XVA Analysis From the Balance Sheet

Claudio Albanese[1,2] and Stéphane Crépey[3]

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Abstract

In the aftermath of the financial crisis, regulators launched a major effort of banking reform aimed at securing the financial system by raising collateralisation and capital requirements. Notwithstanding finance theories, according to which costs of capital and of funding for collateral are irrelevant to investment decisions, banks have introduced an array of XVA metrics to precisely quantify them. In particular, the KVA (capital valuation adjustment) is emerging as a metric of key relevance.

We introduce a capital structure model acknowledging the impossibility for a bank to replicate jump-to-default related cash flows. Because of this counterparty credit risk incompleteness, deals trigger wealth transfers from bank shareholders to bank creditors and shareholders need to set capital at risk. On this basis we devise a theory of XVAs, whereby so-called contra-liabilities and cost of capital are sourced from bank clients at trade inceptions, on top of the fair valuation of counterparty credit risk, in order to compensate shareholders for wealth transfers and risk on capital.

Keywords: Counterparty credit risk, jump-to-default risk, market incompleteness, capital structure of a bank, wealth transfer, risk margin, cost of capital, capital valuation adjustment (KVA).

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[1] *Global Valuation Ltd, London.*

[2] *CASS School of Business, London.*

[3] *Université d'Évry Val d'Essonne, Laboratoire de Mathématiques et Modélisation d'Évry and UMR CNRS 8071.*

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1 Introduction

In the aftermath of the financial crisis, banking regulators launched a major reform effort aimed at reducing counterparty credit risk by raising collateralisation and capital requirements and by incentivising central clearing. The Basel III Accord has been implemented in law in most jurisdictions and is being followed by a stream of other regulatory reforms such as the fundamental review of the trading book (FRTB, see BCBS (2013)), initial margin for uncleared trades (BCBS 261) and standards to ensure total loss-absorbing capacity beyond equity capital (TLAC). Key to the intent of the regulator is the consideration that, assuming complete markets, the level of capital and funding requirements are irrelevant to all investment decisions, including ask prices for contingent claims.

On a parallel and independent track, the regulatory framework for the insurance industry has also been reformed, but on the basis of a different set of principles. Insurance claims are largely unhedgeable and markets are intrinsically incomplete. In particular, the cost of capital is material and is reflected in entry prices. The Swiss Solvency Test (2004), followed by Solvency II for the EU, focus on regulating dividend distribution policies, so as to ensure a sustainable risk remuneration to the shareholders over the lifetime of the portfolio.

Similar to insurance portfolios, cost of capital also plays a key role for derivative portfolios. In addition, since banks are intrinsically leveraged, costs of funding strategies also matter, whose effect is amplified by the onerous collateralization incentives and requirements in Basel III. The unintended consequence of Basel III is that banks reacted by pricing into contingent claims the effects of market incompleteness through so-called XVA metrics, where VA stands for valuation adjustment and X is a catch-all letter to be replaced by C for credit, D for debt, F for funding, M for margin, K for capital and so on.

Counterparty credit risk incompleteness invalidates several of the conclusions of Modigliani-Miller theory but not all. The purpose of this article is to explain and amend the banking XVA metrics from the point of view of a capital structure model acknowledging the impossibility for a bank to replicate jump-to-default related cash flows. This outlines the foundation for a hybrid between Basel III and Solvency II, extending the treatment of counterparty credit risk in Basel III to incorporate cost of funding and of capital consistent with the principles of Solvency II.

Our approach results in an economically sustainable XVA methodology, in two steps. First, the so-called contra-assets are valued as the expected costs of the counterparty credit risk related expenses, i.e. the expected costs of counterparty default losses and funding expenditures. Second, on top of these expected costs, a KVA risk premium is computed as the cost of a sustainable remuneration of the shareholder capital at risk earmarked to absorb the exceptional (beyond expected) losses.

Pricing approaches in incomplete markets include utility indifference, risk minimisation and minimal martingale measures, utility maximisation and minimisation over martingale measures, good-deal pricing, market-consistent valuation, probability distortions, among others (see e.g. Schweizer (2001), Rogers (2001), Cochrane and Saa-Requejo (2000) or Madan (2015)). We follow a cost of capital approach because, thanks to the afore-mentioned two-step procedure, it is the only one that can be implemented in practice at the level of a realistic bank portfolio, as required for XVA purposes.

A cost of capital approach applied to the counterparty credit risk of the derivative portfolio of a bank raises interesting questions, because of a wealth transfer involved in the first step and of a dynamic perspective required in the second step. The ***main contributions*** of this work are a clean concept of the KVA, which is the big issue in investment banks today but lacks serious academic foundation, as well as the justification, based on market incompleteness considerations, that the right counterparty risk correction to derivative entry prices should be CVA+FVA+KVA, rather than a variety of other "VA combinations" (e.g. including DVA) that have been floating around in the literature and in the financial regulation in the last five years.

1.1 Overview of the Paper

A bank is a defaultable entity with shareholders and creditors. Shareholders have the control of the bank and are solely responsible for investment decisions up until the time of bank default. At default time, shareholders are wiped out. Creditors however have no decision power until the time of default, but are protected by laws such as pari-passu forbidding certain trades that would trigger wealth transfers from them to shareholders.

Counterparty credit risk is related to cash flows or valuations linked to either counterparty default or the default of the bank itself. It has capital structure implications and is perceived differently by shareholders and creditors. The risk of financial loss as a consequence of client default is hard to replicate since single name CDS instruments are illiquid and are typically written on bonds, not on swaps with rapidly varying value. The possibility for the bank of hedging its own default is even more questionable since, in order to hedge it, a bank would need to be able to freely trade its own debt. But banks are special firms in that they are intrinsically leveraged and cannot be transformed into a pure equity entity. There is also an argument of scale: Bank liabilities are overwhelming with respect to all other wealth numbers. Last, even if the bank was free to redeem all its debt, bank shareholders could not effectively monetise the hedging benefit, which would be hampered by bankruptcy costs.

Because of this counterparty credit risk incompleteness, deals trigger wealth transfers from bank shareholders to bank creditors and shareholders need to set capital at risk. On this basis we frame a theory of XVAs, whereby so-called contra-liabilities and cost of capital are sourced from bank clients at trade inceptions, on top of the fair valuation of counterparty credit risk, in order to compensate shareholders for these wealth transfers and for the risk on their capital.

We monitor wealth transfer flows across the capital structure of banks and derive the implications to pricing. By so doing, we propose a fundamental shift from a bank-centric view of derivative valuation to a shareholder view. One can mention the related corporate finance notion of debt overhang in Myers (1977), by which a project valuable for the firm as a whole may be rejected by shareholders because the project is mainly valuable to creditors. But such considerations were never considered in the field of derivative pricing before.

The only antecedents to our article seem to be Albanese and Andersen (2014, 2015) and Andersen, Duffie, and Song (2016). But these papers only consider FVA; they do not propose an approach to KVA. Portfolio optimization theory is so far focused on the optimal asset allocation problem for a fund manager. As far as we know, there has been no consideration in the literature dedicated to portfolio optimization for derivative market makers. A market maker has no optionality in asset allocation, so the issue here is not about asset selection. Instead, the challenge we tackle with our KVA approach in this paper is to devise entry prices which keep the position of a derivative market maker on the frontier corresponding to a given return (hurdle rate) on invested equity capital.

A detailed outline is as follows. Section 2 sets the pricing stage and formally introduces the key notions of contra-assets and contra-liabilities. Section 3 yields a preliminary presentation of our approach in a one-period setup. Section 4 provides our model of the capital structure of a bank, which can ultimately be reduced to consideration of the trading loss-and-profit process L of the so-called CA desk of the bank. Section 5 shows that, because of counterparty credit risk incompleteness, windfall benefits at bank default are immaterial to bank shareholders, whereas the costs of capital and funding matter to them and need be reflected in entry prices. Adapting to banks the insurance principles of the Swiss solvency test, Solvency II and IFRS 4 Phase II, Section 6 devises a KVA defined as the cost of remunerating shareholder capital at risk at a sustainable hurdle rate throughout the whole life of the portfolio. This is all done for a portfolio assumed held on a run-off basis, which is discussed in Section 7. Section 8 discusses the main technical insights of the paper.

Key assumptions are emphasized in bold throughout the paper. Once stated they are assumed everywhere unless otherwise specified. Summary lists of these key assumptions with comments and of the main acronyms used in the paper are provided in Sections A and B. As far as financial assets or accounts are concerned, acronyms interchangeably refer to the assets or accounts themselves, or (in equations in particular) to their value or cash amount, as should be clear from the context.

2 Pricing Setup

We consider a pricing stochastic basis $(\Omega, \mathbb{G}, \mathbb{Q})$, with model filtration $\mathbb{G} = (\mathcal{G}_t)_{t\in\mathbb{R}_+}$ and risk-neutral pricing measure $\mathbb{Q}$, such that all the processes in the paper are $\mathbb{G}$ adapted and all the random times are $\mathbb{G}$ stopping times. The corresponding expectation and conditional expectation are denoted by $\mathbb{E}$ and $\mathbb{E}_t$. We denote by r a $\mathbb{G}$ progressive OIS rate process, where OIS rate stands for overnight indexed swap rate, which is together the best market proxy for a risk-free rate and the reference rate for the remuneration of cash collateral. We write $\beta_t = e^{-\int_0^t r_s ds}$ for the corresponding risk-neutral discount factor. **All cash flows are valued by their risk-free discounted $(\mathbb{G}, \mathbb{Q})$ conditional expectation, assumed to exist**. This ensures the internal consistency of our valuation setup. It is also consistent with market practice regarding valuation of inter-dealer fully collateralized transactions, which are typically used as model calibration input data.

Remark 2.1 Valuation in the above sense corresponds to valuation assuming risk-free funding. In practice the funding of client trades, unless they are fully collateralized, is risky. When it comes to the valuation of the funding cash flows of a position, i.e. FVA computations, we work under the convention to only value the funding cash flows in excess over risk-free accrual at rate r of the position, so that the FVA of a position effectively funded at the risk-free rate r (e.g. the FVA of a fully collateralized position) is equal to zero. ∎

Under the cost of capital approach of this paper, **derivative entry prices include, on top of the valuation of the corresponding cash flows, a KVA risk**

premium to be specified in Sect. 6. Valuation in our sense is risk-neutral with respect to some pricing measure $\mathbb{Q}$. By contrast, economic capital and KVA assess risk and its cost, which refer to the historical probability measure $\mathbb{P}$. However, in the context of XVA computations entailing projections over decades, the main source of information is market prices of liquid instruments, which allow the dealer to calibrate a pricing measure $\mathbb{Q}$, and there is little of relevance that can be said about the historical probability measure. Hence, in our model, we assume that **the historical probability measure $\mathbb{P}$ coincides with the pricing measure $\mathbb{Q}$**. The discrepancy between $\mathbb{P}$ and $\mathbb{Q}$ is left to model risk.

All price and value processes are modeled as semimartingales in a càdlàg version. We write $x^{\pm} = \max(\pm x, 0)$ and $\int_a^b = \int_{(a,b]}$.

2.1 Contra-Assets and Contra-Liabilities

In order to focus on counterparty credit risk and XVA analysis, we assume throughout the paper that **the market risk of the bank is perfectly hedged by means of fully collateralized interbank back-to-back hedges to all client trades.**

Hence only the counterparty credit risk related cash flows remain. A key distinction is between the cash flows received by the bank prior its default time and the cash flows received by the bank during its default resolution period. The first stream of cash flows affects the bank shareholders, whereas the second stream of cash flows only affects creditors.

