+\,,\;\; \mathcal{F}^\circ = J\gamma(\mathrm{MtM}-\mathrm{CA})^+\\
&\mathcal{C}^\bullet = (1-J)\big(\mathcal{P}^- - (1-J_1)\mathcal{P}^+\big)\,,\;\; \mathcal{F}^\bullet = (1-J)\big((\mathrm{MtM}-\mathrm{CA})^+ - \gamma(\mathrm{MtM}-\mathrm{CA})^+\big)\\
&L^\circ = \mathcal{C}^\circ + \mathcal{F}^\circ - J\mathrm{CA}\,,\;\; L^\bullet = \mathcal{C}^\bullet + \mathcal{F}^\bullet + (1-J)\mathrm{CA}\,,\;\; L = \mathcal{C} + \mathcal{F} - \mathrm{CA}.
\end{aligned} \tag{8}$$

***Proof***. For the deal to occur, the bank needs to borrow $(\mathrm{MtM}-\mathrm{CA})^+$ unsecured or invest $(\mathrm{MtM}-\mathrm{CA})^-$ risk-free (cf. Remark 3.1). Having assumed zero recovery to the external funder, unsecured borrowing is fairly priced as $\gamma \times$ the amount borrowed by the bank (in line with our assumption that $\mathbb{E}^*\mathcal{F}=0$), i.e. the bank must pay for its risky funding the amount

$$\gamma(\mathrm{MtM}-\mathrm{CA})^+.$$

Moreover, at time 1, under zero recovery upon defaults:

- If the bank is not in default (i.e. $J=1$), then the bank closes its position with the client while receiving $\mathcal{P}$ from its client if the latter is not in default (i.e. $J_1=1$), whereas the bank pays $\mathcal{P}^-$ to its client if the latter is in default (i.e. $J_1=0$). In addition, the bank reimburses its funding debt $(\mathrm{MtM}-\mathrm{CA})^+$ or receives back the amount $(\mathrm{MtM}-\mathrm{CA})^-$ it had lent at time 0;
- If the bank is in default (i.e. $J=0$), then the bank receives back $J_1\mathcal{P}^+$ on the derivative as well as the amount $(\mathrm{MtM}-\mathrm{CA})^-$ it had lent at time 0.

Also accounting for the hedging loss $\mathcal{H}$, the trading loss of the bank over the year is

$$\begin{aligned} L = \gamma(\mathrm{MtM}-\mathrm{CA})^+ - J\big(J_1\mathcal{P} - (1-J_1)\mathcal{P}^- - (\mathrm{MtM}-\mathrm{CA})^+ + (\mathrm{MtM}-\mathrm{CA})^-\big) \\ - (1-J)\big(J_1\mathcal{P}^+ + (\mathrm{MtM}-\mathrm{CA})^-\big) + \mathcal{H}. \end{aligned} \tag{9}$$

In the static setup, the perfect clean hedge condition (see before Section 2.1) writes $\mathcal{H} = \mathcal{P} - \mathrm{MtM}$. Inserting this into the above yields

$$L = (1-J_1)\mathcal{P}^+ + \gamma(\mathrm{MtM}-\mathrm{CA})^+ - \mathrm{CA} - (1-J)(\mathcal{P}^- + (\mathrm{MtM}-\mathrm{CA})^+), \tag{10}$$

as easily checked for each of the four possible values of the pair $(J, J_1)$. That is,

$$\begin{aligned} L^\circ &= \underbrace{J(1-J_1)\mathcal{P}^+}_{\mathcal{C}^\circ} + \underbrace{J\gamma(\mathrm{MtM}-\mathrm{CA})^+}_{\mathcal{F}^\circ} - J\mathrm{CA} \\ L^\bullet &= \underbrace{(1-J)\big(\mathcal{P}^- - (1-J_1)\mathcal{P}^+\big)}_{\mathcal{C}^\bullet} + \underbrace{(1-J)\big((\mathrm{MtM}-\mathrm{CA})^+ - \gamma(\mathrm{MtM}-\mathrm{CA})^+\big)}_{\mathcal{F}^\bullet} + (1-J)\mathrm{CA}, \end{aligned} \tag{11}$$

where the identification of the different terms as part of $\mathcal{C}$ or $\mathcal{F}$ follows from their financial interpretation. ∎

**Remark 3.2** The derivation (9) implicitly allows for negative equity (that arises whenever $L^\circ > \mathrm{CET1}$, cf. (2)), which is interpreted as recapitalization. In a variant of the model excluding recapitalization and negative equity is excluded, the default of the bank would be modeled in a structural fashion as the event $\{L = \mathrm{CET1}\}$, where

$$L = \big((1-J_1)\mathcal{P}^+ + \gamma(\mathrm{MtM}-\mathrm{CA})^+ - \mathrm{CA}\big) \wedge \mathrm{CET1}, \tag{12}$$

and we would obtain, instead of (10), the following trading loss for the bank:

$$\mathbb{1}_{\{\mathrm{CET1}>L\}}L + \mathbb{1}_{\{\mathrm{CET1}=L\}}\big(\mathrm{CET1} - \mathcal{P}^- - (\mathrm{MtM}-\mathrm{CA})^+\big). \tag{13}$$

In this paper we consider a model with recapitalization for the reasons explained in Remark 2.2.

Structural XVA approaches in a static setup have been proposed in Andersen, Duffie, and Song (2019) (without KVA) and Kjaer (2019) (including the KVA). Their marginal limiting results as a new deal size goes to zero are comparable to some of the results that we have here. But then, instead of developing a continuous time version of their corporate finance model and taking the small trade limit, these papers can only start the development of the continuous time model from the single period small trade limit model. By contrast, in our framework, we can have end to end development in the continuous time model of Section 4 and in the present single period model. ∎

### 3.2 Contra-assets And Contra-liabilities

To make shareholder trading losses centered (cf. the next-to-last paragraph of Section 2), clean and CA desks value by expectation their shareholder sensitive cash flows. These include, in case of default of the bank, the transfer of property from the CA desks to the clean desks of the collateral amount MTM on the clean margin account, as well as (cf. Assumptions 2.1.ii and iii) the transfer from shareholders to bondholders of the residual value $\mathrm{RC} = \mathrm{CA}$ on the reserve capital account. Accordingly:

**Definition 3.1** We let

$$\mathrm{MtM} = \mathbb{E}^*\big(\mathcal{P}^\circ + (1-J)\mathrm{MtM}\big) \tag{14}$$

and

$$\mathrm{CA} = \mathrm{CVA} + \mathrm{FVA}, \tag{15}$$

where

$$\begin{aligned}\mathrm{CVA} &= \mathbb{E}^*\big(\mathcal{C}^\circ + (1-J)\mathrm{CVA}\big)\\ \mathrm{FVA} &= \mathbb{E}^*\big(\mathcal{F}^\circ + (1-J)\mathrm{FVA}\big),\end{aligned} \tag{16}$$

hence $\mathrm{CA} = \mathbb{E}^*\big(\mathcal{C}^\circ + \mathcal{F}^\circ + (1-J)\mathrm{CA}\big)$. We also define the contra-liabilities value

$$\mathrm{CL} = \mathrm{DVA} + \mathrm{FDA}, \tag{17}$$

where

$$\mathrm{DVA} = \mathbb{E}^*\big(\mathcal{C}^\bullet + (1-J)\mathrm{CVA}\big) \tag{18}$$

$$\mathrm{FDA} = \mathbb{E}^*\big(\mathcal{F}^\bullet + (1-J)\mathrm{FVA}\big), \tag{19}$$

as well as the firm valuation of counterparty risk,

$$\mathrm{FV} = \mathbb{E}^*(\mathcal{C} + \mathcal{F}). \blacksquare \tag{20}$$

The definitions of $\mathrm{MtM}, \mathrm{CVA}$, and FVA are in fact fix-point equations. However, the following result shows that these equations are well-posed and yields explicit formulas for all the quantities at hand.

**Proposition 3.1** *We have*

$$\begin{aligned}\mathrm{MtM} &= \mathbb{E}\mathcal{P}^\circ\\ \mathrm{CVA} &= \mathbb{E}\big((1-J_1)\mathcal{P}^+\big)\\ \mathrm{FVA} &= \gamma(\mathrm{MtM}-\mathrm{CA})^+ = \frac{\gamma}{1+\gamma}(\mathrm{MtM}-\mathrm{CVA})^+\end{aligned} \tag{21}$$

*and*

$$\begin{aligned}
&\mathbb{E}^* L^\circ = \mathbb{E}L = 0\\
&\mathrm{FDA} = \mathrm{FVA}\\
&\mathrm{FV} = \mathbb{E}^*\mathcal{C} = \mathrm{CVA} - \mathrm{DVA} = \mathrm{CA} - \mathrm{CL}.
\end{aligned} \tag{22}$$

***Proof.*** The first identities in each line of (21) follow from Definition 3.1 by Lemma 3.1 and definition of the involved cash flows in Lemma 3.2. Given (15), the formula $\mathrm{FVA} = \gamma(\mathrm{MtM} - \mathrm{CA})^+$ in (21) is in fact a semi-linear equation

$$\mathrm{FVA} = \gamma(\mathrm{MtM} - \mathrm{CVA} - \mathrm{FVA})^+. \tag{23}$$

But, as $\gamma$ (a probability) is nonnegative, this equation has the unique solution given by the right-hand side in the third line of (21).

Regarding (22), we have

$$\mathbb{E}^* L^\circ = (1-\gamma)\mathbb{E}\big((1-J_1)\mathcal{P}^+ + \gamma(\mathrm{MtM} - \mathrm{CA})^+ - \mathrm{CA}\big) = 0,$$

by application of (5), the first line in (11), (21), and (15). Hence, using (5) again,

$$\mathbb{E}L = (1-\gamma)^{-1}\mathbb{E}^* L^\circ = 0.$$

This is the first line in (22), which implies the following ones by definition of the involved quantities and our assumption that $\mathbb{E}^*\mathcal{F} = 0$. ∎

## 3.3 Capital Valuation Adjustment

Economic capital (EC) is the level of capital at risk that a regulator would like to see on an economic, structural basis. Risk calculations are typically performed by banks "on a going concern", i.e. assuming that the bank itself does not default. Accordingly:

**Definition 3.2** The economic capital (EC) of the bank is given by the 97.5% expected shortfall[1] of the bank trading loss $L$ under $\mathbb{Q}$, which[2] we denote by $\mathbb{ES}(L^\circ)$. ∎

The risk margin (sized by the to-be-defined KVA in our setup) is also loss-absorbing, i.e. part of capital at risk, and the KVA is originally sourced from the client (see Assumption 2.1.iii). Hence, *shareholder* capital at risk only consists of the *difference* between the (total) capital at risk and the KVA. Accordingly (and also accounting, regarding (25), for the last part in Assumption 2.1.iii):

**Definition 3.3** The capital at the risk (CR) of the bank is given by $\max(\mathrm{EC}, \mathrm{KVA})$ and the ensuing shareholder capital at risk (SCR) by

$$\mathrm{SCR} = \max(\mathrm{EC}, \mathrm{KVA}) - \mathrm{KVA} = (\mathrm{EC} - \mathrm{KVA})^+, \tag{24}$$

where, given some hurdle rate (target return-on-equity) $h$,

$$\mathrm{KVA} = \mathbb{E}^*\big(h\mathrm{SCR}^\circ + (1-J)\mathrm{KVA}\big). \;∎ \tag{25}$$

**Proposition 3.2** *We have*

$$\mathrm{KVA} = h\mathrm{SCR} = \frac{h}{1+h}\mathrm{EC} = \frac{h}{1+h}\mathbb{ES}(L^\circ). \tag{26}$$

[1] See e.g. Föllmer and Schied (2016, Section 4.4).

