XVA Analysis From the Balance Sheet

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Abstract

Since the crisis, derivative dealers charge to their clients various add-ons, dubbed XVAs, meant to account for counterparty risk and its capital and funding implications. XVAs deeply affect the derivative pricing task by making it global, nonlinear, and entity dependent. However, before the technical implications, the fundamental points are to understand what deserves to be priced and what does not, and to establish, not only the pricing, but also the corresponding collateralization, dividend, and accounting policy of a bank.

If banks cannot replicate jump-to-default related cash flows, deals trigger wealth transfers from bank shareholders to creditors and shareholders need to set capital at risk. On this basis, we devise a theory of XVAs, whereby so-called contra-liabilities and cost of capital are sourced from bank clients at trade inceptions, on top of the fair valuation of counterparty risk, in order to compensate shareholders for wealth transfer and risk on their capital.

The resulting all-inclusive XVA formula, to be sourced from clients incrementally at every new deal, reads (CVA + FVA + KVA), where C sits for credit, F for funding, and where the KVA is a cost of capital risk premium. This formula corresponds to the cost of the possibility for the bank to go into run-off, while staying in line with shareholder interest, from any point in time onward if wished.

Keywords: Counterparty risk, market incompleteness, wealth transfer, cost of capital, capital valuation adjustment (KVA), balance sheet of a bank.

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Abbreviations

AE Accounting equity.

CA Contra-assets valuation.

CDS Credit default swap.

CM Clean margin.

CR Fair valuation of counterparty risk.

CL Contra-liabilities valuation.

CET1 Core equity tier I capital.

CVA Credit valuation adjustment.

DVA Debt valuation adjustment.

EC Economic capital.

FDA Funding debt adjustment.

FRTB Fundamental review of the trading book.

FTP Funds transfer price.

FVA Funding valuation adjustment.

KVA Capital valuation adjustment.

MtM Mark-to-market.

OIS Overnight index swap.

RC Reserve capital.

RM Risk margin.

SCR Shareholder capital at risk.

UC Uninvested capital.

XVA Generic "X" valuation adjustment.

Unless explicitly specified, an amount paid (received) means effectively paid (received) if positive, but actually received (paid) if negative. A similar convention applies to the notions of cost vs. benefit, loss vs. gain, etc.

1 Introduction

Since the crisis, investment banks charge to their clients, in the form of rebates with respect to the counterparty-risk-free value (dubbed mark-to-market, MtM) of financial derivatives, various add-ons meant to account for counterparty risk and its capital and funding implications. These add-ons are generically termed XVAs, where VA stands for valuation adjustment and X is a catch-all letter to be replaced by C for credit, D for debt, F for funding, M for margin, K for capital (!), and so on.

XVAs deeply affect the derivative pricing task by making it global, nonlinear, and entity dependent. But, before the technical implications and specific models are considered, the fundamental points are to:

1 Understand what deserves to be priced and what does not (which includes a number of doubling counting issues), and

2 Establish, not only the pricing, but also the corresponding collateralization, dividend, and accounting policy of the bank.

These are the two ***objectives*** of this paper.

1.1 Context

Coming after several papers on the valuation of defaultable assets in the 90s (see e.g. Duffie and Huang (1996)), Bielecki and Rutkowski (2002, (14.25) p. 448) yields the formula (CVA − DVA) for the fair valuation of bilateral counterparty risk on a swap assuming risk-free funding. This formula, rediscovered and generalized by others since the crisis (cf. e.g. Brigo and Capponi (2010)), is symmetrical, i.e. it is the negative of the analogous quantity considered from the point of view of the counterparty, consistent with the law of one price and the Modigliani and Miller (1958) theorem.

Around 2010, the materiality of the DVA windfall benefit of a bank at its own default time became the topic of intense debates in the quant and academic finance communities. At least, it seemed reasonable to admit that, if the own default risk of the bank was accounted for in the modeling (in the form of a DVA benefit), then the cost of funding (FVA) implication of this risk should be included as well, which would lead to the modified formula (CVA − DVA + FVA). See for instance Burgard and Kjaer (2011, 2013, 2017), Crépey (2015), Brigo and Pallavicini (2014), or Bichuch, Capponi, and Sturm (2017); see also Bielecki and Rutkowski (2015) for an abstract funding framework (without explicit reference to XVAs), generalizing Piterbarg (2010) to a nonlinear setup.