By linearity of our valuation rule, this decomposition of cash flows immediately translates into a decomposition of value. Namely, under a sign convention where CA (for contra-assets) values shareholders costs and CL (for contra-liabilities) values creditors benefits, so that creditors costs are valued by ($-$CL), the valuation of the costs of counterparty credit risk to the bank as a whole, valuation that we denote by CCR and dub “fair”, satisfies

$$\mathrm{CCR} = \mathrm{CA} - \mathrm{CL}. \tag{1}$$

Contra-assets (contra-liabilities) draw their names from the fact that, from the point of view of the balance sheet of the bank to be developed in Sect. 4.1, they are asset (liability) deductions. Note that CA and CL do not necessarily need to be positive in principle. In case they are not, negative numbers will be involved.

Since market risk is hedged out, we are left with two distinct but intertwined sources of market incompleteness:

- A bank cannot hedge its own jump-to-default exposure.
- A bank cannot replicate counterparty default losses.

As a result, as we will see in detail in the sequel:

- Contra-liabilities are wealth transfers from shareholders to bondholders, for which shareholders can only be compensated by an add-on charged to the client of each deal on top of the fair valuation of its counterparty credit risk.
- Contra-assets related cash flows cannot be replicated, hence capital needs be set at risk by shareholders, which therefore deserve, in the cost of capital approach of this paper, a further KVA add-on as a risk premium.

The all-inclusive XVA charge to bank clients is

$$\mathrm{CCR} + \mathrm{CL} + \mathrm{KVA} = \mathrm{CA} + \mathrm{KVA}. \tag{2}$$

If the bank was able to replicate jump-to-default exposures, then, as we will see, both CL and the KVA would vanish and the XVA charge would reduce to the fair valuation CCR of counterparty credit risk. However, this is not the case, hence add-ons to entry prices are required in order to compensate shareholders for wealth transfers and risk on their capital.

2.1.1 A Market maker Cannot Anticipate Future Trades

In an asymmetric setup with a price maker and a price taker, the price maker passes his costs to the price taker. For transactions between dealers, it is possible that one is the price maker and the other one is the price taker. It is also possible that a transaction triggers gains for the shareholders of both entities. The detailed consideration of these dynamics would lead to an understanding of the drivers to economical equilibrium in a situation where multiple dealers are present. However, in this paper we are not considering this situation and we assume that our bank is a market maker only facing clients. **A bank is a market maker which cannot anticipate future trades**. This is a key assumption as it also underlies our run-off portfolio assumption in later sections, as well as a further assumption that we introduce now on bank funding spreads.

Hull and White (2016) propose that FVA should not be passed to clients, as creditors to banks should recognize the wealth transfers from equity holders to themselves which will occur as a consequence of future trading. Instead we consider that, in the case of a market maker, it is not acceptable to make any assumption about future trade flows and putative profits deriving from them. Specifically, we assume that **the positive impact of trades on the realized recovery of the bank is not reflected in the bank funding spreads.**

At the bottom of this work lies the fact that a bank cannot replicate jump-to-default exposures. However we do not state this as standing assumption, because it is instructive to see what would happen if a bank could replicate jump-to-default exposures. We will then find that CL=KVA=0. Same conclusions would follow if the positive recovery impact of future trades could be anticipated, but this is ruled out by the assumption above.

3 Preliminary Approach in a Static Setup

In this section we present the main ideas of our XVA approach in an elementary static one-year setup, with r set equal to 0.

Assume that at time 0 a bank, with equity $\mathrm{E} = w_0$ corresponding to its initial wealth, enters a derivative position (or portfolio) with a client. The counterparty credit risk related cash flows affecting the bank before its default time τ are its counterparty default losses and funding expenditures, respectively denoted by $\mathcal{C}^\star$ and $\mathcal{F}^\star$ (the analogs, in a one-period setup, of the processes $\mathcal{C}^{\tau-}$ and $\mathcal{F}^{\tau-}$ stopped before τ in later sections). Let P_0 denote the mark-to-market of the deal ignoring counterparty credit risk and assuming risk-free funding. The bank wants to charge to its client an add-on, or obtain from its client a rebate, denoted by CA, accounting for its expected counterparty default losses and funding expenditures.

3.1 Cash Flows

Accounting for the to-be-determined add-on CA, in order to enter the position, the bank needs to borrow $(P_0 - \mathrm{CA})^+$ unsecured or invest $(P_0 - \mathrm{CA})^-$ risk-free (we write $x^{\pm} = \max(\pm x, 0)$), depending on the sign of $(P_0 - \mathrm{CA})$, in order to pay $(P_0 - \mathrm{CA})$ to its client. We assume that the bank and its client are both default prone with zero recovery. We denote by J and J' the survival indicators of the bank and its client at time 1 (both being assumed alive at time 0), with default probability of the bank $\mathbb{Q}(J = 0) = \gamma$ and no joint default for simplicity, i.e $\mathbb{Q}(J = J' = 0) = 0$. We assume that unsecured borrowing is fairly priced as γ × the amount borrowed by the bank, so that the funding expenditures of the bank amount to

$$\mathcal{F}^{\star} = \gamma(P_0 - \mathrm{CA})^+,$$

deterministically in this one-period setup. At time 1:

- If alive (i.e. $J = 1$), then the bank closes the position while receiving P_1 if its client is alive (i.e. $J' = 1$) or pays P_1^- if its client is in default (i.e. $J' = 0$).
  - Note $J'P_1 - (1 - J')P_1^- = P_1 - (1 - J')P_1^+$. Hence the counterparty default loss of the bank appears as the random variable

    $$\mathcal{C}^{\star} = (1 - J')P_1^+. \tag{3}$$

  In addition, the bank reimburses its funding debt $(P_0 - \mathrm{CA})^+$ or receives back the amount $(P_0 - \mathrm{CA})^-$ it had lent at time 0.
- If in default (i.e. $J = 0$), then the bank receives back P_1^+ on the derivative as well as the amount $(P_0 - \mathrm{CA})^-$ it had lent at time 0.

We assume further that a fully collateralized back-to-back market hedge is set up by the bank in the form of a deal with a third party, with no entrance cost and a payoff to the bank $-(P_1 - P_0)$ at time 1, irrespective of the default status of the bank and the third party at time 1.

Observe that, as joint defaults are excluded,

$$J(J'P_1 - (1 - J')P_1^-) = JP_1 - J(1 - J')P_1^+ = JP_1 - (1 - J')P_1^+ = JP_1 - \mathcal{C}^{\star}. \tag{4}$$

Collecting all the cash flows, the wealth of the bank at time 1 is, using (4) in the second equality

$$\begin{aligned} w_1 \quad &= \mathrm{E} - \mathcal{F}^{\star} + (1 - J)\big(P_1^+ + (P_0 - \mathrm{CA})^-\big) \\ &\qquad + J\big(J'P_1 - (1 - J')P_1^- - (P_0 - \mathrm{CA})^+ + (P_0 - \mathrm{CA})^-\big) - (P_1 - P_0) \\ &= \mathrm{E} - (\mathcal{C}^{\star} + \mathcal{F}^{\star}) + (1 - J)\big(P_1^+ + (P_0 - \mathrm{CA})^-\big) \\ &\qquad + J\big(P_1 - (P_0 - \mathrm{CA})^+ + (P_0 - \mathrm{CA})^-\big) - (P_1 - P_0) \\ &= \big(\mathrm{E} - (\mathcal{C}^{\star} + \mathcal{F}^{\star} - \mathrm{CA})\big) + (1 - J)(P_1^- + (P_0 - \mathrm{CA})^+). \end{aligned} \tag{5}$$

The result of the bank over the year is

$$w_1 - w_0 = w_1 - \mathrm{E} = -(\mathcal{C}^{\star} + \mathcal{F}^{\star} - \mathrm{CA}) + (1 - J)(P_1^- + (P_0 - \mathrm{CA})^+).$$

However, the cash flow $(1 - J)(P_1^- + (P_0 - \mathrm{CA})^+)$ is only received by the bank if it is in default at time 1, so that it only benefits bank creditors. Hence the profit-and-loss of bank shareholders reduces to $-(\mathcal{C}^\star + \mathcal{F}^\star - \mathrm{CA})$, i.e. their trading loss-and-profit, which we denote by L, appears as

$$L = \mathcal{C}^\star + \mathcal{F}^\star - \mathrm{CA}. \tag{6}$$

Remark 3.1 The derivation (5) allows for negative equity, which is interpreted as recapitalization. In a variant of the model excluding recapitalization, where the default of the bank would be modeled in a structural fashion as $\mathrm{E} - L < 0$ and negative equity is excluded, we would get instead of (5)

$$w_1 \quad = (\mathrm{E} - L)^+ + \mathbb{1}_{\{\mathrm{E} < L\}}(P_1^- + (P_0 - \mathrm{CA})^+). \tag{7}$$

In this paper we consider a model with recapitalization for reasons explained in Sect. 4.2. The alternative (7) is considered from a different angle in Capponi and Crépey (2016). ∎

3.2 Funds Transfer Price

In order to account for expected counterparty default losses and funding expenditures, the bank charges to its client the add-on

$$\mathrm{CA} = \underbrace{\mathbb{E}\mathcal{C}^\star}_{\mathrm{CVA}} + \underbrace{\mathbb{E}\mathcal{F}^\star}_{\mathrm{FVA}}. \tag{8}$$

Since

$$\mathrm{FVA} = \mathbb{E}\mathcal{F}^\star = \mathcal{F}^\star = \gamma(P_0 - \mathrm{CA})^+$$

(all deterministically in a one-period setup), (8) is in fact an equation for CA. Equivalently, we have the following semi-linear equation for FVA = CA − CVA :

$$\mathrm{FVA} = \gamma(P_0 - \mathrm{CVA} - \mathrm{FVA})^+,$$

which has the unique solution

$$\mathrm{FVA} = \frac{\gamma}{1+\gamma}(P_0 - \mathrm{CVA})^+.$$

Substituting this and (3) into (8), we obtain

$$\mathrm{CA} = \underbrace{\mathbb{E}[(1 - J')P_1^+]}_{\mathrm{CVA}} + \underbrace{\frac{\gamma}{1+\gamma}(P_0 - \mathrm{CVA})^+}_{\mathrm{FVA}}. \tag{9}$$

In view of (6) and (8), observe that charging to the client a CA add-on corresponding to expected counterparty default losses and funding expenditures is equivalent to setting the add-on CA such that, in expectation, the trading loss-and-profit of bank shareholders is zero ($\mathbb{E}L = 0$), as it would also be the case without the deal. However, without the deal, the loss-and-profit of bank shareholders would be zero not only in expectation, but deterministically. Hence, to compensate shareholders for the risk on their equity triggered by the deal, under the cost of capital approach of this paper, the bank charges to its client an additional amount (risk margin)

$$\mathrm{KVA} = h\mathrm{E}, \tag{10}$$

where h is some hurdle rate, e.g. 10%. Moreover, since the initial equity E of the bank can be interpreted as capital at risk earmarked to absorb the losses $(\mathcal{C}^\star + \mathcal{F}^\star)$ of the bank above CA, it is natural to size E by some risk measure of the bank shareholders loss-and-profit L. The all-inclusive XVA add-on to the entry price for the deal, which we call funds transfer price (FTP), appears as (cf. the right-hand side in (2))

$$\mathrm{FTP} = \underbrace{\mathrm{CA}}_{\text{Expected costs}} + \underbrace{\mathrm{KVA}}_{\text{Risk premium}} . \tag{11}$$

3.3 Monetizing the Contra-Liabilities?

Let us now assume, for the sake of the argument, that the bank would be able to hedge its own jump-to-default exposure through a further deal, whereby the bank would deliver a payment $(1-J)(P_1^- + (P_0 - \mathrm{CA})^+)$ at time 1 in exchange of an upfront fee fairly valued as

$$\mathrm{CL} = \underbrace{\mathbb{E}[(1-J)P_1^-]}_{\mathrm{DVA}} + \underbrace{\mathbb{E}[(1-J)(P_0-\mathrm{CA})^+]}_{\mathrm{FDA}=\gamma(P_0-\mathrm{CA})^+=\mathrm{FVA}} . \tag{12}$$

Here DVA and FDA stand for debt valuation adjustment and funding debt adjustment, which are the contra-liability counterparts of the CVA and the FVA. As unsecured borrowing is assumed fairly priced ignoring the positive impact of the trade on the effective recovery of the bank (cf. Sect. 2.1.1), the value $\mathrm{FDA} = \mathbb{E}[(1-J)(P_0-\mathrm{CA})^+]$ of the default funding cash flow $(1-J)(P_0-\mathrm{CA})^+$ equals the cost $\mathrm{FVA} = \gamma(P_0-\mathrm{CA})^+$ of funding the position. But the FVA and the FDA do not impact the same economic agent, namely the FVA hits bank shareholders whereas the FDA benefits creditors. Hence the net effect of funding is not nil to shareholders, but reduces to an FVA cost. Note that, instead of the zero recovery rate that was anticipated at the time when the bank issued the debt, the realized recovery is $(1-J)(P_1^- + (P_0-\mathrm{CA})^+)$ because of the trade that occurred, but this was not anticipated and not reflected in the price of borrowing.