[2] As, by definition of $\mathbb{Q}$, this quantity does not depend on $L^\bullet$.

***Proof.*** The first identity follows from Lemma 3.1. The resulting KVA semi-linear equation (in view of (24)) is solved similarly to the FVA equation (23). ∎

The KVA formula (26) (as well as its continuous-time analog (50)) can be used either in the direct mode, for computing the KVA corresponding to a given $h$, or in the reverse-engineering mode, for defining the "implied hurdle rate" associated with the actual level on the risk margin account of the bank. Cost of capital proxies have always been used to estimate return-on-equity. The KVA is a refinement, fine-tuned for derivative portfolios, but the base return-on-equity concept itself is far older than even the CVA. In particular, the KVA is very useful in the context of collateral and capital optimization.

**KVA Risk Premium and Indifference Pricing Interpretation** The CA component of the FTP corresponds to the expected costs for the shareholders of concluding the deal. This CA component makes the shareholder trading loss $L^\circ$ centered (cf. the first line in (22)). On top of expected shareholder costs, the bank charges to the clients a risk margin (RM). Assume the bank shareholders endowed with a utility function $U$ on $\mathbb{R}$ such that $U(0) = 0$. In a shareholder indifference pricing framework, the risk margin arises as per the following equation:

$$\mathbb{E}^* U(J(\mathrm{RM} - L)) = \mathbb{E}^* U(0) = 0 \tag{27}$$

(the expected utility of the bank shareholders without the deal), where

$$\mathbb{E}^* U(J(\mathrm{RM} - L)) = \mathbb{E}^* \big( J U(\mathrm{RM} - L) \big) = (1-\gamma) \mathbb{E} U(\mathrm{RM} - L),$$

by (5). Hence

$$\mathbb{E} U(\mathrm{RM} - L) = 0. \tag{28}$$

The corresponding RM is interpreted as the minimal admissible risk margin for the deal to occur, seen from bank shareholders' perspective.

Taking for concreteness $U(-\ell) = \frac{1 - e^{\rho \ell}}{\rho}$, for some risk aversion parameter $\rho$, (28) yields $\mathrm{RM} = \rho^{-1} \ln \mathbb{E} e^{\rho L} = \rho^{-1} \ln \mathbb{E} e^{\rho L^\circ}$, by the observation following (5). In the limiting case where the shareholder risk aversion parameter $\rho \to 0$ and $\mathbb{E} U(-L) \to -\mathbb{E}(L) = 0$ (by the first line in (22)), then $\mathrm{RM} \to 0$.

In view of (3) and (26), the corresponding implied KVA and hurdle rate $h$ are such that

$$\mathrm{KVA} = \rho^{-1} \ln \mathbb{E} e^{\rho L^\circ}, \quad \frac{h}{1+h} = \frac{\rho^{-1} \ln \mathbb{E} e^{\rho L^\circ}}{\mathbb{E}\mathbb{S}(L^\circ)}. \tag{29}$$

The hurdle rate $h$ in our KVA setup plays the role of a risk aversion parameter, like $\rho$ in the exponential utility framework. The above shows that having set the historical probability measure equal to the pricing probability measure in our model (see before Section 2.1) does not imply that our setup is "risk-neutral", in the sense that shareholders would have no aversion to risk: It is only so when $h$ is equal to 0 (i.e. $\rho = 0$). In fact, there are two layers of risk premium in our model. The first one is embedded in the risk-neutral pricing measure $\mathbb{Q}^*$ and the second one is the KVA.

An indifference price has a competitive interpretation: Assume that the bank is competing for the client with other banks. Then, in the limit of a continuum of competing banks with a continuum of indifference prices, whenever a bank makes a deal, this can only be at its indifference price. Our stylized indifference pricing model of a KVA defined by a constant hurdle rate $h$ exogenizes (by comparison with the endogenous hurdle rate $h$ in (29)) the impact on pricing of the competition between banks. It does so in a way that generalizes smoothly to a dynamic setup (see Section 4), as required to deal with a real derivative banking portfolio. It then provides a refined notion of return-on-equity for derivative portfolios, where a full-fledged optimization approach would be impractical.

### 3.4 Initial Margin and Fungibility of Capital at Risk as a Funding Source

In case of initial margin that would be received (RIM) and posted (PIM) by the bank, at the level of, say, some $\mathbb{Q}$ value-at-risk of $\pm(\mathcal{P} - \mathrm{MtM})$, then $\mathcal{P}$ needs be replaced by $(\mathcal{P} - \mathrm{RIM})$ everywhere in the above, whence:

- an accordingly diminished CVA;
- additional initial margin related cash flows in $\mathcal{F}^\circ$ and $\mathcal{F}^\bullet$ given as $J\gamma\mathrm{PIM}$ and $(1 - J)(\mathrm{PIM} - \gamma\mathrm{PIM})$, triggering additional adjustments MVA in CA and MDA = MVA in CL, where

$$\mathrm{MVA} = \mathbb{E}^*\big(J\gamma\mathrm{PIM} + (1 - J)\mathrm{MVA}\big) = \gamma\mathrm{PIM}; \tag{30}$$

- the second FVA formula in (21) modified into $\mathrm{FVA} = \frac{\gamma}{1+\gamma}(\mathrm{MtM} - \mathrm{CVA} - \mathrm{MVA})^+$.

Accounting further for the additional free funding source provided by capital at risk (cf. Remark 3.1), then, in view of the specification given in the first sentence of Definition 3.3 for the latter, one needs replace $(\mathrm{MtM} - \mathrm{CA})^\pm$ by $(\mathrm{MtM} - \mathrm{CA} - \max(\mathrm{EC}, \mathrm{KVA}))^\pm$ everywhere before. This results in the same CVA and MVA as in the previous paragraph, but in the following *system* for the random variable $L^\circ$ and the FVA and the KVA numbers (cf. the corresponding lines in (11), (21), (26), and recall (15)):

$$\begin{aligned}
L^\circ &= J(1 - J_1)\mathcal{P}^+ + J\gamma(\mathrm{MtM} - \mathrm{CA} - \max(\mathrm{EC}, \mathrm{KVA}))^+ + J\gamma\mathrm{PIM} - J\mathrm{CA}\\
\mathrm{FVA} &= \gamma(\mathrm{MtM} - \mathrm{CA} - \max(\mathrm{EC}, \mathrm{KVA}))^+\\
\mathrm{KVA} &= \frac{h}{1+h}\mathbb{ES}(L^\circ).
\end{aligned} \tag{31}$$

This system entails a coupled dependence between, on the one hand, the FVA and KVA numbers and, on the other hand, the shareholder loss process $L^\circ$. However, once CVA, PIM, RIM, and MVA computed as in the above, the system (31) can be addressed numerically by Picard iteration, starting from, say, $L^{(0)} = \mathrm{KVA}^{(0)} = 0$ and $\mathrm{FVA}^{(0)}$ as per the last line in (21), then looping into (31) until numerical convergence.

### 3.5 Funds Transfer Price

The funds transfer price (all-inclusive XVA rebate to MtM) aligning the deal to shareholder interest (in the sense of a given hurdle rate $h$, cf. the next-to-last paragraph before Section 2.1) is

$$\begin{aligned}
\mathrm{FTP} &= \underbrace{\mathrm{CVA} + \mathrm{FVA}}_{\text{Expected shareholder costs CA}} + \underbrace{\mathrm{KVA}}_{\text{Shareholder risk premium}}\\
&= \underbrace{\mathrm{CVA} - \mathrm{DVA}}_{\text{Firm valuation FV}} + \underbrace{\mathrm{DVA} + \mathrm{FDA}}_{\text{Wealth transfer CL}} + \underbrace{\mathrm{KVA}}_{\text{Shareholder Risk premium}},
\end{aligned} \tag{32}$$

where all terms are explicitly given in Propositions 3.1 and 3.2 (or the corresponding variants of Section 3.4 in the refined setup considered there).

The above results implicitly assumed that the bank cannot hedge jump-to-default cash flows (cf. Section 1.1). To understand this, let us temporarily suppose, for the sake of the argument, that the bank would be able to hedge its own jump-to-default through a further deal, whereby the bank would deliver a payment $L^{\bullet}$ at time 1 in exchange of a fee fairly valued as

$$\mathrm{CL} = \mathbb{E}^{*} L^{\bullet} = \mathrm{DVA} + \mathrm{FDA}, \tag{33}$$

deposited in the reserve capital account of the bank at time 0.

We include this hedge and assume that the client would now contribute at the level of $\mathrm{FV} = \mathrm{CA} - \mathrm{CL}$ (cf. (22)), instead of CA before, to the reserve capital account of the bank at time 0. Then the amount that needs by borrowed by the bank for implementing its strategy is still $\gamma(\mathrm{MtM} - \mathrm{CA})^{+}$ as before. But the trading loss of the bank becomes, instead of $L$ before,

$$\mathcal{C} + \mathcal{F} - \mathrm{FV} + (L^{\bullet} - \mathrm{CL}) = \mathcal{C} + \mathcal{F} - \mathrm{CA} + L^{\bullet} = L + L^{\bullet} = L^{\circ}, \tag{34}$$

where the last line in (22) and the last identity in (8) were used in the first and second equality. By comparison with the situation from previous sections without own-default hedge by the bank:

- the client is better off by the amount $\mathrm{CA} - \mathrm{FV} = \mathrm{CL}$,
- the shareholders are still indifferent to the deal in expected counterparty default and funding expenses terms,
- the bondholders are completely wiped out.

The CL originating cash flow $L^{\bullet}$ has been hedged and monetized by the shareholders, who have passed the corresponding benefit to the client.

Under a cost-of-capital pricing approach, the bank would still charge to its client a KVA add-on $\frac{h}{1+h}\mathbb{ES}(L^{\circ})$, as risk compensation for the nonvanishing shareholder trading loss $L^{\circ}$ still triggered by the deal. If, however, the bank could also hedge the (zero-valued, by the first line in (22)) loss $L^{\circ}$, hence the totality of $L = L^{\circ} - L^{\bullet}$ (instead of $L^{\bullet}$ only in the above), then the trading loss and the KVA would vanish. As a result, the all-inclusive XVA add-on (rebate from MtM valuation) would boil down to

$$\mathrm{FV} = \mathrm{CVA} - \mathrm{DVA},$$

the value of counterparty risk and funding to the bank as a whole.

**Connection With the Modigliani-Miller Theory** The Modigliani-Miller invariance result, with Modigliani and Miller (1958) as a seminal reference, consists in various facets of a broad statement that the funding and capital structure policies of a firm are irrelevant to the profitability of its investment decisions. Modigliani-Miller (MM) irrelevance, as we put it for brevity hereafter, was initially seen as a pure arbitrage result. However, it was later understood that there may be market incompleteness issues with it. So quoting Duffie and Sharer (1986, page 9), "generically, shareholders find the span of incomplete markets a binding constraint [...] shareholders are not indifferent to the financial policy of the firm if it can change the span of markets (which is typically the case in incomplete markets)"; or Gottardi (1995, page 197): "When there are derivative securities and markets are incomplete the financial decisions of the firm have generally real effects".

A situation where shareholders may "find the span of incomplete markets a binding constraint" is when market completion is legally forbidden. This corresponds to the XVA case, which is also at the crossing between market incompleteness and the presence of derivatives pointed out above as the MM non irrelevance case in Gottardi (1995). Specifically, the contra-assets and contra-liabilities that emerge endogenously from the impact of counterparty risk on the derivative portfolio of a bank cannot be "undone" by shareholders, because jump-to-default risk cannot be replicated by a bank.

As a consequence, MM irrelevance is expected to break down in the XVA setup. In fact, as visible on the trade incremental FTP (counterparty risk pricing) formula (32) (cf. also (40) and Proposition 4.2 in a dynamic and trade incremental setup below), cost of funding and cost of capital are material to banks and need be reflected in entry prices for ensuring shareholder indifference to the trades, i.e. preserving their hurdle rate throughout trades.

# 4 XVA Analysis in a Dynamic Setup

We now consider a dynamic, continuous-time setup, with model filtration $\mathbb{G}$ and a (positive) bank default time $\tau$ endowed with an intensity $\gamma$. The bank survival probability measure associated with the risk-neutral pricing measure $\mathbb{Q}^*$ is then the probability measure $\mathbb{Q}$ with $(\mathbb{G}, \mathbb{Q}^*)$ density process $Je^{\int_0^\cdot \gamma_s ds}$ (assumed integrable), where $J = \mathbb{1}_{[0,\tau)}$ is the bank survival indicator process (cf. Schönbucher (2004)). In particular, writing $Y^\circ = JY + (1-J)Y_{\tau-}$, for any left-limited process $Y$, we have by application of the results of Crépey and Song (2017) (cf. the condition (A) there):

**Lemma 4.1** *For every $\mathbb{Q}$ (resp. sub-, resp. resp. super-) martingale $Y$, the process $Y^\circ$ is a $\mathbb{Q}^*$ (resp. sub-, resp. resp. super-) martingale.* ∎

**Remark 4.1** In the dynamic setup, the survival measure formulation is a light presentation, sufficient for the purpose of the present paper (skipping the related integrability issues), of an underlying reduction of filtration setup, which is detailed in the above-mentioned reference (regarding Lemma 4.1, cf. also Collin-Dufresne, Goldstein, and Hugonnier (2004, Lemma 1)). ∎

## 4.1 Case of a Run-Off Portfolio

First, we consider the case of a portfolio held on a run-off basis (cf. Section 2.1). We denote by $T$ the final maturity of the portfolio and we assume that all prices and XVAs vanish at time $T$ if $T < \tau$. Then the results of Albanese and Crépey (2019) show that all the qualitative insights provided by the one-period XVA analysis of Section 3 are still valid. The trading loss of the bank is now given by the process

$$L = \mathcal{C} + \mathcal{F} + \mathrm{CA} - \mathrm{CA}_0 \tag{35}$$

and the bank *shareholder* trading loss by the $\mathbb{Q}$ (hence $\mathbb{Q}^*$, by Lemma 4.1) martingale

$$L^\circ = \mathcal{C}^\circ + \mathcal{F}^\circ + \mathrm{CA}^\circ - \mathrm{CA}_0. \tag{36}$$

In (35)-(36), we have CA = CVA + FVA as in (15); the processes $\mathcal{C}$, $\mathcal{F}$, CVA, and FVA are continuous-time processes analogs, detailed in the case of bilateral trade portfolios in Section A.1-A.2, of the eponymous quantities in Section A.1-3 (which were constants or random variables there).

**Proposition 4.1** *The core equity tier 1 capital of the bank is given by*

$$\mathrm{CET1} = \mathrm{CET1}_0 - L. \tag{37}$$

*Shareholder equity (i.e. wealth, cf. Definition 2.1 and the following comments) is given by*

$$\mathrm{SHC} = \mathrm{SHC}_0 - (L + \mathrm{KVA} - \mathrm{KVA}_0). \tag{38}$$

***Proof.*** In the continuous-time setup, Assumption 2.1.i is written as

$$\mathrm{RC} + \mathrm{RM} + \mathrm{SCR} + \mathrm{UC} - \mathrm{CM} - (\mathrm{RC} + \mathrm{RM} + \mathrm{SCR} + \mathrm{UC} - \mathrm{CM})_0 = \mathcal{P} - (\mathcal{C} + \mathcal{F} + \mathcal{H}).$$

Given the definition of CET1 in (2), the perfect clean hedge condition written in the dynamic setup as $\mathcal{P} + \mathrm{MtM} - \mathrm{MtM}_0 - \mathcal{H} = 0$, and the balance conditions (3), this is equivalent to

$$\mathrm{CA} + \mathrm{CET1} - (\mathrm{CA} + \mathrm{CET1})_0 = -(\mathcal{C} + \mathcal{F}).$$

In view of (35), we obtain (37).

As $\mathrm{SHC} = \mathrm{CET1} - \mathrm{RM}$ (cf. (2)), we have by (37):

$$\mathrm{SHC} = \mathrm{CET1}_0 - L - \mathrm{RM} = \mathrm{CET1}_0 - \mathrm{RM}_0 - (L + \mathrm{RM} - \mathrm{RM}_0),$$

which, by the third balance condition in (3), yields (38). ∎

By Lemma 4.1, the continuous-time $\mathrm{KVA}^{\circ}$ that stems from (48)-(49) is a $\mathbb{Q}^*$ supermartingale with terminal condition $\mathrm{KVA}^{\circ}_T = 0$ on $\{T < \tau\}$ and drift coefficient $h\mathrm{SCR}$, where SCR is given as in (24), but for EC there dynamically defined as the time-t conditional, 97.5% expected shortfall of $(L^{\circ}_{t+1} - L^{\circ}_t)$ under $\mathbb{Q}$, killed at $\tau$.

**Remark 4.2** It is only before $\tau$ that the right-hand-sides in the definitions (2) really deserve the respective interpretations of shareholder equity of the bank and core equity tier 1 capital. Hence, it is only the parts of (37) and (38) stopped before $\tau$, i.e.

$$\mathrm{CET1}^{\circ} = \mathrm{CET1}_0 - L^{\circ}\,,\;\; \mathrm{SHC}^{\circ} = \mathrm{SHC}_0 - (L^{\circ} + \mathrm{KVA}^{\circ} - \mathrm{KVA}_0), \tag{39}$$

which are interesting financially.

## 4.2 Trade Incremental Cost-of-Capital XVA Strategy

In Albanese and Crépey (2019) and in Section 4.1 above, the derivative portfolio of the bank is assumed held on a run-off basis. By contrast, real-life derivative portfolios are incremental.

Assume a new deal shows up at time $\theta \in (0, \tau)$. We denote by $\Delta\cdot$, for any portfolio related process, the difference between the time $\theta$ values of this process for the run-off versions of the portfolio with and without the new deal.

**Definition 4.1** We apply the following trade incremental pricing and accounting policy:

- The clean desks pay $\Delta\mathrm{MtM}$ to the client and the CA desks add an amount $\Delta\mathrm{MtM}$ on[3] the clean margin account;
- The CA desks charge to the client an amount $\Delta\mathrm{CA}$ and add it on[4] the reserve capital account;

[3] i.e. remove $(-\Delta\mathrm{MtM})$ from, if $\Delta\mathrm{MtM} < 0$.
[4] i.e. remove $(-\Delta\mathrm{CA})$ from, if $\Delta\mathrm{CA} < 0$.

- The management of the bank charges the amount $\Delta$KVA to the client and adds it on[5] the risk margin account. ∎

**Remark 4.3** Thus, it is the client who provides all the amounts to the clean margin, reserve capital, and risk margin accounts of the bank required for resetting the accounts to their theoretical target levels (3) corresponding to the updated portfolio.

In an asymmetric setup with a price maker and a price taker, the price maker passes his costs to the price taker. For transactions between dealers, it is possible that one is the price maker and the other one is the price taker. It is also possible that a transaction triggers gains for the shareholders of both entities. The detailed consideration of these dynamics would lead to an understanding of the drivers to economical equilibrium in a situation where multiple dealers are present (as opposed to our setup where only one market maker bank is considered). ∎

The funds transfer price of a deal is the all-inclusive XVA add-on charged by the bank to the client in the form of a rebate with respect to the mark-to-market $\Delta$MtM of the deal. Under the above scheme, the overall price charged to the client for the deal is $\Delta\mathrm{MtM}-\Delta\mathrm{CA}-\Delta\mathrm{KVA}$, i.e.

$$\begin{aligned}\mathrm{FTP} &= \Delta\mathrm{CA}+\Delta\mathrm{KVA}=\Delta\mathrm{CVA}+\Delta\mathrm{FVA}+\Delta\mathrm{KVA}\\ &=\Delta\mathrm{FV}+\Delta\mathrm{CL}+\Delta\mathrm{KVA},\end{aligned}\tag{40}$$

by (15) and the last line in (22) (which still hold in continuous time, see Albanese and Crépey (2019, (5) and (16))) applied to the portfolios with and without the new deal.