Let $\widetilde{\mathrm{CA}}$ denote the modified CA charge to be passed to the client when the hedge is assumed. Accouting for the hedging gain $\mathcal{H} = \mathrm{CL} - (1-J)(P_1^- + (P_0-\mathrm{CA})^+)$, the wealth of the bank at time 1 now appears as (cf. (5))

$$\begin{aligned}\widetilde{w}_1 \quad &= (\mathrm{E} - (\mathcal{C}^\star + \mathcal{F}^\star - \widetilde{\mathrm{CA}})) + (1-J)(P_1^- + (P_0-\mathrm{CA})^+) + \mathcal{H}\\ &= \mathrm{E} - (\mathcal{C}^\star + \mathcal{F}^\star - \widetilde{\mathrm{CA}}) + \mathrm{CL}.\end{aligned} \tag{13}$$

By comparison with (5), the CL originating cash flow $(1-J)(P_1^- + (P_0-\mathrm{CA})^+)$ is hedged out and monetized as an amount CL received by the bank at time 0. The trading loss-and-profit of bank shareholders now appears as

$$\widetilde{L} = w_0 - \widetilde{w}_1 = \mathrm{E} - \widetilde{w}_1 = \mathcal{C}^\star + \mathcal{F}^\star - \widetilde{\mathrm{CA}} - \mathrm{CL}.$$

The amount $\widetilde{\mathrm{CA}}$ making $\widetilde{L}$ centered is

$$\widetilde{\mathrm{CA}} = \mathbb{E}(\mathcal{C}^\star + \mathcal{F}^\star) - \mathrm{CL} = (\mathrm{CVA} + \mathrm{FVA}) - (\mathrm{DVA} + \mathrm{FDA}) = \mathrm{CVA} - \mathrm{DVA}, \tag{14}$$

because FVA=FDA (cf. (12)).

Hence, if the bank was able to hedge its own jump-to-default risk, in order to satisfy its shareholders in expectation, it would be enough for the bank to charge to its client an add-on $\widetilde{\mathrm{CA}} = \mathrm{CVA} - \mathrm{DVA}$. This difference also coincides with the fair valuation of counterparty credit risk when market completeness and no trading restrictions are assumed (cf. Duffie and Huang (1996)). However, in the present setup, under the approach of this paper, the bank would still charge to its client a KVA add-on $h\widetilde{\mathrm{E}}$ as risk compensation for the non flat loss-and-profit $\widetilde{L}$ triggered by the deal (unless $\widetilde{L}$ can be hedged out as well). But $\widetilde{\mathrm{E}}$ would be sized by some risk measure of $\widetilde{L}$, instead of L for E in (10).

3.4 Wealth Transfer Interpretation and Funds Transfer Price Decomposition

As seen in Sect. 1.1, a bank cannot hedge its own jump-to-default risk in practice. But the findings of Sect. 3.3 are important from an interpretive point of view.

We see from (8) and (12) that CA can be viewed as the sum between CL and the fair valuation CCR = CVA − DVA of counterparty credit risk. In view of the above, CL can be interpreted as an add-on that the bank needs to source from its client, on top of the fair valuation of counterparty credit risk, in order to compensate the loss of value to shareholders due to the inability of the bank to hedge its own jump-to-default risk. In other words, due to this market incompleteness (or trading restriction), the deal triggers a wealth transfer from bank shareholders to creditors equal to CL, which then needs be sourced by the bank from its client in order to put shareholders back at value equilibrium in expected terms.

In conclusion of this section, the FTP (11) can be decomposed as (cf. (2))

$$\begin{aligned}\mathrm{FTP} &= \underbrace{\mathrm{CVA}+\mathrm{FVA}}_{\text{Expected costs CA}} + \underbrace{\mathrm{KVA}}_{\text{Risk premium}}\\ &= \underbrace{\mathrm{CVA}-\mathrm{DVA}}_{\text{Fair valuation CCR}} + \underbrace{\mathrm{DVA}+\mathrm{FDA}}_{\text{Wealth transfer CL}} + \underbrace{\mathrm{KVA}}_{\text{Risk premium}},\end{aligned} \tag{15}$$

where CA is given by (9) and where the random variable L used to size the equity E in the KVA formula (10) is the bank shareholders loss-and-profit L as of (6).

4 Capital Structure Model

The sequel of the paper generalizes to a dynamic and incremental setup the static FTP formulas (15) (cf. (50)).

In order to show that the CA and KVA equations make together a self-contained and well-posed problem, we need to identify the connection between the different XVA metrics. Toward this end, this section recasts CA and CL in the perspective of the balance sheet of the bank. The main outputs of the setup will be synthesized in the form of Proposition 4.1, which shows that, for XVA analysis purposes, a dynamic model of the balance sheet can ultimately be reduced to consideration of the trading loss-and-profit process L of the so-called CA desk of the bank.

4.1 Balance sheet of a Bank

The accounting result of the bank as a whole is called accounting equity (AE). To reflect the capital structure of the bank, AE is represented as follows:

$$\mathrm{AE} = \mathrm{A} - \mathrm{CA} - (\mathrm{L} - \mathrm{CL}), \tag{16}$$

where

A, L: Assets (A) and liabilities (L), are computed ignoring counterparty credit risk. In the assets (liabilities) accounts, one places, in particular, default free valuation of all unsecured derivative receivables and derivative payable hedges (unsecured derivative payables and derivative receivable hedges).

CA: The contra-assets (CA) account entails the valuation of all cash flows related to the credit risk of either the counterparties or the bank and occurring before the default of the bank itself.

CL: The contra-liabilities (CL) account entails the valuation of all the counterparty credit risk related cash flows received by the bank during the resolution process after its default.

Assets (A) also include

RC: Reserve capital, is capital sourced from bank clients and reserved to compensate for expected but unhedgeable risks, i.e. counterparty default losses and funding expenditures;

SCR: Shareholder capital at risk, is earmarked to absorb exceptional losses.

RM: Risk margin (or KVA) account, is capital sourced from bank clients and retained for future distribution as a dividend and risk compensation. It is also loss-absorbing.

UC: Uninvested capital, is sourced as either equity or debt, but does not contribute to regulatory capital. It is not considered to be exposed to the risk of exceptional losses.

Note that the risk margin is also loss-absorbing. Hence the economic capital (EC) of the bank, i.e. its resource devoted to cope with exceptional losses (beyond the expected levels of losses taken care of by RC), is the sum between SCR and RM.

We emphasize that, since the risk margin consists of retained earnings meant to be released to the bank shareholders, we do not put their theoretical KVA target value as a liability (or contra-asset) on the balance sheet. This is consistent with the treatment of the risk margin in Swiss solvency test capital at risk calculations.

Liabilities (L) also include

D: Debt, is the value of the bank to creditors.

The reader should not think that individual trades are accounted for as contributing exclusively to one item or the other in the above. These items only indicate a repartition of value, not of actual trades. In particular, a derivative is considered as asset or liability depending on whether it is in-the-money or out-of-the-money (when valued on a default-free basis). This may fluctuate dynamically in time for any given trade.

In Figure 1 we represent the difference FC=UC−D, the so called “free capital” in a Solvency II actuarial terminology, as a single line on the asset side.

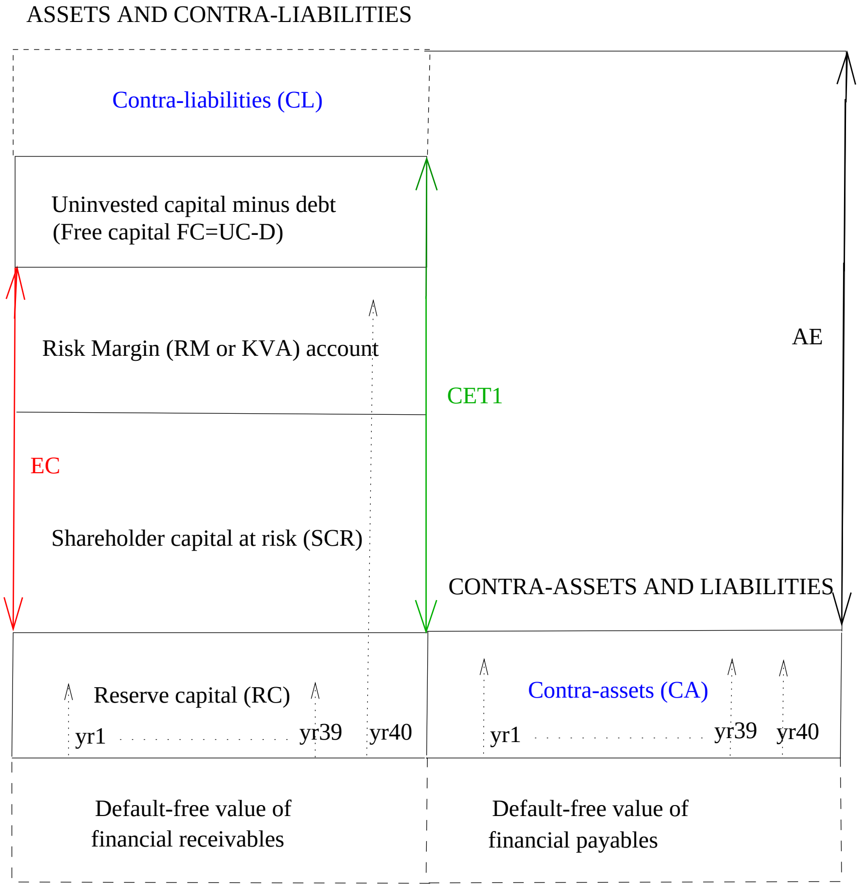

Figure 1: Capital structure model of a bank. Default-free valuation of financial payables (assets) and receivables (liabilities) at the bottom are shown in dashed boxes because, under our back-to-back market hedge assumption, they constantly match each other. Contra-liabilities at the left top are shown in a dashed box because they should be ignored from the point of view of the shareholders. The difference between the left side and the right side of the balance sheet corresponds to the accounting equity (AE). The difference between AE and contra-liabilities corresponds to the core equity tier I capital of the bank (CET1, see Sect. 4.3). The determination of CA and KVA is the goal of a counterparty credit risk model. The KVA is obtained from economic capital (EC) by the KVA formula (45) and SCR is obtained from them as (EC−KVA). At each new deal, the RC and the RM accounts are refilled by the client of the deal. Between deals, the RC account is depleted by counterparty default losses and funding expenditures as they occur, whereas risk margins gradually flow from the RM account into the shareholders dividend stream. The arrows in the left column represent losses in “normal years 1 to 39” and in an “exceptional year 40” with full depletion of RC, SCR and RM (the numberings yr1 to yr40 are fictitious yearly scenarios in line with ES, the 97.5% expected shortfall that underlies economic capital). The arrows in the right column symbolize the average depreciation in time of contra-assets (assuming a portfolio held on a run-off basis for the reasons to be explained in Sect. 7). Whenever reserves are depleted and below the theoretical level, banks raise equity capital to realign them.