**Remark 4.4** As opposed to the $\Delta$XVA terms, which all entail portfolio-wide computations, $\Delta$MtM reduces to the so-called clean valuation of the new deal, by trade-additivity of MtM (as follows from Equations (41) and (29) in Albanese and Crépey (2019)). ∎

Obviously, the legacy portfolio of the bank has a key impact on the FTP. It may very well happen that the new deal is risk-reducing with respect to the portfolio, in which case $\mathrm{FTP}<0$, i.e. the overall, XVA-inclusive price charged by the bank to the client would be $\Delta\mathrm{MtM}-\mathrm{FTP}>\Delta\mathrm{MtM}$ (subject of course to the commercial attitude adopted by the bank under such circumstance).

In order to exclude for simplicity jumps of our $L$ and KVA processes at $\theta$ (the ones related to the initial portfolio, but also those, starting at time $\theta$, corresponding to the augmented portfolio), we assume a quasi-left continuous model filtration $\mathbb{G}$ and a $\mathbb{G}$ predictable stopping time $\theta$. The first assumption excludes that martingales can jump at predictable times; It is satisfied in all practical models and, in particular, in all models with Lévy or Markov chain driven jumps. The second assumption is reasonable regarding the time at which a financial contract is concluded. Note that it was actually already assumed regarding the (fixed) time 0 at which the portfolio of the bank is supposed to have been set up in the first place.

**Lemma 4.2** *Assuming the new trade at time $\theta$ handled by the trade incremental policy of Definition 4.1 after the balance conditions* (3) *have been held before $\theta$, then shareholder equity (i.e. wealth)* $\mathrm{SHC}^{\circ}$ *(see Remark 4.2) is a $\mathbb{Q}^*$ submartingale on $[0,\theta]\cap\mathbb{R}_+$, with drift coefficient* $h\mathrm{SCR}$ *killed at $\tau$.*

***Proof.*** In the case of a trade incremental portfolio, in principle, the second identity in (39) is only guaranteed to hold *before* $\theta$. However, in view of the observation made in Remark 4.3 and because, under our (harmless) technical assumptions, there can be no dividends arising

[5] i.e. removes $(-\Delta\mathrm{KVA})$ from, if $\Delta\mathrm{KVA}<0$.

from the portfolio expanded with the new deal (i.e. jumps in the related processes $L$ and KVA, defined on $[\theta, +\infty)$) at time $\theta$ itself, the process SHC does not jump at $\theta$. The process $L$ and KVA related to the legacy portfolio cannot jump at $\theta$ either. As a result, the second identity in (39) still holds at $\theta$. It is therefore valid on $[0, \theta] \cap \mathbb{R}_+$. The result then follows from the respective martingale and supermartingale properties of the (original) processes $L^\circ$ and $\mathrm{KVA}^\circ$ recalled before and after Proposition 4.1. ∎

The above XVA strategy can be iterated between and throughout every new trades. We call this approach the ***trade incremental cost-of-capital XVA strategy***. By an iterated application of Lemma 4.2 at every new trade, we obtain the following:

**Proposition 4.2** *Under a dynamic and trade incremental cost-of-capital XVA strategy, shareholder equity (i.e. wealth)* $\mathrm{SHC}^\circ$ *is a* $\mathbb{Q}^*$ *submartingale on* $\mathbb{R}_+$*, with drift coefficient* $h\mathrm{SCR}$ *killed at* $\tau$. ∎

Thus, a trade incremental cost-of-capital XVA strategy results in a sustainable strategy for profits retention, both between and throughout deals, which was already the key principle behind Solvency II (see Section 1.1). Note that, without the KVA (i.e. for $h = 0$), the (risk-free discounted) shareholder wealth process $\mathrm{SHC}^\circ$ would only be a risk-neutral martingale, which could only be acceptable to shareholders without risk aversion (cf. Section 3.3).

## 4.3 Computational challenges

Figure 2 yields a picturesque representation, in the form of a corresponding XVA dependence tree, of the continuous-time XVA equations.

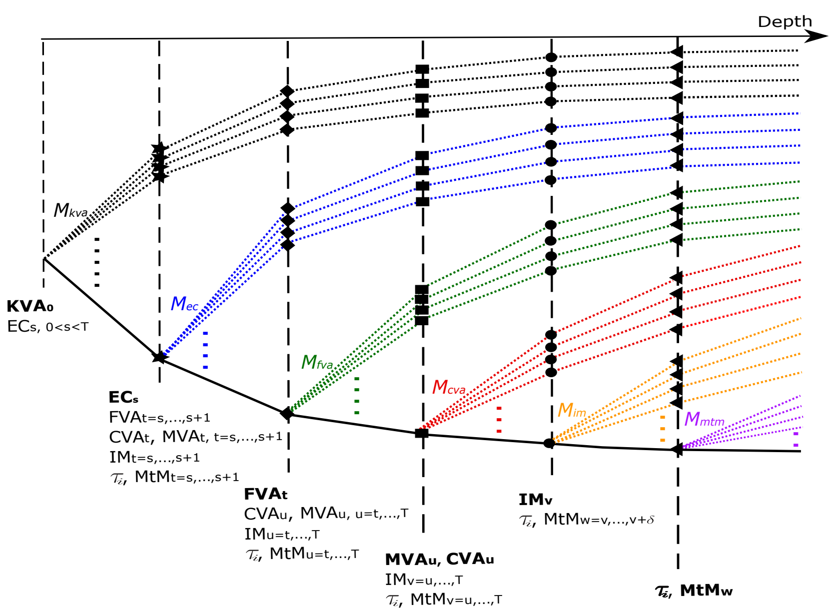


Figure 2: The XVA equations dependence tree (*Source*: Abbas-Turki, Diallo, and Crépey (2018)).

For concreteness, we restrict ourselves to the case of bilateral trading in what follows, referring the reader to Armenti and Crépey (2019, Section 6.2) for the more general (realistic) situation of a bank also involved into centrally cleared trading. As visible from the corresponding equations in Section A, the CVA of the bank can then be computed as the sum of its CVAs restricted to each netting set (or client $i$ of the bank, with default time denoted by $\tau_i$ in Figure 2). The (initial margins and) the MVA are also most accurately calculated at each netting set level. By contrast, the FVA is defined in terms of a semilinear equation that can only be solved at the level of the overall client portfolio of the bank. The KVA can only be computed at the level of the overall portfolio and relies on conditional risk measures of future fluctuations of the shareholder trading loss process $L^{\circ}$, which itself involves future fluctuations of the other XVA processes (as these are part of the bank liabilities).

Moreover, the fungibility of capital at risk with variation margin induces a coupling between, on the one hand, the "backward" FVA and KVA processes and, on the other hand, the "forward" shareholder loss process $L^{\circ}$. As in the static case of Section 3.4 (cf. the last paragraph there), the ensuing forward backward system needs be decoupled by Picard iteration.

These are heavy computations encompassing all the derivative contracts of the bank. Yet these computations require accuracy so that trade incremental XVA computations, which are required as XVA add-ons to derivative entry prices (cf. Section 4.2), are not in the numerical noise of the machinery.

As developed in Abbas-Turki, Diallo, and Crépey (2018), computational strategies for (each Picard iteration of) the XVA equations involve a mix of nested Monte Carlo and of simulation/regression schemes, optimally implemented on GPUs. In view of Figure 2, a pure nested Monte Carlo approach would involve five nested layers of simulation (with respective numbers of paths $M_{xva} \sim \sqrt{M_{mtm}}$). Moreover, nested Monte Carlo implies intensive repricing of the mark-to-market cube, i.e. pathwise MtM valuation for each netting set, or/and high dimensional interpolation. In this work, we use no nested Monte Carlo or conditional repricing of future MtM cubes: each successive layer beyond the base MtM layer in the XVA dependence tree (from right to left in Figure 2, at each Picard iteration) is "learned" instead.

## 4.4 Deep (Quantile) Regression XVA Framework

We denote by $\mathbb{E}_t$, $\mathbb{V}\mathrm{a}\mathbb{R}_t$, and $\mathbb{ES}_t$ (and simply, in case $t = 0$, $\mathbb{E}$, $\mathbb{V}\mathrm{a}\mathbb{R}$, and $\mathbb{ES}$) the time-$t$ conditional expectation, value-at-risk, and expected shortfall with respect to the bank survival measure $\mathbb{Q}$.

We compute the mark-to-market cube using CUDA routines. The pathwise XVAs are obtained by deep learning regression, i.e. extension of Longstaff and Schwartz (2001) kind of schemes to deep neural network regression bases as also considered in Huré, Pham, and Warin (2019) or Beck, Becker, Cheridito, Jentzen, and Neufeld (2019). The conditional value-at-risks and expected shortfalls involved in the embedded pathwise EC and IM computations are obtained by deep quantile regression, as follows.

Given features $X$ and labels $Y$ (random variables), we want to compute the conditional value-at-risk and expected shortfall functions $q(\cdot)$ and $e(\cdot)$ such that $\mathbb{V}\mathrm{a}\mathbb{R}(Y|X) = q(X)$ and $\mathbb{ES}(Y|X) = e(X)$. Recall from Fissler, Ziegel, and Gneiting (2016) and Fissler and Ziegel (2016) that value-at-risk is *elicitable*, expected shortfall is not, but their pair is *jointly elicitable*. Specifically, we consider loss functions $\rho$ of the form

$$\begin{aligned}\rho(q(\cdot), e(\cdot); X, Y) = (\mathbb{1}_{\{Y<q(X)\}} - \alpha) f(q(X)) - (\mathbb{1}_{\{Y<q(X)\}}) f(Y) + \\ \dot{g}(e(X))(e(X) - q(X) + \alpha^{-1}(q(X) - Y))\mathbb{1}_{\{Y<q(X)\}} - g(e(X)).\end{aligned}$$

One can show (cf. also Dimitriadis and Bayer (2019)) that, for a suitable choice of the functions $f$, $g$ including $f(z) = z$ and $g = \ln(1 + e^z)$ (our choice in our numerics), the pair of the conditional value-at-risk and expected shortfall functions is the minimizer, over all measurable pair-functions $(q(\cdot), e(\cdot))$, of the error

$$\mathbb{E}\rho(q(\cdot), e(\cdot); X, Y). \tag{41}$$

In practice, one minimizes numerically the error (41), based on $m$ independent simulated values of $(X, Y)$, over a parametrized family of functions $(q, e)(x) \equiv (q, e)_\theta(x)$. Dimitriadis and Bayer (2019) restrict themselves to multilinear functions. In our case we use a feedforward neural network parameterization (see e.g. Goodfellow, Bengio, and Courville (2017)). The minimizing pair $(q, e)_{\widehat{\theta}}$ then represents the two scalar neural network approximation of the conditional value-at-risk and expected shortfall functions pair.

The left and right panels of Figure 3 show the respective deep neural networks for pathwise value-at-risk/expected shortfall (with error (41)) and pathwise XVAs (with classical quadratic norm error). Deep learning methods typically show “unreasonably good” generalization and scalability performances (cf. Section 5.4). In the case of conditional value-at-risk and expected shorfall computations, deep learning quantile regression is also easier to implement than more naive methods. Compare for instance with the resimulation and sort-based scheme of Barrera, Crépey, Diallo, Fort, Gobet, and Stazhynski (2019) for the value-at-risk and expected shorfall at each outer node of a nested Monte Carlo simulation.

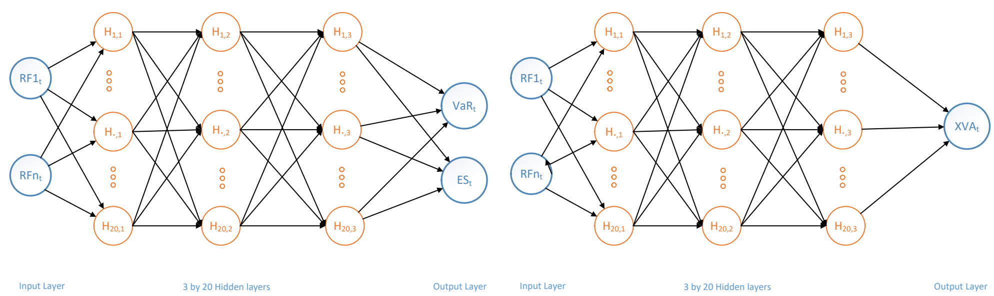


Figure 3: Neural networks with state variables (realizations of the risk factors at the considered pricing time) as features, 100 epochs, full batch training, 3 by 20 hidden nodes. *(Left)* Joint value-at-risk/expected shortfall neural network: output is joint estimate of pathwise conditional value-at-risk and expected shorfall, at a selected confidence level, of the label (inputs to initial margin or economic capital) given the features; Hyperbolic tangent activation. *(Right)* XVAs neural network: output is estimate of pathwise conditional mean of the label (XVA generating cash flows) given the features at selected alpha level; ReLU activation.

Algorithm 1 yields our fully discrete (time and space) scheme for simulating the Picard iteration (52) until numerical convergence to the XVA processes. Note that, as opposed to more rudimentary, expected exposure based XVA computational approaches (see Section 1 in Abbas-Turki, Diallo, and Crépey (2018)), this algorithm requires the simulation of the client defaults.

# 5 Swap Portfolio Case Study

We consider an interest rate swap portfolio case study with 10 clients in different economies, involving 10 one-factor Hull White interest-rates, 9 Black-Scholes exchange rates, and 11 Cox-Ingersoll-Ross default intensity processes. The default times of the clients and the bank itself are

**Algorithm 1** Deep XVAs algorithm.

- Simulate forward $m$ realizations (Euler paths) of the market risk factor processes and of the client survival indicator processes (i.e. default times) on a refined time grid;
- For each pricing time $t = t_i$ (of a pricing time grid with coarser step $h$) and client $c$:
  - Learn the corresponding $\mathbb{V}\mathrm{a}\mathbb{R}_t$ and $\mathbb{ES}_t$ terms visible in (53) or (under the time-discretized outer integral in) (55);
  - Learn the corresponding $\mathbb{E}_t$ terms visible in (54) through (56);
  - Compute the ensuing pathwise CVA and MVA as per (54)–(56);
- For $\mathrm{FVA}^{(0)}$, consider the following time discretization of (51) with time step $h$:

$$\mathrm{FVA}^{(0)}_t \approx \mathbb{E}_t[\beta_t\beta^{-1}_{t+h}\mathrm{FVA}^{(0)}_{t+h}] + h\lambda_t\Big(\sum_c J^c_t(P^c_t - \mathrm{VM}^c_t) - \mathrm{CA}^{(0)}_{\mathrm{t}}\Big)^+ \tag{42}$$

  and, for each $t = t_i$, learn the corresponding $\mathbb{E}_t$ in (42), then solve the semi-linear equation for $\mathrm{FVA}^{(0)}_t$ (recall $\mathrm{CA}^{(0)}_{\mathrm{t}}$ includes $\mathrm{FVA}^{(0)}_t$);
- For each Picard iteration $k$ (until numerical convergence), simulate forward $L^{(k)}$ as per the first line in (52) (which only uses known or already learned quantities), and:
  - For economic capital $\mathrm{EC}^{(k)}$, for each $t = t_i$, learn $\mathbb{ES}_t\big((L^{(k)})^\circ_{t_i+1} - (L^{(k)})^\circ_{t_i}\big)$ (cf. Definition A.1);
  - $\mathrm{KVA}^{(k)}$ and $\mathrm{FVA}^{(k)}$ then require a backward recursion solved by deep learning approximation much like the one for $\mathrm{FVA}^{(0)}$ above.

---

jointly modeled by a "common shock" or dynamic Marshall-Olkin copula model as per Crépey, Bielecki, and Brigo (2014, Chapt. 8–10) and Crépey and Song (2016) (see also Elouerkhaoui (2007, 2017)). This whole setup results in up to 40 risk factors used as deep learning features (including the client default indicators).

In this model we consider a bank portfolio of 10,000 randomly generated swap trades, with

- swap rates uniformly distributed on $[0.005, 0.05]$,
- number of six-monthly coupon resets uniform on $[5 \dots 60]$,
- trade currency and counterparty both uniform on $[1, 2, 3 \dots, 10]$,
- notional uniform on $[10000, 20000, \dots, 100000]$,
- direction: either "asset heavy" bank 75% likely to pay fixed in the swaps, or "liability-heavy" bank 75% likely to receive fixed,
- collateralization (cf. Section A.4): either "no CSA portfolio" without initial margin (IM) nor variation margin (VM), or "CSA portfolio" with VM = MtM and posted initial margin (PIM) pledged at 99% gap risk value-at-risk, received initial margin (RIM) covering 75% gap risk and leaving excess as residual gap CVA,
- for economic capital, 97.5% expected shortfall of 1-year ahead trading loss of the bank shareholders.

We use Monte Carlo simulation with with 50K paths of 8 coarse (pricing) and 32 fine (risk factors) time steps per year. The figures that follow only display profiles, i.e. term structures, that is, expectations as a function of time of the corresponding processes. But all these processes are computed pathwise, based on the deep learning regression and quantile regression methodology of Section 4.4, allowing for all XVA inter-dependencies. XVA profiles (or pathwise XVAs if wished) are of course much more informative for traders than the spot XVA values (or time 0 confidence intervals) returned by most XVA systems.

## 5.1 Validation Results

Figure 4 is a sanity check that the deeply regressed CVA in one year, $\mathrm{CVA}_1^c$ (here in the case of a no CSA netting set $c$), is consistent with the output of a nested Monte Carlo. In fact, the design of our neural network architecture is guided by $k$-fold cross-validation and benchmarking with nested Monte Carlo.

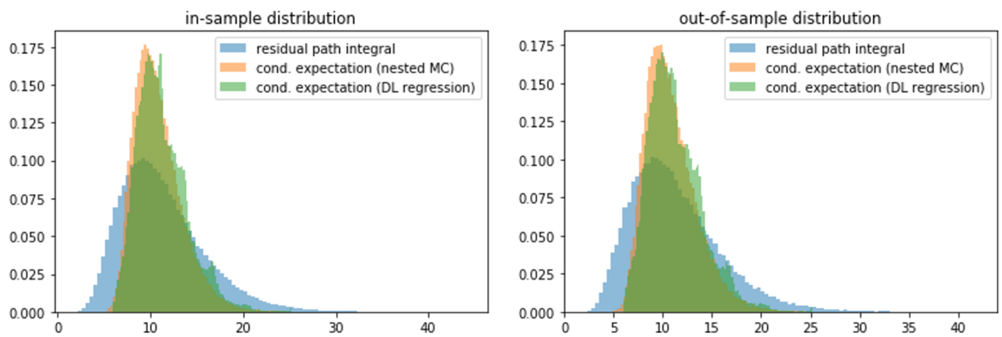


Figure 4: Random variable $\mathrm{CVA}_1^c$ (in the case of a no CSA netting set $c$) obtained by deep regression (green histogram) versus nested Monte Carlo (orange histogram) applied to the corresponding integrands (blue histogram) visible in (54). *(Left)*: in-sample distributions; *(Right)*: out-of-sample distributions.

Figure 5 *(left)* is a further sanity check that the profiles of the successives iterates $L^{(k)}$ of the shareholder trading loss process $L^\circ$ converge rapidly with $k$, with the profile for $k = 3$ already visually indistinguishable from the one for $k = 2$ (see before Section 5). Figure 5 *(right)* shows the loss process $L^{(3)}$, displayed as its mean and mean $\pm$ 2 stdev profiles. Consistent with its martingale property, the loss process $L^{(3)}$ appears numerically centered around zero. The latter holds, at least, beyond $t \sim 5$ years. For earlier times, the regression errors, accumulated backward across pricing times since the final maturity of the portfolio, induce a non negligible bias (the corresponding confidence intervals no longer contains 0). This is the reason why we use a coarser pricing time step than simulation time step (see Algorithm 1).

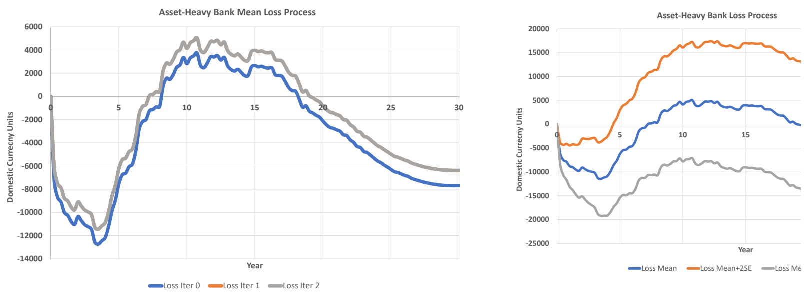


Figure 5: *(Left)* Profiles of the processes $L^{(k)}$, for $k = 1, 2, 3$; *(Right)* Mean $\pm$ 2 stdev profiles of the process $L^{(3)}$.