4.2 Losses and Earnings

We consider a derivative portfolio held on a run-off basis, i.e. such that no new trades enter the portfolio in the future until its final maturity T (the case of incremental portfolios will be discussed in Sect. 7). Accounting for the bank default time τ, the time horizon of the model is $\bar{\tau} = \tau \wedge T$.

It is useful to introduce the concept of the CA desk, which is a central XVA desk of the bank in charge of absorbing counterparty default losses and funding expenditures. The CA desk, however, would not be in charge of the KVA, which under our approach is of a different nature and is therefore treated separately. The CA desk would only sell the contra-assets to the clients of the bank, deposit the corresponding payment into the RC account and then be exposed to the corresponding payoffs (as long as the bank is alive). As time proceeds, counterparty default losses and funding expenditures occur and are covered by the CA desk with the RC account. The CA desk may also setup a related (typically imperfect) hedge. We call CA desk loss process, denoted by L, the trading loss-and-profit process of the CA desk. In parallel, risk margins are sourced by the bank from clients at deals inception, deposited into the RM (or KVA) account and then gradually released from the RM account into the shareholders dividend stream.

Losses-and-earnings realization times are typically quarter ends for bank profits, released as dividends, versus recapitalization managerial decision times for losses. However, there is no way to “calibrate” a realization time schedule of losses-and-earnings. In our model, we assume for simplicity that **losses-and-earnings are marked to model and realized in real time,** which we call the continuous reset (or instantaneous recapitalisation) assumption. This is also in the spirit of the regulation, which imposes regular realignments of the different banking accounts to their theoretical target values.

In particular, the trading gains $(-dL_t)$ continuously flow into the shareholders dividend stream. Equivalently, the RC account is continuously reset to its theoretical target CA level so that, much like with futures, the position of the CA desk is reset to zero at all times, but it generates the gains $(-dL_t)$.

Similarly, the RM (or KVA) account and economic capital EC are continuously reset to their theoretical target levels, which, within our proposed treatment in this paper, will be given as (38) and (44) below. In particular, $(-d\mathrm{KVA}_t)$ amounts continuously flow from the RM account to the shareholder dividend stream, into which the bank also sends the $r_t\mathrm{KVA}_t dt$ OIS accrual payments on the KVA account.

In sum, shareholders dividends net to

$$-(dL_t + d\mathrm{KVA}_t - r_t\mathrm{KVA}_t dt) = -(dL_t + \beta_t^{-1} d(\beta_t \mathrm{KVA}_t)). \tag{17}$$

We emphasize that negative dividends are possible in our setup (cf. Remark 3.1 in the static case). They are interpreted as recapitalisation, i.e. equity dilution. In particular, we refrain from treating states of zero or negative equity as triggers of bank default as in Andersen et al. (2016, Sect. III-IV) because recapitalisation decisions, possibly due to government intervention, empirically play a major role to stave off defaults.

Our continuous reset assumption means that, instead of viewing losses as depletions of the different banking accounts, until the point of default where one of them (with possible reshuffling between them) would become negative, we view losses as continuously compensated by shareholders; instead of viewing losses as money flowing away from the balance sheet, we view them as money flowing into it as refill, i.e. replenishment of the different bank accounts at their theoretical target level, until the point of default where the payers cease willing to do so. When this happens is modeled as a totally unpredictable event at some exogenous time calibrated to the bank CDS spread, which we view as the most reliable and informative credit data regarding anticipations of markets participants about future recapitalization, government intervention and other (including TLAC bail-in) bank failure resolution policies.

As demonstrated by the well-known difficulties in calibrating structural default time models, this view on the default time of the bank is actually more realistic and it is definitely more suitable for XVA analysis purposes, which requires a proper calibration to the CDS curve of the bank.

Remark 4.1 In a Merton mindset, the default of the bank in our setup could be modeled as the first time when UC goes below D, i.e. when the free capital FC=UC−D becomes negative (cf. Remark 3.1 in the one-period setup). Merton (1974)'s purpose was to develop an option-theoretic view on equity and corporate debt. For this of course a structural model of the default time of a firm is required. In our case the reason why we introduce a capital structure model of the bank is not to come up with such a structural model of the bank default time, which for the above-explained reasons would be unrealistic. Instead, the motivation for our capital structure model is to put the contra-assets and contra-liabilities of a bank in a balance sheet perspective in order to identify the structural connection between the different XVA metrics and show that the overall XVA problem is self-contained and well-posed. In particular, in the analysis of counterparty credit risk, the free capital process FC=UC−D has a very passive role. The key feature is the dividing line between contra-assets and contra-liabilities, items which are not present in the Merton model. ∎

4.3 Economic Capital

Core equity tier I capital (CET1) is the regulatory metric that represents the core financial strength of a bank. Since contra-liabilities do not benefit to the shareholders, CET1 is given by the difference between the overall result AE of the bank and CL, i.e.

$$\mathrm{CET1} = \mathrm{AE} - \mathrm{CL} = \mathrm{A} - \mathrm{L} - \mathrm{CA}, \tag{18}$$

by (16).

Under our back-to-back market hedge assumption, default-free valuation of financial payables (assets) and receivables (liabilities) constantly match each other, so that the difference between assets and liabilities reduces to

$$\mathrm{A} - \mathrm{L} = \mathrm{RC} + \mathrm{EC} + \mathrm{FC} \tag{19}$$

(recalling EC=SCR+RM). Hence,

$$\begin{aligned} \mathrm{CET1} &= \mathrm{A} - \mathrm{L} - \mathrm{CA} \\ &= (\mathrm{RC} - \mathrm{CA}) + (\mathrm{EC} + \mathrm{FC}). \end{aligned} \tag{20}$$

In the case of a back-to-back hedged portfolio held on a run-off basis with instantaneous recapitalization, the first term is constantly reset to 0 (any discrepancy between RC and CA is instantaneously realized into $(-dL)$). Moreover, the impacts of the derivative portfolio on the different entries in

$$\mathrm{CET1} = \mathrm{EC} + \mathrm{FC} = \mathrm{EC} + \mathrm{UC} - \mathrm{D} \tag{21}$$

are interconnected.

Example 4.1 If the risk decreases on the market, i.e. if EC decreases (for instance because the portfolio amortizes, simply, or because of a favorable evolution of the underlying risk factors), then, assuming a constant debt D, this can be compensated by a corresponding increase in uninvested capital UC (see Figure 2 for an illustrative scenario), hence CET1 does not need to change. If the bank borrows collateral, this increases UC and D by the same amounts, hence CET1 does not need to change. If the default of a counterparty occurs, then the counterparty default loss is instantaneously realized as a loss by the CA desk and a continuously reset CET1 as of (21) does not need to change. ∎

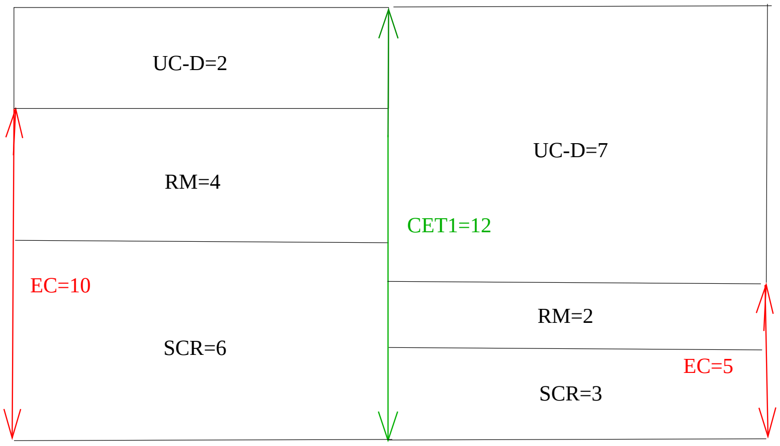

Figure 2: Illustrative impacts of a risk decrease on the different entries in (19).

Accordingly, **we assume that**

$$\mathrm{CET1} = \mathrm{EC} + \mathrm{UC} - \mathrm{D} = y, \tag{22}$$

a given constant (working under the convention that OIS interest payments on EC and FC go to the shareholders directly rather than accruing the EC and FC amounts themselves).

Basel II Pillar II defines economic capital as the 99% value-at-risk of the negative variation over a one-year period of core equity tier I capital. However, given our continuous reset assumption, by core equity tier I capital in this sentence, we must understand not only the continuously reset constant CET1 process as of (22), but we also need to account for the amounts $(-L)$ that are gained as profits or absorbed as losses by the shareholders in the resets of the RC account, for the interest payments to the shareholders on the EC and FC accounts and for discounting under the risk-measure. In sum, since CET1 is the constant y in (22) and the effects of OIS payments and risk-free discounting on CET1 cancel out each other, the risk to which shareholders are exposed corresponds exactly to the fluctuations of the process $(-\int_0^{\cdot} \beta_t dL_t)$.

Moreover, the FRTB requires a shift from value-at-risk defined as the 99% quantile to an expected shortfall measure (ES) with a 97.5% confidence. The interpretation of this expected shortfall is that economic capital is sized to a level sufficient to absorb average losses deriving by extreme levels of depletion of reserve capital occurring in the worst out of every forty years (cf. Figure 1).

Last, according to regulatory guidelines, expected shortfall is computed on a going-concern basis, i.e. by conditioning to the non-occurrence of bank default.

In conclusion, our reference definition for economic capital is

$$\mathrm{ES}_t(L) = \frac{\mathbb{E}\big[\int_t^{t+1} \beta_t^{-1}\beta_s dL_s \mathbb{1}_{\{\int_t^{t+1} \beta_t^{-1}\beta_s dL_s \geq \mathrm{VaR}_t(L)\}} \,\big|\, \mathcal{G}_t \vee \{\tau > t+1\}\big]}{\mathbb{Q}\big[\int_t^{t+1} \beta_t^{-1}\beta_s dL_s \geq \mathrm{VaR}_t(L) \,\big|\, \mathcal{G}_t \vee \{\tau > t+1\}\big]}, \qquad (23)$$

where

$$\mathrm{VaR}_t(L) = \inf\Big\{\ell : \mathbb{Q}\big[\int_t^{t+1} \beta_t^{-1}\beta_s dL_s > \ell \,\big|\, \mathcal{G}_t \vee \{\tau > t+1\}\big] \leq 2.5\%\Big\}.$$

Solvency II introduces a further modification as economic capital is required to be in excess of cost of capital (KVA in our setup). This modification is considered in Sect. 6.2.

Note that, as it is only the fluctuations of L that matter in (23), the value of the unknown constant y in (22) is immaterial.

Summarizing key features of our setup:

Proposition 4.1 *Assuming a back-to-back hedged portfolio held on a run-off basis satisfying* (22) *with continuous recapitalization until bank default:*
(i) *Shareholder dividends are given by*

$$-(dL_t + d\mathrm{KVA}_t - r_t\mathrm{KVA}_t dt) = -dL_t - \beta_t^{-1} d(\beta_t \mathrm{KVA}_t); \qquad (24)$$

(ii) *The reference definition for economic capital is* $\mathrm{ES} = \mathrm{ES}_t(L)$ *as of* (23).

Proof. This holds by construction in our setup (cf. the justifications for (17) and (23) given above). ∎

The balance sheet perspective developed in this section has been key in identifying the economic meaning of the XVA accounting terms as well as the connections between them. In particular, Proposition 4.1(ii) is telling us that the data required as input in economical capital and later in turn KVA computations correspond to the contra-assets mis-hedge process L.

Thanks to this connection, the XVA problem as a whole is self-contained and, as it will therefore turn out in concrete setups (see e.g. Albanese, Caenazzo, and Crépey (2016)), well-posed. This is of course good news. But another good news is that, now that Proposition 4.1 has been established, in view of it, we can forget most of the cumbersome balance sheet data in the sequel of this paper and in any actual XVA modeling, focusing on the key quantities CA, CL and on the process L, the latter playing the role of a "dynamic reduced model" of the balance sheet.

5 Wealth Transfer Analysis

We denote by $J = \mathbb{1}_{[0,\tau)}$ the survival indicator process of the bank. For any left-limited process Y, we denote by $\Delta_\tau Y = Y_\tau - Y_{\tau-}$ the jump of Y at τ and by $Y^{\tau-} = JY + (1-J)Y_{\tau-}$ the process Y stopped before time τ, so that

$$dY_t = dY_t^{\tau-} + (-\Delta_\tau Y)\, dJ_t,\ 0 \leq t \leq \bar{\tau}. \qquad (25)$$

We denote by $\mathcal{C}$ the cumulative stream of counterparty credit (as opposed to funding or hedging) cash flows. Hence counterparty default losses contributing to CA correspond to $\mathcal{C}^{\tau-}$, whereas $(-\Delta_\tau \mathcal{C})$ contributes to CL.

The CA desk may setup a (partial) counterparty credit risk hedge. The hedge is assumed to be funded separately from the derivative portfolio, being for instance implemented through repo markets or through assets with no upfront payment such as CDS contracts. The hedging gain process of the CA desk, including the cost of setting the hedge, is modeled as $\mathcal{H}^{\tau-}$, for some risk neutral local martingale $\mathcal{H}$ starting from 0. The rationale here is that hedging gains arise in practice as the stochastic integral of predictable hedging ratios against (funding cost inclusive) wealth processes of individual hedging assets. Under our standing valuation rule, each hedging asset is valued as risk-free discounted expectation of its future cash flows. Hence, the individual wealth process related to a long position in any of the hedging assets is a risk-neutral local martingale, as is in turn the stochastic integral $\mathcal{H}$. However, it is only the $\mathcal{H}^{\tau-}$ component of $\mathcal{H}$ that contributes to CA, whereas $(-\Delta_\tau \mathcal{H})$ contributes to CL.

Example 5.1 In the Black-Scholes model with constant interest rate r on a stock S with volatility σ, assuming that the hedge is implemented through a repo market with zero repo basis on S, then (assuming no dividends on S)

$$d\mathcal{H}_t = \zeta_t \big(dS - rS_t dt\big) = \zeta_t \sigma S_t dW_t, \tag{26}$$

where W is the risk-neutral Brownian motion driving S and ζ is the hedging ratio in S. The instantaneous cost of funding the hedge is $\zeta_t r S_t dt$, which is included in (26). ∎

Likewise, the funding expenditures of the CA desk are modeled as

$$-r_t \mathrm{CA}_t dt + d\mathcal{F}^{\tau-}, \tag{27}$$

for some risk neutral local martingale $\mathcal{F}$ starting from 0. Here the rationale is that funding is implemented in practice as the stochastic integral of predictable hedging ratios against funding assets. Given our standing valuation rule, the value process of each of these assets is a martingale modulo risk-free accrual. Therefore the funding costs generated by the funding strategy of the CA desk accumulate into a local martingale $\mathcal{F}$ stopped before τ, i.e. $\mathcal{F}^{\tau-}$, minus the risk-free accrual of the value CA that sits on the reserve capital account (under our continuous reset assumption). The cash flow $(-\Delta_\tau \mathcal{F})$ contributes to CL.

Example 5.2 Let

$$dB_t = r_t B_t dt$$
$$dD_t = (r_t + \lambda_t) D_t dt + (1-R) D_{t-} dJ_t = r_t D_t dt + D_{t-}\big(\lambda_t dt + (1-R)\, dJ_t\big)$$

represent the risk-free OIS deposit asset and a risky bond issued by the bank for its investing and unsecured borrowing purposes. Here λ represents an unsecured funding spread and R is some recovery coefficient, taken as an exogenous constant (as the positive impact of trades on the realized recovery of the bank is not reflected in the bank funding spreads, cf. Sect. 2.1.1). The risk-neutral martingale condition that applies to (βD) under our standing valuation framework implies that $\lambda = (1-R)\gamma$, where γ is the risk-neutral default intensity of the bank, so that

$$\lambda_t dt + (1-R)\, dJ_t = (1-R) d\mu_t,$$

where $d\mu_t = \gamma dt + dJ_t$ is the compensated jump-to-default martingale of the bank. We assume a dynamic amount of collateral C_t on the derivative portfolio, borrowed unsecured and posted by the bank if $C_t > 0$, received and invested at OIS rate by the bank if $C_t < 0$, remunerated OIS by the receiving party to the posting party. The corresponding funding policy of the bank is represented by a splitting of the amount CA_t on the RC account (under our continuously reset convention) as

$$\mathrm{CA}_t = \underbrace{\big(C_t + (\mathrm{CA}_t - C_t)^+\big)}_{\text{Invested at the risk-free rate as } \nu_t B_t} - \underbrace{(\mathrm{CA}_t - C_t)^-}_{\text{Unsecurely funded as } \eta_t D_t} . \tag{28}$$

A standard self-financing condition expressing the conservation of cash flows at the level of the bank as a whole yields

$$\begin{aligned} d\left(\nu_t B_t - \eta_t D_t\right) &= \nu_t dB_t - \eta_{t-} dD_t \\ &= \nu_t r_t B_t dt - \eta_t (r_t + \lambda_t) D_t dt - (1-R)\eta_{\tau-} D_{\tau-} dJ_t \\ &= r_t \mathrm{CA}_t dt - (1-R)\eta_{t-} D_{t-} d\mu_t,\ 0 \le t \le \bar{\tau} \end{aligned}$$

(a left-limit in time is required in η because D jumps at time τ, so that the process η, which is defined implicitly through $(\mathrm{CA}^- - C)$ in (28), is not predictable). Equivalently viewed in terms of costs, i.e. flipping signs in the above, we obtain

$$-\,d\left(\nu_t B_t - \eta_t D_t\right) = -r_t \mathrm{CA}_t dt + d\mathcal{F}_t = \left(-r_t \mathrm{CA}_t dt + d\mathcal{F}_t^{\tau-}\right) + \left(-\Delta_\tau \mathcal{F}\right) dJ_t,$$

where we set $d\mathcal{F}_t = (1-R)(\mathrm{CA}_{t-} - C_{t-})^- d\mu_t$. ■

Lemma 5.1 *Assuming a back-to-back hedged portfolio held on a run-off basis satisfying* (22) *with continuous recapitalization until the bank default time* τ*, given the credit, funding and hedging cash flows stream* $\mathcal{C}, \mathcal{F}$ *and* $\mathcal{H}$ *:*
(i) *Assuming integrability, we have, for* $0 \le t \le \bar{\tau}$*,*

$$\begin{aligned} \mathrm{CA}_t &= \mathbb{E}_t \int_t^{\bar{\tau}} \beta_t^{-1} \beta_s d\mathcal{C}_s^{\tau-} + \mathbb{E}_t \int_t^{\bar{\tau}} \beta_t^{-1} \beta_s d\mathcal{F}_s^{\tau-} + \mathbb{E}_t[\beta_t^{-1} \beta_\tau \mathbb{1}_{\{\tau<T\}} \Delta_\tau \mathcal{H}] \\ \mathrm{CL}_t &= \mathbb{E}_t\big[\beta_t^{-1} \beta_\tau \mathbb{1}_{\{\tau<T\}} (-\Delta_\tau \mathcal{C} - \Delta_\tau \mathcal{F} + \Delta_\tau \mathcal{H})\big] \\ \mathrm{CCR}_t &= \mathbb{E}_t \int_t^{\bar{\tau}} \beta_t^{-1} \beta_s d\mathcal{C}_s. \end{aligned} \tag{29}$$

(ii) *The CA desk loss process* L *is given as the risk-neutral local martingale such that*

$$\beta_t dL_t = d(\beta_t \mathrm{CA}_t) + \beta_t (d\mathcal{C}_t^{\tau-} + d\mathcal{F}_t^{\tau-} - d\mathcal{H}_t^{\tau-}),\ 0 \le t \le \bar{\tau}, \tag{30}$$

starting from the accrued loss $L_0 = z$ *of the CA desk.*

Proof. **(i)** In view of (27), the funding expenditures in excess over risk-free accrual at rate r of reserve capital, i.e. in excess over a cost $(-r_t \mathrm{CA}_t dt)$ (typically negative cost, i.e. benefit), appear as $\mathcal{F}^{\tau-}$. Hence, accounting for Remark 2.1 as far as the valuation of the funding cash flows in CA is concerned, by definition, we have

$$\begin{aligned} \mathrm{CA}_t &= \mathbb{E}_t \int_t^{\bar{\tau}} \beta_t^{-1} \beta_s (d\mathcal{C}_s^{\tau-} + d\mathcal{F}_s^{\tau-} - d\mathcal{H}_s^{\tau-}) \\ \mathrm{CL}_t &= \mathbb{E}_t\big[\beta_t^{-1} \beta_\tau \mathbb{1}_{\{\tau<T\}} (-\Delta_\tau \mathcal{C} - \Delta_\tau \mathcal{F} + \Delta_\tau \mathcal{H})\big] \\ \mathrm{CCR}_t &= \mathrm{CA}_t - \mathrm{CL}_t = \mathbb{E}_t \int_t^{\bar{\tau}} \beta_t^{-1} \beta_s (d\mathcal{C}_s + d\mathcal{F}_s - d\mathcal{H}_s), \end{aligned} \tag{31}$$

from which (29) is deduced by the martingale properties of $\mathcal{F}$ and $\mathcal{H}$.
(ii) The different cash flows contributing to the CA desk loss process L are the counterparty default losses prior to time τ, which correspond to $d\mathcal{C}^{\tau-}$, and the funding expenditures given by (27), minus the CA hedging gains $d\mathcal{H}^{\tau-}$. Also accounting for the mark-to-model of the liabilities, valued by the CA process, of the CA desk, the loss-and-profit process L appears as

$$dL_t = d\mathrm{CA}_t + d\mathcal{C}_t^{\tau-} - r_t\mathrm{CA}_t dt + d\mathcal{F}_t^{\tau-} - d\mathcal{H}_t^{\tau-},$$

which is equivalent to (30), where the right-hand side is a local martingale in view of the first identity in (29). ∎

Remark 5.1 Note that, assuming integrability, the CA risk-neutral valuation formula in (29) is equivalent to a martingale condition on L joint with a terminal condition $\mathrm{CA}_{\bar{\tau}} = 0$ (cf. the comments following (9) in the static setup). This underlies another point of view on valuation, where no valuation opertor is introduced, but L is instead assumed to be a risk-neutral local martingale, jointly with a terminal condition $\mathrm{CA}_{\bar{\tau}} = 0$. Under these alternative assumptions the ensuing CA formulas would be the same. A martingale condition on L can be interpreted as a shareholder no-arbitrage condition. ∎

The following immediate corollary to Lemma 5.1 shows that contra-liabilities can be interpreted as a wealth transfer triggered by deals away from shareholders, due to the impossibility for the bank to hedge its own jump-to-default risk.