Figure 6 *(left)* shows out-of-samples error profiles related to the deep learning of pathwise CVAs for five of the counterparties, with corresponding CVA integrands (labels) denoted by $I_t$ (each curve stops at the final maturity of the corresponding client portfolio). The CVA error profiles reveal more difficulty in learning the earlier CVAs. This is because of a higher variance of the corresponding cash flows (integrated over longer time frames) in conjunction with a lower variance of the features (risk factors diffused over shorter time horizons). Figure 6 *(right)* shows the learned FVA profiles. The blue FVA curve represents the mean FVA originating cash flows, which, in principle as on the picture, should match the orange mean FVA itself learned from these cash flows. The 5th and 95th percentiles FVA estimates are a bit less smooth in time then the mean profiles, as expected.

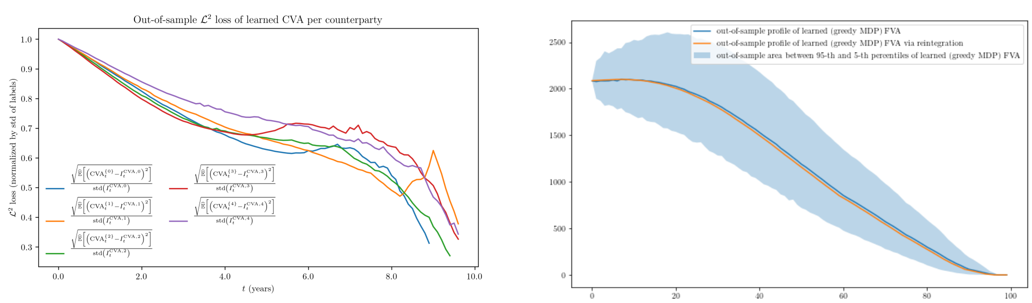


Figure 6: Out-of-sample learning performance. *(Left)* CVA learning error profile. *(Right)* Learned FVA.

## 5.2 Portfolio-Wide XVA Profiles

Figure 7 shows the GPU generated profiles of $\mathrm{MtM} = \sum_c P^c \mathbb{1}_{[0,\tau_c^\delta)}$ in the case of the asset-heavy portfolio and of the liability-heavy portfolio.

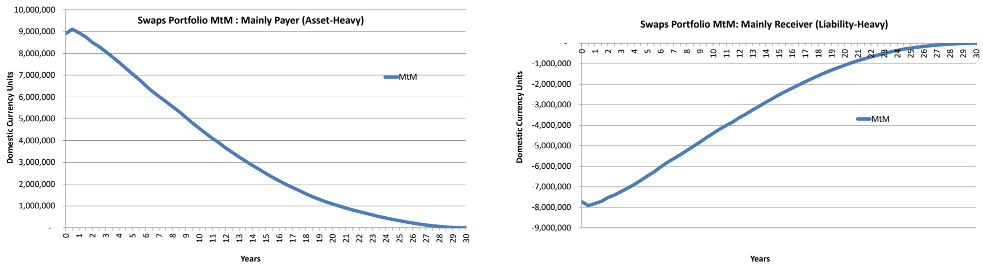


Figure 7: MtM profiles. *(Left)* Asset-heavy portfolio. *(Right)* Liability-heavy portfolio.

Figure 8 shows the porftolio-wide XVA profiles of the asset-heavy *(top)* vs. liability–heavy *(bottom)* portfolio and of the no CSA *(left)* vs. CSA portfolio *(right)*. Obviously, asset–heavy or no CSA means more CVA. The correponding curves also emphasize the transfer from counterparty credit into liquidity funding risk prompted by extensive collateralisation. Yet FVA/MVA risk is ignored in current derivatives capital regulation.

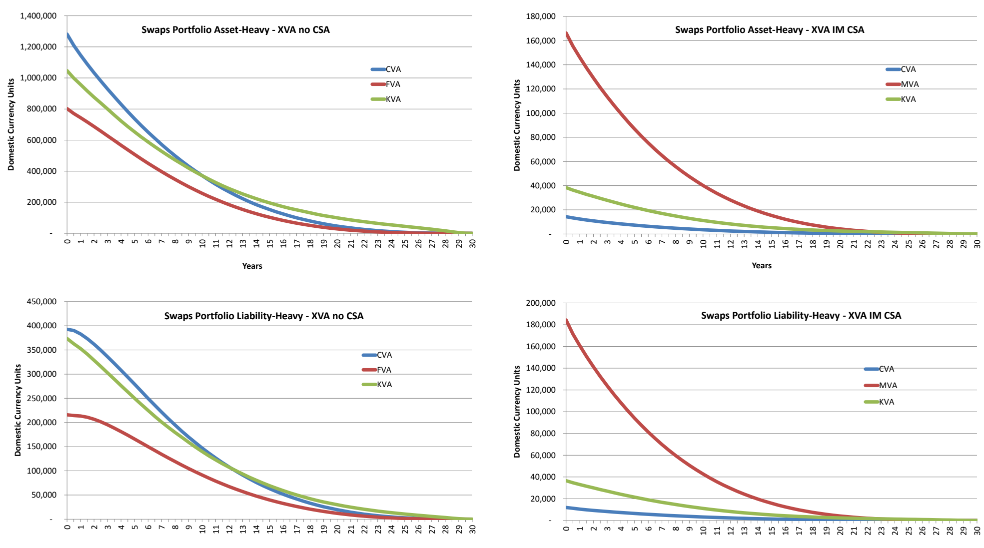


Figure 8: *(Top left)* Asset-heavy portfolio, no CSA. *(Top right)* Asset-heavy portfolio under CSA. *(Bottom left)* Liability–heavy portfolio, no CSA. *(Bottom right)* Liability-heavy portfolio under CSA.

Figure 9 shows that *(top left)* capital at risk as funding (cf. Section 3.4) has a material impact on the already (reserve capital as funding) reduced FVA, *(top right)* treating KVA as a risk margin (cf. (25)) gives a huge discounting impact, *(bottom left)* deep learning detects material initial margin convexity in the asset-heavy CSA portfolio, and *(bottom right)* deep learning detects material economic capital convexity in the asset-heavy no CSA portfolio.

The above findings demonstrate the necessity of pathwise capital and margin calculations for accurate FVA, MVA, and KVA calculations.

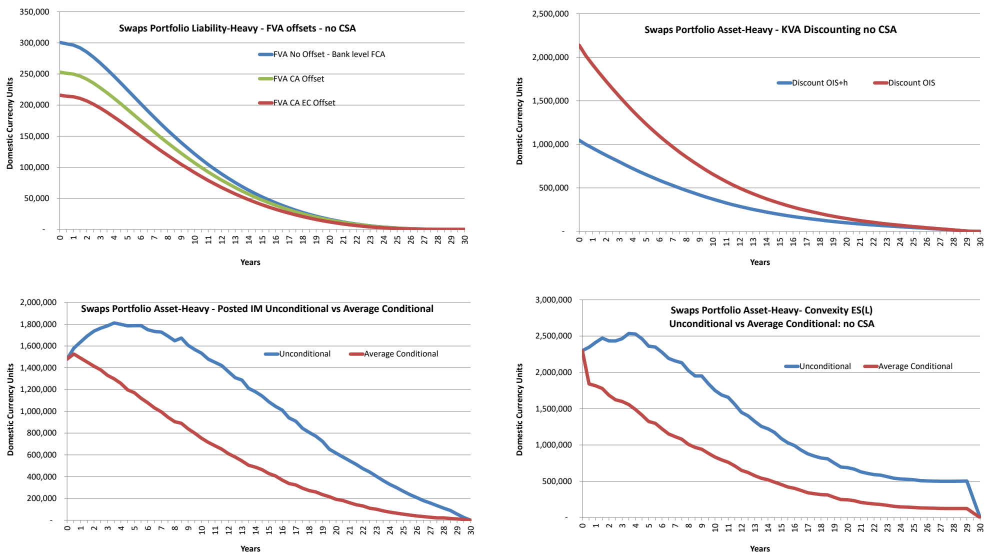


Figure 9: *(Top left)* FVA ignoring the off-setting impact of reserve capital and capital at risk, cf. Section 3.4 (blue), FVA as per (51) accounting for the off-setting impact of reserve capital but ignoring the one of capital at risk (green), refined FVA as per (46) accounting for both impacts (red). *(Top right)* KVA ignoring the off-setting impact of the risk margin, i.e. with CR instead of $(\mathrm{CR} - \mathrm{KVA})$ in (50) (red), refined KVA as per (48)–(49) (blue). *(Bottom left)* In the case of the asset-heavy portfolio under CSA, unconditional PIM profile, i.e. with $\mathbb{V}\mathrm{a}\mathbb{R}_t$ replaced by $\mathbb{V}\mathrm{a}\mathbb{R}$ in (53) (blue), vs. pathwise PIM profile, i.e. mean of the pathwise PIM process as per (53) (red). *(Bottom right)* In the asset-heavy portfolio no CSA case, unconditional economic capital profile, i.e. EC profile ignoring the words "time-$t$ conditional" in Definition A.1 (blue), vs. pathwise economic capital profile, i.e. mean of the pathwise EC process as per Definition A.1 (red).

## 5.3 Trade Incremental XVA Profiles

Next, we consider, on top of the previous portfolios, an incremental trade given as a par 30 year (receive fix or pay fix) swap with 100k notional. Figure 10 shows the trade incremental XVA profiles produced by our deep learning approach. Note that, for obtaining such smooth incremental profiles, it has been key to use common random numbers, as much as possible, between the original portfolio XVA computations and the ones regarding the portfolio expanded with the new trade.

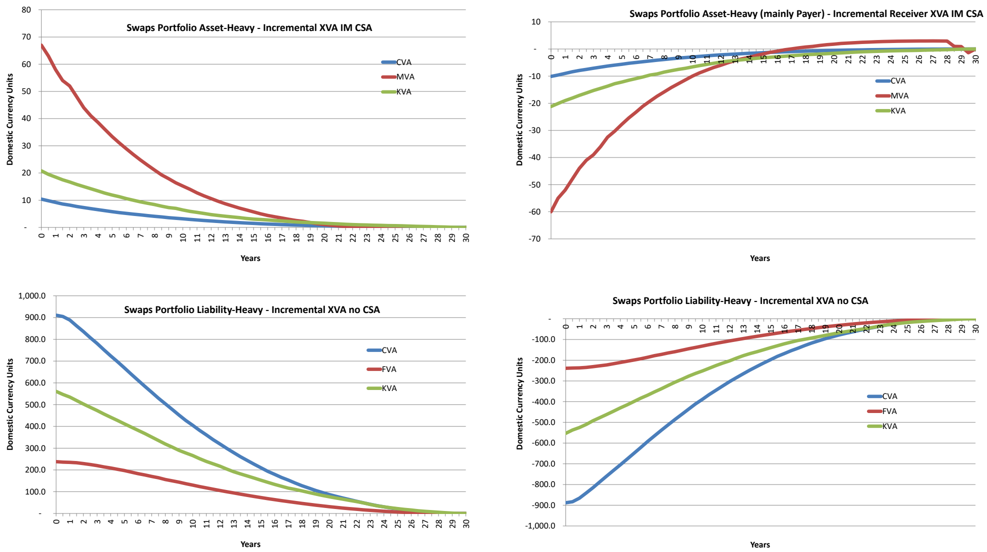


Figure 10: *(Top left)* Asset-heavy portfolio, no CSA. Incremental receive fix trade. *(Top right)* Liability-heavy portfolio, no CSA. Incremental pay fix trade. *(Bottom left)* Asset-heavy portfolio under CSA. Incremental Pay Fix Trade. *(Bottom right)* Liability-heavy portfolio under CSA. Incremental receive fix trade.