Theorem 5.1 The formula $\mathrm{CCR}_t = \mathbb{E}_t \int_t^{\bar{\tau}} \beta_t^{-1}\beta_s d\mathcal{C}_s$ holds independently of the funding and hedging policy of the bank. Moreover:
(i) If the bank could replicate its own default, i.e. for $\Delta_\tau\mathcal{H} = \Delta_\tau\mathcal{C} + \Delta_\tau\mathcal{F}$, then we would have

$$\mathrm{CL} = 0,\ \mathrm{CA} = \mathrm{CCR}. \tag{32}$$

(ii) But the bank cannot hedge its own default, i.e. $\Delta_\tau\mathcal{H} = 0$, hence

$$\begin{aligned} \mathrm{CA}_t &= \mathbb{E}_t \int_t^{\bar{\tau}} \beta_t^{-1}\beta_s d\mathcal{C}_s^{\tau-} + \mathbb{E}_t \int_t^{\bar{\tau}} \beta_t^{-1}\beta_s d\mathcal{F}_s^{\tau-}, \\ \mathrm{CL}_t &= \mathbb{E}_t\big[\beta_t^{-1}\beta_\tau \mathbb{1}_{\{\tau<T\}}(-\Delta_\tau\mathcal{C})\big] + \mathbb{E}_t\big[\beta_t^{-1}\beta_\tau \mathbb{1}_{\{\tau<T\}}(-\Delta_\tau\mathcal{F})\big]. \end{aligned} \tag{33}$$

In the special case where

$$d\mathcal{H}_t = \beta_t^{-1} d(\beta_t \mathrm{CA}_t) + d\mathcal{C}_t^{\tau-} + d\mathcal{F}_t^{\tau-},\ 0 \le t \le \bar{\tau} \tag{34}$$

(assuming this would be an achievable hedging gain process), then the process L is constant and (23) yields ES = 0. ∎

The default exposure of the derivative portfolio as well as the collateralization and funding policies of the bank determine the data $\mathcal{C}$ and $\mathcal{F}$ of the CA, CL and L processes. Once $\mathcal{C}$ and $\mathcal{F}$ are specified, Theorem 5.1 can be turned into concrete formulas for all the XVA components and for the dynamics of the process L which is required as input data in KVA computations. See Albanese et al. (2016) for illustration in the case of a bank engaged into bilateral portolios. Note that the process $\mathcal{F}$ typically involves the CA process, hence the CA formula in (33) is in fact an equation for the CA process (cf. the developments following (8) in a static setup and see Example 5.2).

The industry terminology tends to distinguish an FVA in the technical sense of the cost of funding cash collateral for variation margin from an MVA defined as the cost of funding segregated collateral posted as initial margin (see Albanese et al. (2016)). The academic literature tends to merge the two in an overall FVA meant in the broader sense of the cost of funding the derivative trading strategy of the bank. Such an overall FVA would correspond to the second term in the CA formula in (33), where the first term corresponds to the CVA (cf. the static CA formula (8)). The first and second terms in the CL formula in (33) correspond to the continuous-time analogs of the DVA and FDA in the static CL formula (12).

5.1 Connection with the Modigliani and Miller (1958) Theorem

The Modigliani and Miller (1958) theorem includes two key assumptions. One is that, as a consequence of trading, total wealth is conserved. The second assumption is that markets are complete. In our setup we keep the wealth conservation hypothesis but we lift the completeness. This ensures the validity of the part of the theorem stating that the wealth of the bank as a whole should not depend on the funding policy of the bank, which corresponds to the first statement in Theorem 5.1. But the part stating that the interests of shareholders and creditors are aligned with each other, i.e. that (32) holds, is only valid if the bank can hedge its own default. Since this is not the case in practice, the derivative portfolio of the bank triggers a wealth transfer from the shareholders to the creditors by the amount CL in (33).

Besides, the impossibility of replicating counterparty default losses (the hedging gain process in (34) is typically not achievable in practice) implies that the process L is not constant but fluctuates in time. Hence, capital needs to be set at risk by the shareholders, which therefore deserve a risk premium. This risk premium is gradually released to shareholders in the form of the RM (or KVA) payments, which is the topic of the next section.

Remark 5.2 The situation of market incompleteness that we consider in this paper is that of a bank subject to trading constraints preventing it from hedging jump-to-default risk. For different (unrelated to counteparty credit risk) extensions of the Modigliani and Miller (1958) theorem in incomplete markets, see Gottardi (1995) and the references there. ∎

6 Cost of Capital

To generate a dividend flow as a risk compensation to the bank shareholders, the bank clients are asked to pay an additional amount, which in the insurance literature is called risk margin, while in the banking literature it is currently called capital valuation adjustment (KVA). The KVA is not treated as reserve capital, but rather as a retained earning which contributes to economic capital and is different from CA in that it is not a capital deduction. The level of compensation required by shareholders is driven by market considerations. Typically, investors in banks expect a hurdle rate h of about 10% to 12%. In this paper we work with an exogenous and constant hurdle rate h. An endogenous h would arise in a situation of competitive equilibrium between different banks (as opposed to our setup where only one bank is considered).

When a bank charges cost of capital to clients, these revenues are accounted for as profits. However, since prevailing accounting standards for derivative securities are based on the theoretical assumption of market completeness, they do not envision a mechanism to retain these earnings for the purpose of remunerating capital across the entire life of transactions that can be as long as decades. In complete markets, there is no justification for market and credit risk capital (the only justification for reserving capital is to cover operational risk, which we do not consider in this paper). Hence, mark-to-market profits are immediately distributable. A strategy of earning retention beyond the end of the ongoing accounting year (or quarter) is still possible as in all firms, but this would be regarded as purely a business decision, not subject to financial regulation under the Basel III Accord.

This leads to an explosive instability characteristic of a leverage ratchet effect. For instance, if a bank starts off today by entering a 30-year swap with a client, the bank books a profit. Assuming the trade is perfectly hedged, the profit is distributable at once. The following year, the bank still needs capital to absorb the risk of the 29-year swap in the portfolio. In order to remunerate shareholders, given that the profits from this trade have already been distributed the previous year, the bank has no other option than to lever up, i.e. sell and hedge another swap, book a new profit and distribute the dividend to shareholders that are now posting capital for both swaps. As long as trading volumes grow exponentially, the scheme self-sustains. When exponential growth stops, the bank return on equity crashes. The financial crisis of 2007-2008 can be largely explained along these lines (see Figure 3). In the aftermath of the crisis, the first casualty was the return on equity for the fixed income business as profits had already been distributed and market-level hurdle rates could not be sustained by portfolio growth.

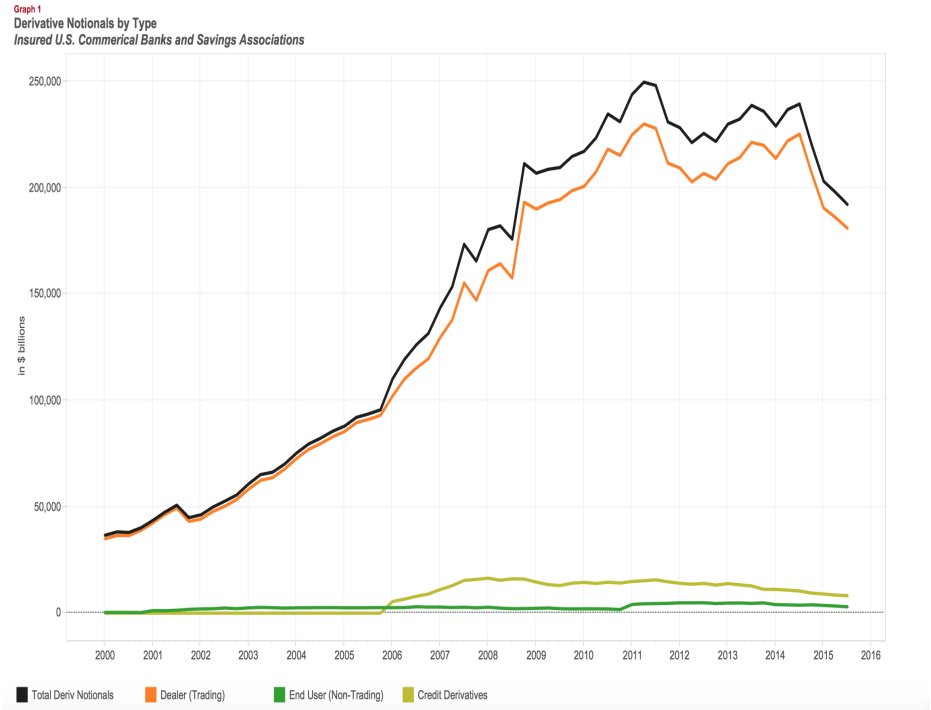

Figure 3: Global financial crisis Ponzi scheme (Source: Office of the comptroller of the currency, Q3 2015 quarterly bank trading revenue report).

Interestingly enough, however, in the insurance domain, the Swiss Solvency Test (2004) and Solvency II, unlike Basel III, do regulate the distribution of retained earnings through a mechanism tied to so called risk margins envisioned as cost of capital. Also, the accounting standards set out in IFRS 4 Phase II (see IFRS (2012, 2013) are consistent with Solvency II and include a treatment for risk margins that has no analogue in the banking domain. See Wüthrich and Merz (2013), Eisele and Artzner (2011) and Salzmann and Wüthrich (2010) regarding the risk margin and cost of capital actuarial literature.

The purpose of this section is to discuss a Solvency II inspired framework for assessing cost of capital (KVA) for a bank, pass it on to the bank clients and distribute it gradually to the bank shareholders through a dividend policy which would be sustainable even in the limit case of a portfolio held on a run-off basis, with no new trades ever entered in the future.

Solvency II requires in addition that EC must be higher than the ensuing risk margin (i.e. the KVA). In this paper, we show how to handle this constraint and how to handle it optimally.

Remark 6.1 The purpose of this article is to discuss general principles and, from this viewpoint, it seems clear that Solvency II establishes a strong precedent which one cannot ignore. Even under the current banking regulatory environment, it would be perfectly possible for the board of directors of a bank to decide to implement the KVA strategy of this paper on a voluntary basis, even without a prescriptive regulatory environment, as a way to implement a sustainable dividend distribution policy. ∎

6.1 KVA Equations

Let $C \geq \mathrm{ES}$ represent a putative economic capital process for the bank. To size the KVA we ask the following question: What should be the level $K = K_t(C)$ of a KVA account ending up with $K_{\bar{\tau}} = 0$ and such that the ensuing dividend stream

$$-(dL_t + d\mathrm{KVA}_t - r_t\mathrm{KVA}_t dt) \tag{35}$$

(cf. (24)) corresponds to an average (expected) remuneration $h(C_t - K_t)dt$ at each point in time $t \in [0, \bar{\tau}]$ to the shareholders?