Our model assumes the market risk of trades to be fully hedged (see the next-to-last paragraph before Section 2.1 and the proof of Lemma 3.2). We can also assess the XVA on the market risk hedges by including the relevant bilateral hedge counterparties in the model (we refer the reader to Armenti and Crépey (2019) for the case of centrally cleared hedges). In what follows, we consider:

- 10 counterparties: 8 no CSA and 2 bilateral VM/IM CSA hedge counterparties,
- portfolios of 5,000 randomly generated swap trades as before, plus 5,000 corresponding hedge trades,
- incremental trade given as a par 30 year swap with 100k notional, along with the corresponding hedge trade.

The 8 no CSA counterparties are primarily asset or liability heavy. One bilateral CSA hedge counterparty is asset-heavy and one liability-heavy. Figure 11 provides the trade incremental XVA profiles of the bilateral hedge alternatives in combination with those for the initial counterparty trade. The main XVA impact of the hedge is then a corresponding incremental MVA term, which can contribute to make the global FTP related to the trade+hedge package more or less positive or negative, depending on the data (cf. the four panels in Figure 11), as can only be inferred by a refined XVA computation.

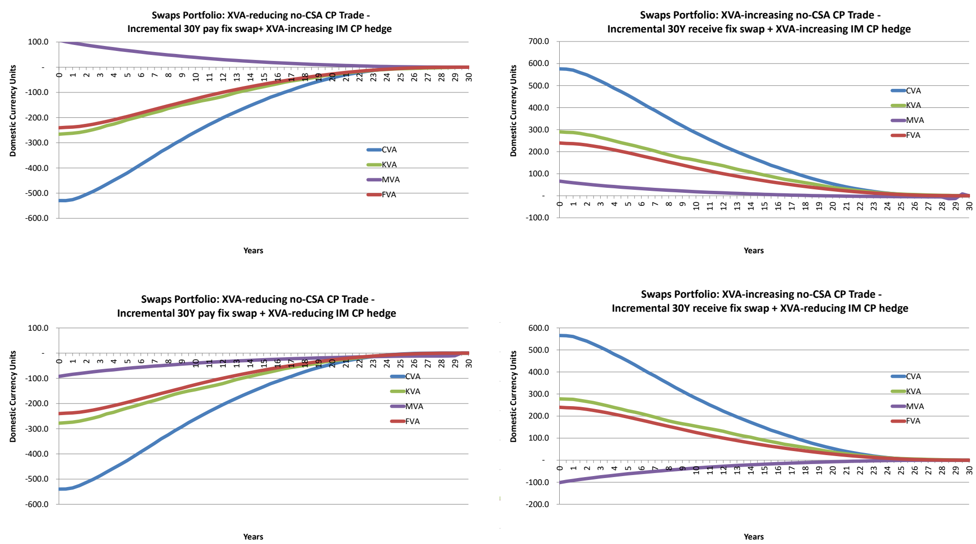


Figure 11: *(Top left)* XVA-reducing trade + XVA-increasing bilateral hedge *(Top right)* XVA-increasing trade + XVA-increasing bilateral hedge. *(Bottom left)* XVA-reducing trade + xva-reducing bilateral hedge *(Bottom right)* XVA-increasing trade + XVA-reducing bilateral hedge.

## 5.4 Scalability

Our deep learning XVA implementation uses CNTK, the Microsoft Cognitive Toolkit. CNTK is written in core C++/CUDA (with wrappers for Python, C#, and Java). This is key for XVA applications, which are usually developed in C++: CNTK automatic differentiation in C++/CUDA means C++ training, as opposed to automatic differentiation in Python (hence no C++ training) with Tensorflow. CNTK allows embedding the deep learning task within XVA processing, hence leveraging GPU: our ensuing deep learning calculations achieve around 80 to 90% Cuda occupancy; the GPU acceleration factor is of the order of 15.

Additional results obtained by doubling the numbers of counterparties and risk factors (to 20 counterparties and 80 risk factors) indicate that scaling to realistically high dimension is achievable. The timings displayed in Table 2 are taken on Lenovo P52 laptop with NVidia Quadro P3200 GPU @ 5.5 Teraflops peak FP32 performance, and 14 streaming multiprocessors. NVidia Tesla V100 should be at least 6-10 times faster. Cache the computations and scale over multiple V100s for acceptable trade incremental pricing performance in production.

| Computation Time (seconds) | | | | |
|---|---|---|---|---|
| | 10 clients 40 risk factors | | 20 clients 80 risk factors | |
| | No CSA | CSA | No CSA | CSA |
| Initial Risk Factor &Trade Pricing Simulation Cuda | 118.0 | 118.0 | 134.0 | 134.0 |
| Counterparty Learning Calculations | 386.7 | 1,890.0 | 435.8 | 2,211.1 |
| Bank Level Learning Calculations | 525.3 | 156.0 | 687.4 | 229.9 |
| Total Initial Batch | 1,030.0 | 2,164.0 | 1,257.2 | 2,574.9 |
| | | | | |
| Re-simulate 1 counterparty trade pricing Cuda | 19.0 | 19.0 | 25.0 | 25.0 |
| Counterparty Learning Calculations | 38.7 | 189.0 | 43.6 | 221.1 |
| Bank Level Learning Calculations | 525.3 | 156.0 | 687.4 | 229.9 |
| Total Incremental Trade | 583.0 | 364.0 | 756.0 | 476.0 |

Table 2: GPU simulation + CNTK learning indicative timings

# A Continuous-Time XVA Equations

We recall from Crépey, Élie, Sabbagh, and Song (2019) the continuous-time XVA equations for bilateral trade portfolios when capital at risk is deemed fungible with variation margin, also adding here initial margin and MVA as in the refined static setup of Section 3.4.

We write $\boldsymbol{\delta}_\eta(dt) = d\mathbb{1}_{\{\eta \le t\}}$ for the Dirac measure at a random time $\eta$.

## A.1 Cash Flows

We suppose that the client derivative portfolio of the bank is partitioned into bilateral netting sets of contracts which are jointly collateralized and liquidated upon clients or bank default. Given a netting set $c$ of the client portfolio, we denote by:

- $\mathcal{P}^c$ and $P^c$, the corresponding contractually promised cash flows and clean value processes;
- $\tau_c$, $J^c$, and $R_c$, the corresponding default times, survival indicators, and recovery rates, whereas $\tau$, $J$, and $R$ are the analogous data regarding the bank itself, with bank credit spread process $\lambda$ taken as a proxy of its risky funding spread process[6];
- $\tau_c^\delta = \tau_c + \delta$ and $\tau^\delta = \tau + \delta$, where $\delta$ is a positive margin period of risk, in the sense that the liquidation of the netting set $c$ happens at time $\tau_c^\delta \wedge \tau^\delta$;
- $\mathrm{VM}^c$, the variation margin (re-hypotecable collateral) exchanged between the bank and client $c$, counted positively when received by the bank;
- $\mathrm{PIM}^c$ and $\mathrm{RIM}^c$, the related initial margin (segregated collateral) posted and received by the bank;
- RC and CR, the reserve capital and capital at risk of the bank (see Definition 2.1).

The contractually promised cash flows are supposed to be hedged out by the bank but one conservatively assumes no XVA hedge, so that the bank is left with the following trading cash flows $\mathcal{C}$ and $\mathcal{F}$ (cf. (35) and see Albanese and Crépey (2019, Lemmas 5.2 and 5.3) for detailed derivations of analogous equations in a slightly simplified setup):

[6] See Armenti and Crépey (2019, Section 5) for the discussion of cheaper funding schemes for initial margin.

- The (counterparty) credit cash flows

$$
\begin{aligned}
&d\mathcal{C}_t = \\
&\sum_{c;\tau_c\leq\tau^\delta}(1-R_c)\Big((P^c+\mathcal{P}^c)_{\tau_c^\delta\wedge\tau^\delta}-(\mathcal{P}^c+\mathrm{VM}^c+\mathrm{RIM}^c)_{(\tau_c\wedge\tau)-}\Big)^+\boldsymbol{\delta}_{\tau_c^\delta\wedge\tau^\delta}(dt)\\
&-(1-R)\sum_{c;\tau\leq\tau_c^\delta}\Big((P^c+\mathcal{P}^c)_{\tau^\delta\wedge\tau_c^\delta}-(\mathcal{P}^c-\mathrm{VM}^c+\mathrm{PIM}^c)_{(\tau\wedge\tau_c)-}\Big)^-\boldsymbol{\delta}_{\tau^\delta\wedge\tau_c^\delta}(dt);
\end{aligned}
\tag{43}
$$

- The (risky) funding cash flows

$$
\begin{aligned}
d\mathcal{F}_t &= J_t\lambda_t\Big(\sum_c J^c(P^c-\mathrm{VM}^c)-\mathrm{RC}{-}\mathrm{CR}\Big)_t^+dt\\
&-(1-R)\Big(\sum_c J^c(P^c-\mathrm{VM}^c)-\mathrm{RC}{-}\mathrm{CR}\Big)_{\tau-}^+\boldsymbol{\delta}_\tau(dt)\\
&+J_t\lambda_t\sum_c J_t^c\mathrm{PIM}_t^c dt-(1-R)\sum_c J_{\tau-}^c\mathrm{PIM}_{\tau-}^c\boldsymbol{\delta}_\tau(dt),
\end{aligned}
\tag{44}
$$

  where the RC and CR terms account for the fungibility of reserve capital and capital at risk with variation margin.