The reason why $K_{\bar{\tau}} = 0$ is because less would mean insufficient and more would be wasteful. More precisely, if the KVA was negative at any point in time before $\bar{\tau}$, this would mean that the RM account was insufficiently provisioned to satisfy shareholders remuneration requirement for their capital at risk. A jump to a negative value at the exact time τ in case $\tau < T$ might not be an issue. Such a scenario looks a bit artificial anyway. We exclude it for the sake of the well-posedness of the KVA equation. As we will prove in Theorem 6.1(ii), proceeding along the above lines yields a KVA process which is nonnegative on $[0, \bar{\tau}]$, as desired.

The reason why $(C_t - K_t)$ appears rather than C_t after (35) is because risk margins are loss-absorbing and therefore part of the economic capital. Therefore, shareholder capital at risk, which by assumption is remunerated at the hurdle rate h, only corresponds to the difference $(C_t - K_t)$.

The above-sketched approach can be contrasted with alternative KVA approaches in the literature, developed by Green, Kenyon, and Dennis (2014) and Green and Kenyon (2016) or Elouerkhaoui (2016), where all the pricing adjustments are viewed as part of fair valuation and where, in particular, the KVA is treated as a contra-asset for reserve capital such as CVA or FVA, as if the KVA was a capital deduction. By contrast, we consider that, as risk margin is retained earnings meant to be released to the bank shareholders, it does not belong to the balance sheet as a liability. This view is consistent with the treatment of the risk margin in Swiss solvency test capital at risk calculations.

Under our standing modelling assumptions, we have $\mathbb{P} = \mathbb{Q}$, under which the CA desk loss process L is a local martingale. Hence the above conditions on the process K reduce to

$$K_{\bar{\tau}} = 0 \text{ and } dK_t + \big(h(C_t - K_t) - r_t K_t\big)dt \text{ is a local martingale on } [0, \bar{\tau}]. \tag{36}$$

This differential specification is in the form of a linear backward stochastic differential equation (BSDE, see El Karoui, Hamadène, and Matoussi (2009) for a survey), equivalent to the integrated form (37) below. As we demonstrate in Lemma 6.1 below, the solution $K = K_t(C)$ to this equation is unique. Furthermore, since the equation is linear, the solution is given by the explicit formula (39). However, the difference $(C_t - K_t)$ represents shareholder capital at risk and must therefore be nonnegative. If one accounts for the resulting consistency condition $C \geq K(C)$, then, as we will see in detail in Sect. 6.2, the KVA BSDE becomes of Lipschitz type (38), which, as our next result shows, is also well-posed.

We denote by $\mathcal{L}^p$ the space of $\cdot^p$-integrable processes over $[0, \bar{\tau}]$, for any $p \geq 1$.

Lemma 6.1 *Consider the following BSDEs:*

$$K_t = \mathbb{E}_t \int_t^{\bar{\tau}} \big(hC_s - (r_s + h)K_s\big)ds,\ t \in [0, \bar{\tau}], \tag{37}$$

$$\mathrm{KVA}_t = \mathbb{E}_t \int_t^{\bar{\tau}} \big(h \max(\mathrm{ES}_s, \mathrm{KVA}_s) - (r_s + h)\mathrm{KVA}_s\big)ds,\ t \in [0, \bar{\tau}] \tag{38}$$

to be solved for respective processes K and KVA. *Assuming that r is bounded from below and that C (respectively* ES*) and r are in $\mathcal{L}^2$, then the BSDE* (37) *(respectively* (38)*) is well posed in $\mathcal{L}^2$, where well-posedness includes existence, uniqueness, and so-called comparison.*

The $\mathcal{L}^2$ solution K to (37) *admits the explicit representation*

$$K_t = h\mathbb{E}_t \int_t^{\bar{\tau}} e^{-\int_t^s (r_u + h)du} C_s ds,\ t \in [0, \bar{\tau}]. \tag{39}$$

In case the process L is constant, then ES *and the solution* KVA *to* (38) *vanish.*

Proof. In terms of the coefficient

$$f_t(k) = h\Big(\max(\mathrm{ES}_t, k) - k\Big) - r_t k = h \max\big(\mathrm{ES}_t, k\big) - (r_t + h)k,\ k \in \mathbb{R}, \tag{40}$$

the KVA BSDE (38) appears as

$$\mathrm{KVA}_t = \mathbb{E}_t \int_t^{\bar{\tau}} f_s(\mathrm{KVA}_s)ds,\ t \in [0, \bar{\tau}]. \tag{41}$$

For any real $k, k' \in \mathbb{R}$ and $t \in [0, \bar{\tau}]$, we have

$$\begin{aligned}\big(f_t(k) - f_t(k')\big)(k - k') &= -(r_t + h)(k - k')^2 + h\big(\max(\mathrm{ES}_t, k) - \max(\mathrm{ES}_t, k')\big)(k - k')) \\ &\leq -r_t(k - k')^2 \leq C(k - k')^2,\end{aligned}$$

for some constant C (having assumed r bounded from below), so that the coefficient f satisfies the so-called monotonicity condition. Moreover, for $|k| \leq \bar{k}$, we have:

$$|f_{\cdot}(k) - f_{\cdot}(0)| \leq h \max\left(|\mathrm{ES}|, \bar{k}\right) + |h + r|\bar{k} + h\mathrm{ES}^{+}.$$

Hence, assuming that ES and r are in $\mathcal{L}^2$, the following integrability conditions hold:

$$\sup_{|k|\leq \bar{k}} |f_{\cdot}(k) - f_{\cdot}(0)| \in \mathcal{L}^1, \text{ for any } \bar{k} > 0, \qquad \text{and} \qquad f_{\cdot}(0) \in \mathcal{L}^2.$$

Therefore, by application of the general filtration BSDE results of Kruse and Popier (2016, Sect. 5), the BSDE (41) is well-posed in $\mathcal{L}^2$, where well-posedness includes existence, uniqueness and comparison. Even simpler computations prove the analogous statements regarding the linear BSDE (37). Moreover, (39) obviously solves (37). Finally, in case L is constant, then ES vanishes and KVA = 0 obviously solves (38). ∎

6.2 The KVA Constrained Optimization Problem

Under the Solvency II capital requirement, economic capital is the sum between shareholder capital at risk (SCR) and risk margin (the insurance analog of the KVA, which are also loss-absorbing). Some actuarial literature dwells with the puzzle according to which the calculation of the risk margin depends on economic capital projections in the future, while economic capital itself depends on the risk margin, an apparently circular dependency (see e.g. Salzmann and Wüthrich (2010, Sect. 4.4) and Robert (2013)).

This paper addresses the problem of circular dependency as follows. First we compute the economic capital according to some risk measure. Then we define KVA using economic capital projections discounted at a hurdle rate as of (39). Our SCR is defined a posteriori as the difference (EC − KVA).

However, for so doing, we need to account for the additional constraint that EC ≥ KVA. Otherwise this would break the consistency condition

$$\mathrm{SCR} \geq 0. \tag{42}$$

Remark 6.2 In our notation, the Solvency II accounting condition in Wüthrich and Merz (2013, Definition 9.15 (a)) reads SCR ≥ CA − RC. In our continuously reset framework where CA = RC at all times, this condition is the same as (42). ∎

A BSDE based KVA approach allows addressing the constraint (42) as follows. To emphasize the dependence of K on C, we henceforth denote by $K(C)$ the solution (39) to the linear BSDE (37). We define the set of admissible economic capital processes

$$\mathcal{C} = \{C \in \mathcal{L}^2; C \geq \max\left(\mathrm{ES}, K(C)\right)\}, \tag{43}$$

where $C \geq$ ES is the risk acceptability condition and $C \geq K(C)$ is the consisteny condition (cf. the respective conditions (b) and (a) and their discussion in Wüthrich and Merz (2013, pages 270 and 271)). Assuming ES in $\mathcal{L}^2$, we define

$$\mathrm{EC} = \max(\mathrm{ES}, \mathrm{KVA}), \tag{44}$$

where KVA is the $\mathcal{L}^2$ solution to the BSDE (38).

Lemma 6.2 *Assuming that r is bounded from below and that* ES *and r are in $\mathcal{L}^2$, the solution* KVA *to* (38) *solves the linear BSDE* (37) *for the implicit data $C = \mathrm{EC}$, i.e. we have* $\mathrm{KVA} = K(\mathrm{EC})$, *that is,*

$$\mathrm{KVA}_t = h\mathbb{E}_t \int_t^{\bar{\tau}} e^{-\int_t^s (r_u+h)du} \mathrm{EC}_s ds,\ t \in [0, \bar{\tau}]. \tag{45}$$

In particular, the KVA *process discounted at the OIS rate is a supermartingale.*

Proof. The process KVA is in $\mathcal{L}^2$ and, by virtue of (38) and (44), we have, for $t \in [0, \bar{\tau}]$,

$$\begin{aligned}\mathrm{KVA}_t &= \mathbb{E}_t \int_t^{\bar{\tau}} \Big(h \max\big(\mathrm{ES}_s, \mathrm{KVA}_s\big) - (r_s + h)\mathrm{KVA}_s\Big) ds \\ &= \mathbb{E}_t \int_t^{\bar{\tau}} \Big(h\mathrm{EC}_s - (r_s + h)\mathrm{KVA}_s\Big) ds.\end{aligned} \tag{46}$$

Hence, the process KVA solves the linear BSDE (37) for $C = \mathrm{EC} \in \mathcal{L}^2$. The identity $\mathrm{KVA} = K(\mathrm{EC})$ follows by uniqueness of an $\mathcal{L}^2$ solution to the linear BSDE (37) established in Lemma 6.1. Equation (45) follows by an application of (39). The supermartingale property of the KVA discounted at the OIS rate is visible in the following differential form of (46): $\mathrm{KVA}_{\bar{\tau}} = 0$ and

$$d\mathrm{KVA}_t + \big(h(\mathrm{EC}_t - \mathrm{KVA}_t) - r_t\mathrm{KVA}_t\big)dt \text{ is a local martingale on } [0, \bar{\tau}], \tag{47}$$

i.e.

$$d(\beta\mathrm{KVA})_t + h\beta_t(\mathrm{EC}_t - \mathrm{KVA}_t)dt \text{ is a local martingale on } [0, \bar{\tau}], \tag{48}$$

where the drift is nonnegative by definition (44) of EC. ∎

The next result shows that the consistency condition $\mathrm{SCR} \geq 0$ is optimally handled by defining the KVA through the BSDE (38) and the ensuing EC process as (44). In fact, EC thus defined is the minimal admissible economic capital process, with the cheapest ensuing cost of capital given by the KVA, which is also shown to be nondecreasing in the hurdle rate h.

Theorem 6.1 *Under the assumptions of Lemma 6.2, we have:*
(i) $\mathrm{EC} = \min \mathcal{C}, \mathrm{KVA} = \min_{C \in \mathcal{C}} K(C)$;
(ii) *The process* KVA *is nonnegative and it is nondecreasing in h.*

Proof. **(i)** We saw in Lemma 6.2 that $\mathrm{KVA} = K(\mathrm{EC})$, hence

$$\mathrm{EC} = \max(\mathrm{ES}, \mathrm{KVA}) = \max\big(\mathrm{ES}, K(\mathrm{EC})\big),$$

therefore $\mathrm{EC} \in \mathcal{C}$. Moreover, for any $C \in \mathcal{C}$, we have (cf. (40)):

$$f_t(K_t(C)) = h \max\big(\mathrm{ES}_t, K_t(C)\big) - (r_t + h)K_t(C) \leq hC_t - (h + r_t)K_t(C).$$

Hence, the coefficient f of the KVA BSDE (38) never exceeds the coefficient of the linear BSDE (37) when both coefficients are evaluated at the solution $K_t(C)$ of (37). Since these are BSDEs with equal (null) terminal condition, the BSDE comparison theorem applied to the BSDEs (37) and (38) (see Kruse and Popier (2016, Proposition 4 and

Remark 3)) yields KVA $\leq K(C)$. Consequently, KVA $= \min_{C\in\mathcal{C}} K(C)$ and, for any $C \in \mathcal{C}$,

$$C \geq \max(\mathrm{ES}, K(C)) \geq \max(\mathrm{ES}, \mathrm{KVA}) = \mathrm{EC}.$$

Hence EC $= \min \mathcal{C}$.

(ii) The 97.5% expected shortfall of a centered random variable is nonnegative. Hence ES is nonnegative and the KVA is nonnegative, by (45) and (44). Since ES is nonnegative, then, as visible in (40), the coefficient $f_t(k)$ of the KVA BSDE (38) is nondecreasing in the hurdle rate parameter h. So is therefore in turn the $\mathcal{L}^2$ solution KVA to (38), by the BSDE comparison theorem of Kruse and Popier (2016, Proposition 4 and Remark 3) applied to the BSDE (38) for different values of h. ∎

7 Incremental XVA Methodology

In all the above the derivative portfolio of the bank is assumed held on a run-off basis, i.e. set up at time 0 and assuming that no new trades will ever enter the portfolio in the future until its final maturity T. In practice derivative portfolios are incremental and models are precisely required for computing incremental XVA values at every new trade. This last section of the paper relates the run-off assumption to a pricing approach myopic to the trade-flow, as this cannot be anticipated by a market maker, and which would be sustainable even in the limit case where no new trades would be entered into the future. This way, we arrive to a sustainable strategy for profit retention, which is the key principle behind Solvency II.

Banks are market makers and, as such, they are price makers. Bank clients are price takers willing to accept a loss in a trade for the sake of receiving benefits that become apparent only once one includes their real investment portfolio, which cannot be done explicitly in a pricing model.

The manager of a market maker portfolio cannot decide on asset selection: trades are proposed by clients and the market maker needs to stand ready to bid for a trade at a suitable price no matter what the trade is and when it arrives. A new trade has two impacts: it triggers a wealth transfer from shareholders to bondholders and alters the risk profile of the portfolio. This is reflected by a jump of the balance sheet, from the one related to the endowment (pre-trade portfolio) right before the time $t = 0$ (say) the new deal is considered, to the one related to the portfolio including the new deal. Let us denote by "$\Delta\cdot$" the corresponding jump of any of our balance sheet metrics in Figure 1. Again, the arrival process for new trades and the client decision whether to accept the bank bid or not follow stochastic processes which cannot be anticipated. Therefore the corresponding jumps in the balance sheet cannot be predicted or offset ex-ante. Once a trade happens, it has to be computed at the moment and, for upgrading from one balance sheet to the other at the new deal, a market maker has no other option than asking clients to refill the RC and RM accounts by the required amounts of

$$\Delta\mathrm{RC} = \Delta\mathrm{CA} \text{ and } \Delta\mathrm{RM} = \Delta\mathrm{KVA}, \tag{49}$$

where the equalities hold by virtue of the continuous reset assumption in each of the balance sheets.

If future trades could be anticipated (this is the argument in Hull and White (2016)), then one could optimise further and assess the fair valuation of debt ahead of time to compensate for the anticipated occurrence of wealth transfers. This would lead to greater efficiencies and allow banks to bid for trades at entry prices given by unadjusted fair valuations. However, the impossibility of anticipating new trades and otherwise offsetting wealth transfers, justifies the use of incremental XVAs computed under the run-off assumption in the balance sheets with and without the new deal.

In view of (49) and (1), consistent with the preliminary static setup formulas (15), the all-inclusive XVA add-on to the entry price for a new deal, dubbed funds transfer price (FTP), appears as

$$\mathrm{FTP} = \Delta\mathrm{CA} + \Delta\mathrm{KVA} = \Delta\mathrm{CCR} + \Delta\mathrm{CL} + \Delta\mathrm{KVA}, \tag{50}$$

where ΔCCR would be the counterparty credit risk add-on fair to the bank as a whole, ΔCL is the compensation for the wealth triggered by the new deal away from shareholders and ΔKVA is their premium for the risk on their equity arising from the mis-hedge of the contra-assets (mis-hedge of counterparty default losses and funding expenditures), all computed in an incremental run-off basis as explained above.

Obviously, the endowment has a key impact on the FTP of a new trade. It may very well happen that a new deal is risk-reducing with respect to the pre-existing portfolio, in which case $\mathrm{FTP} < 0$.

8 Conclusion

To conclude this paper we emphasize its main technical insights.

In order to focus on counterparty credit risk and XVAs, we assume that the market risk of the bank is perfectly hedged by means of perfectly collateralized back-to-back trades. This back-to-back hedge perspective does not only result in more concise derivation of the XVA equations, direct as opposed to two-step in most other XVA references in the literature, where XVA equations are obtained as the difference between a linear equation for the mark-to-market of the portfolio ignoring counterparty credit risk and another equation for the "risky value" of the portfolio. The back-to-back hedge perspective also yields a much clearer view on the cash flows that generate the contra-liabilities. In particular, Section 3 clarifies in an elementary static setup how, accounting for the back-to-back hedge, these cash flows are not a fiction or an abstract compensation of others, but actual cash flows that fall into the estate of the defaulted bank and increase the realized recovery rate of creditors.

We root our XVA approach on a capital structure model, depicted in Figure 1 in Section 4.2, acknowledging the impossibility for a bank to replicate jump-to-default related cash flows. Such a balance sheet perspective is key in identifying the economic meaning of the XVA accounting terms as well as the connections between them. As synthetized in Proposition 4.1 and Theorem 5.1, due to counterparty credit risk incompleteness, the derivative portfolio of the bank triggers a wealth transfer from shareholders to creditors that can only be compensated by a corresponding add-on to entry prices.

Moreover, shareholders bear the trading risk of a central XVA desk (dubbed CA desk in this paper) in charge of counterparty risk and funding within the bank. Our KVA is formalized as the cost (45) of a remuneration policy of shareholder capital at risk, which would be sustainable even in the limit case of a portfolio held on a run-off basis, with no new trades ever entered in the future. Conceived in the spirit of a portfolio optimization tool for a derivative market maker in incomplete counterparty credit risk markets, the KVA in this sense is a risk premium tuned in order to keep the shareholders on an "efficient frontier" such that

$$\text{“Average instantaneous return}_t = \text{Risk aversion } h \times \text{ Risk measure}_t \times dt\text{”}$$

(cf. (36)). Our KVA definition is genuinely dynamic, as opposed to multi-period simply in the Solvency II actuarial literature. This allows us to solve the puzzle according to which the calculation of the risk margin depends on economic capital projections in the future while economic capital itself depends on the risk margin, an apparently circular dependency. Specifically, Theorem 6.1 states and solves the ensuing KVA problem as a constrained optimization problem, where economic capital and its cost are minimized under a nonnegativity consistency condition on the ensuing shareholder capital at risk (SCR, which in our model appears endogenously as the difference between economic capital and the KVA).

Under the structural approach of this paper, the KVA is a risk premium taking as input data the contra-assets mis-hedge loss-and-profit process L. Thanks to this connection, the CA equation for contra-assets valuation and the KVA equation, i.e. the XVA equations as a whole, are a self-contained problem.

As opposed to the other XVAs, our KVA is not the valuation of some cash flows, but a risk premium. As risk margin is retained earnings meant to be released to the bank shareholders, the KVA in our sense does not belong to the balance sheet as a liability, at least not statically as part of contra-assets. But, in some sense, the KVA is a measure of the fluctuations of the balance sheet.

A last important output of this work is that the ensuing XVA methodology, even though rooted in the analysis of a bank balance sheet, does not require problematic balance sheet data. All it requires in practice is the modeling of the trading loss-and-profit process L, which plays the role of a reduced dynamic model of the balance sheet. We refer the reader to Albanese, Caenazzo, and Crépey (2016) for illustration in the concrete setup of a bank engaged in bilateral trade portfolios.

A Assumptions

We list the key assumptions by order of appearance in the body of the paper, with comments. At the bottom of this work lies the fact that a bank cannot replicate jump-to-default exposures. However we do not state this as a standing assumption, because it is instructive to see what would happen if a bank could replicate jump-to-default exposures (we then find that CL=KVA=0).

A key assumption is (A5), as it also underlies (A6) through (A9).

(A1) **All cash flows are valued by their risk-free discounted $(\mathbb{G}, \mathbb{Q})$ conditional expectation, assumed to exist.**

Ensures the internal consistency of the valuation setup.

(A2) **Derivative entry prices include, on top of the valuation of the corresponding cash flows, a KVA risk premium.**

Remunerates bank shareholders for their capital at risk that needs be reserved against trading risk (mis-hedge of contra-assets, under (A4)).

(A3) **The historical probability measure $\mathbb{P}$ coincides with the pricing measure $\mathbb{Q}$.**

As little of relevance can be said about the historical probability measure for XVA computations entailing projections over decades, the discrepancy between $\mathbb{P}$ and $\mathbb{Q}$ is left to model risk.

(A4) **The market risk of the bank is perfectly hedged by means of perfectly collateralized back-to-back trades.**

Made in order to focus on counterparty credit risk and XVAs. Yields direct derivations of the XVA equations. Harmless as far as the equations themselves are concerned.

(A5) **A bank is a market maker which cannot anticipate future trades.**

Implies that banks do have trading costs and have no other option than passing these costs to their clients to survive in the long run.

(A6) **The positive impact of trades on the realized recovery of the bank is not reflected in the bank funding spreads.**

A consequence of (A5). The positive impact of future trades on the realized recovery of the bank is not anticipated and therefore not reflected in the price of borrowing for funding these trades.

(A7) **We consider a derivative portfolio held on a run-off basis.**

The right assumption to make, incrementally at every new trade, in order to achieve pricing and dividend policies that are robust, i.e. sustainable even in the limit case where no new trades would ever be entered in the future (cf. (A5)).

(A8) **Losses-and-earnings are marked to model and realized in real time.**

Until the payers cease willing to do so and the bank defaults. This is in the spirit of the regulation which imposes regular realignments of the different banking accounts to their theoretical target values.

(A9) $\mathrm{CET1} = \mathrm{EC} + \mathrm{UC} - \mathrm{D} = y$, **a given constant**.

A natural complement of the run-off and continuous reset assumptions (A7) and (A8).

B Acronyms

A	Assets.
AE	Accounting equity.
BSDE	Backward stochastic differential equation.
CA	Contra-assets (or their valuation).
CCR	Counterparty credit risk (or its valuation).
CL	Contra-liabilities (or their valuation).
CDS	Credit default swap.
CET1	Core equity tier I capital.
CVA	Credit valuation adjustment.
D	Debt.

DVA	Debt valuation adjustment.
EC	Economic capital.
ES	Expected shortfall.
FC	Free capital (UC−D).
FDA	Funding debt adjustment (the contra-liability counterpart of the FVA).
FRTB	Fundamental review of the trading book.
FTP	Funds transfer price.
FVA	Funding valuation adjustment.
IFRS	International financial reporting standards.
KVA	Capital valuation adjustment (the banking notion of risk margin).
L	Liabilities.
OIS	Overnight index swap.
RC	Reserve capital (or CA account).
RM	Risk margins (or KVA) account.
SCR	Shareholder capital at risk.
TLAC	Total loss-absorbing capacity.
UC	Uninvested capital.
XVA	Generic "X" valuation adjustment

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