## A.2 Valuation

We distinguish between a (strict) FVA, in the strict sense of the cost of raising variation margin, and an MVA for the cost of raising initial margin (see Remark 2.1). The (other than K)VA equations are then

$$
\mathrm{RC}=\mathrm{CA}=\mathrm{CVA}+\mathrm{FVA}+\mathrm{MVA}, \tag{45}
$$

the so-called "contra-assets valuation" sourced from the clients and deposited in the reserve capital account of the bank, where, for $t<\tau$,

$$
\begin{aligned}
\mathrm{CVA}_t &= \mathbb{E}_t\sum_{t<\tau_c^\delta}(1-R_c)\Big((P^c+\mathcal{P}^c)_{\tau_c^\delta}-(\mathcal{P}^c+\mathrm{VM}^c+\mathrm{RIM}^c)_{\tau_c-}\Big)^+\\
\mathrm{FVA}_t &= \mathbb{E}_t\int_t^T\lambda_s\Big(\sum_c J^c(P^c-\mathrm{VM}^c)-\mathrm{CA}-\mathrm{CR}\Big)_s^+ds\\
\mathrm{MVA}_t &= \mathbb{E}_t\int_t^T\lambda_s\sum_c J_s^c\mathrm{PIM}_s^c ds.
\end{aligned}
\tag{46}
$$

The corresponding trading loss and profit process $L$ of the bank is such that

$$
\begin{aligned}
&L_0=0 \text{ and, for } t<\tau,\\
&dL_t=\sum_c(1-R_c)\Big((P^c+\mathcal{P}^c)_{\tau_c^\delta}-(\mathcal{P}^c+\mathrm{VM}^c+\mathrm{RIM}^c)_{\tau_c-}\Big)^+\boldsymbol{\delta}_{\tau_c^\delta}(dt)\\
&+\lambda_t\Big(\sum_c J^c(P^c-\mathrm{VM}^c)-\mathrm{CA}-\mathrm{CR}\Big)_t^+dt\\
&+\lambda_t\sum_c J_t^c\mathrm{PIM}_t^c dt\\
&+d\mathrm{CA}_t,
\end{aligned}
\tag{47}
$$

so that $L$ is a $\mathbb{Q}$ martingale, hence (by Lemma 4.1) $L^\circ$ is a $\mathbb{Q}^*$ martingale.

By the same rationale as Definitions 3.2 and 3.3 in the static setup:

**Definition A.1** $\mathrm{EC}_t$ is the time-$t$ conditional 97.5% expected shortfall of $(L^\circ_{t+1} - L^\circ_t)$ under $\mathbb{Q}$. ∎

Given a positive target hurdle rate $h$:

**Definition A.2** We set

$$\mathrm{CR} = \max(\mathrm{EC}, \mathrm{KVA}), \tag{48}$$

for a KVA process such that, for $t < \tau$,

$$\mathrm{KVA}_t = \mathbb{E}_t\Big[\int_t^T h\big(\mathrm{CR}_s - \mathrm{KVA}_s\big)ds\Big]. \ \blacksquare \tag{49}$$

That is, for $t < \tau$,

$$\begin{aligned}\mathrm{KVA}_t &= \mathbb{E}_t\Big[\int_t^T he^{-h(s-t)}\mathrm{CR}_s ds\Big] \\ &= \mathbb{E}_t\Big[\int_t^T he^{-h(s-t)}\max(\mathrm{EC}_s, \mathrm{KVA}_s\big)ds\Big].\end{aligned} \tag{50}$$

The next-to-last identity is the continuous-time analog of the risk margin formula under the Swiss solvency test cost of capital methodology: see Swiss Federal Office of Private Insurance (2006, Section 6, middle of page 86 and top of page 88).

## A.3 The XVA Equations are Well-Posed

In view of (45), the second line in (46) is in fact an FVA *equation*. Likewise, the second line in (50) is a KVA equation. Moreover, as capital at risk is fungible with variation margin, i.e. in consideration of the CR term in (46)-(47), where $\mathrm{CR} = \max(\mathrm{EC}, \mathrm{KVA})$, we actually deal with an $(\mathrm{FVA}, \mathrm{KVA})$ *system*, and even, as EC depends on $L$ (cf. Definition A.1), with a forward backward system for the forward loss process $L$ and the backward pair $(\mathrm{FVA}, \mathrm{KVA})$.

However, as in the refined static setup of Section 3.4, the coupling between $(\mathrm{FVA}, \mathrm{KVA})$ and $L$ can be disentangled by the following Picard iteration:

- Let CVA and MVA be as in (46), $L^{(0)} = \mathrm{KVA}^{(0)} = 0$, and , for $t < \tau$,

$$\mathrm{FVA}^{(0)}_t = \mathbb{E}_t \int_t^T \lambda_s\Big(\sum_c J^c(P^c - \mathrm{VM}^c) - \mathrm{CA}^{(0)}\Big)^+_s ds, \tag{51}$$

  where $\mathrm{CA}^{(0)} = \mathrm{CVA} + \mathrm{FVA}^{(0)} + \mathrm{MVA}$;

- For $k \geq 1$, writing explicitly $\mathrm{EC} = \mathrm{EC}(L)$ to emphasize the dependence of EC on $L$, let

$L_0^{(k)} = 0$ and, for $t < \tau$,

$$
\begin{aligned}
dL_t^{(k)} &= \sum_c (1-R_c)\Big((P^c + \mathcal{P}^c)_{\tau_c^\delta} - (\mathcal{P}^c + \mathrm{VM}^c + \mathrm{RIM}^c)_{\tau_c -}\Big)^+ \boldsymbol{\delta}_{\tau_c^\delta}(dt) \\
&\quad + \lambda_t \Big( \sum_c J^c (P^c - \mathrm{VM}^c) - \mathrm{CA}^{(k-1)} - \max\big(\mathrm{EC}(L^{(k-1)}), \mathrm{KVA}^{(k-1)}\big)\Big)_t^+ dt \\
&\quad + \lambda_t \sum_c J_t^c \mathrm{PIM}_t^c dt + d\mathrm{CA}_t^{(k-1)}, \\
\mathrm{KVA}_t^{(k)} &= h \mathbb{E}_t \int_t^T e^{-h(s-t)} \max\big(\mathrm{EC}_s(L^{(k)}), \mathrm{KVA}_s^{(k)}\big) ds, \\
\mathrm{CA}_t^{(k)} &= \mathrm{CVA}_t + \mathrm{FVA}_t^{(k)} + \mathrm{MVA}_t \text{ where } \mathrm{FVA}_t^{(k)} = \\
&\quad \mathbb{E}_t \int_t^T \lambda_s \Big( \sum_c J^c (P^c - \mathrm{VM}^c) - \mathrm{CA}^{(k)} - \max\big(\mathrm{EC}(L^{(k)}), \mathrm{KVA}^{(k)}\big)\Big)_s^+ ds.
\end{aligned}
\tag{52}
$$

**Theorem 4.1 in Crépey, Élie, Sabbagh, and Song (2019)** *Assuming square integrable data, the XVA equations are well-posed within square integrable solution (including when one accounts for the fact that capital at risk can be used for funding variation margin). Moreover, the above Picard iteration converges to the unique square integrable solution of the XVA equations.* ∎

## A.4 Collateralization Schemes

We denote by $\Delta_t^c = \mathcal{P}_t^c - \mathcal{P}_{(t-\delta)-}^c$ the cumulative contractual cash flows with the client $c$ accumulated over a past period of length $\delta$. In our case study, we consider both "no CSA" netting sets $c$, with VM = RIM = PIM = 0, and "(VM/IM) CSA" netting sets $c$, with $\mathrm{VM}_t^c = P_t^c$ and, for $t \le \tau_c$,

$$
\mathrm{RIM}_t^c = \mathbb{V}\mathrm{a}\mathbb{R}_t\Big((P_{t^\delta}^c + \Delta_{t^\delta}^c) - P_t^c\Big),\ \ \mathrm{PIM}_t^c = \mathbb{V}\mathrm{a}\mathbb{R}_t\Big(-(P_{t^\delta}^c + \Delta_{t^\delta}^c) + P_t^c\Big), \tag{53}
$$

for some PIM and RIM quantile levels $a_{pim}$ and $a_{rim}$ (and $t^\delta = t + \delta$).

The following result can be derived by similar computations as the ones in Armenti and Crépey (2019, Section A).

**Proposition A.1** *In a common shock default model of the clients and the bank itself (see the beginning of Section 5), with pre-default intensity processes $\gamma^c$ of the clients and $\gamma$ of the bank, then* $\mathrm{CVA} = \mathrm{CVA}^{nocsa} + \mathrm{CVA}^{csa}$*, where, for $t < \tau$,*

$$
\begin{aligned}
\mathrm{CVA}_t^{nocsa} &= \sum_{c\ nocsa} \mathbb{1}_{t<\tau_c}(1-R_c)\mathbb{E}_t \int_t^T (P_{s^\delta}^c + \Delta_{s^\delta}^c)^+ \gamma_s^c e^{-\int_t^s \gamma_u^c du} ds \\
&\quad + \sum_{c\ nosca} \mathbb{1}_{\tau_c < t < \tau_c^\delta}(1-R_c)\mathbb{E}_t (P_{\tau_c^\delta}^c + \Delta_{\tau_c^\delta}^c)^+,
\end{aligned}
\tag{54}
$$

$$
\begin{aligned}
\mathrm{CVA}_t^{csa} &= \sum_{c\ csa} \mathbb{1}_{t<\tau_c}(1-R_c)(1-a_{rim}) \times \\
&\qquad \mathbb{E}_t \int_t^T (\mathbb{ES}_s - \mathbb{V}\mathrm{a}\mathbb{R}_s)\left((P_{s^\delta}^c + \Delta_{s^\delta}^c) - P_s^c\right) \gamma_s^c e^{-\int_t^s \gamma_u^c du} ds
\end{aligned}
\tag{55}
$$

$$
+ \sum_{c\ csa} \mathbb{1}_{\tau_c < t < \tau_c^\delta}(1-R_c)\mathbb{E}_t\Big((P_{\tau_c^\delta}^c + \Delta_{\tau_c^\delta}^c) - (P_{\tau_c}^c + \mathrm{RIM}_{\tau_c}^c)\Big)^+,
$$

*where* $(\mathbb{ES}_s - \mathbb{VaR}_s)$ *in* (55) *is computed at the* $a_{rim}$ *confidence level. Assuming its posted initial margin borrowed unsecured by the bank, then* $\mathrm{MVA} = \mathrm{MVA}^{csa}$*, where, for* $t < \tau$,

$$\mathrm{MVA}_t^{csa} = \sum_{c\ csa} J_t^c \mathbb{E}_t \int_t^T (1-R)\gamma_s \mathrm{PIM}_s^c e^{-\int_t^s \gamma_u^c du} ds. \blacksquare \tag{56}$$

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