Then Hull and White (2012) objected that the FVA was only the compensator of another windfall benefit of the bank at its own default, corresponding to the non-reimbursement by the bank of its funding debt. Accounting for the corresponding "DVA2" (akin to the FDA in Albanese and Andersen (2014) or in the present paper) would bring back to the original fair valuation formula:

$$\mathrm{CVA} - \mathrm{DVA} + \mathrm{FVA} - \mathrm{FDA} = \mathrm{CVA} - \mathrm{DVA}.$$

However, it was soon understood (cf. Burgard and Kjaer (2013, end of Section 3.1)) that this argument implicitly assumes that the bank can perfectly hedge its own default, which, as a bank is an intrinsically leveraged entity, is not the case in practice. On the (still ongoing, to some extent) FVA debate, see also Castagna (2012), Burgard and Kjaer (2012), Albanese and Andersen (2015, 2014), Albanese, Andersen, and Iabichino (2015), and Andersen, Duffie, and Song (2017).

1.2 Contents of the Paper

In fact, our view is that counterparty risk entails two distinct but intertwined sources of market incompleteness (see Section 2.1):

- A bank cannot[1] hedge its own jump-to-default exposure,
- A bank cannot perfectly hedge counterparty default losses.

In this article we specify the banking XVA metrics that align derivative entry prices to shareholder interest, given this impossibility for a bank to replicate the jump-to-default related cash flows. Pricing approaches in incomplete markets include cost-of-capital, utility indifference, risk minimisation and minimal martingale measures, utility maximisation and minimisation over martingale measures, good-deal pricing, market-consistent valuation or probability distortions, among others (see e.g. Schweizer (2001), Rogers (2001), Cochrane and Saa-Requejo (2000) or Madan (2015)). We develop a counterparty risk cost-of-capital pricing approach, whereby so-called contra-liabilities and cost of capital are sourced from bank clients at trade inceptions, on top of the fair valuation of counterparty risk, in order to compensate shareholders for wealth transfer and risk on their capital.

This research fits into a stream of XVA literature reflecting an increasing shift, regarding derivative valuation, from the viewpoint of the bank as a whole to a bank shareholder-centric view. One can mention the related corporate finance notion of debt overhang in Myers (1977), by which a project valuable for the firm as a whole may be rejected by shareholders because the project is mainly valuable to creditors. But, until recently, such considerations were hardly considered in the field of derivative pricing. Related articles in this regard are Burgard and Kjaer (2011, 2013, 2017), Albanese and Andersen (2015, 2014), Albanese et al. (2015), Andersen et al. (2017), Green, Kenyon, and Dennis (2014), and Green and Kenyon (2015).

However, except for the last two, these papers only consider FVA (or its avatar MVA); they do not propose an approach to KVA. Not only a bank cannot hedge its own jump-to-default: It cannot replicate its counterparty default losses either. Hence the trading of the bank generates a non-vanishing loss-and-profit process L. Then the regulator comes and requires that capital be set at risk by the shareholders, which therefore require a risk premium.

Based on these principles, regarding ***objective 1*** above, we show that the counterparty risk rebate to the mark-to-market of a deal that aligns its entry price to the

[1] Or, at least, does not, cf. Remark 9.1.

interest of bank shareholders is, instead of the fair valuation (CVA − DVA) of counterparty risk:

$$\mathrm{CVA} + \mathrm{FVA} + \mathrm{KVA}, \tag{1}$$

all to be computed unilaterally in a certain sense (even though we do include the default of the bank itself in our modeling) and charged to clients on an incremental run-off basis for every new deal. Our KVA is a risk premium, in the line of the risk margin in the Swiss Solvency Test (2004) and in the eurozone Solvency II insurance regulations. This comes in contrast with other approaches in the literature discussed in detail in Section 9.2, such as Green et al. (2014) and Elouerkhaoui (2016), and some theoretical actuarial literature, where the KVA is treated as a liability (contra-asset like the CVA and the FVA). For instance, in their perspective, KVA (dividend) payments contribute to the trading profit-and-loss of the bank. But dividends are meant to remunerate risk, i.e. unhedged trading losses. Hence the XVA loop does not close. Instead, our KVA approach devises entry prices which keep the position of a derivative market maker on an "efficient frontier" corresponding to a given return (hurdle rate) on invested equity capital. Cf. also Brigo, Francischello, and Pallavicini (2017) for a recent KVA paper in the direction of indifference pricing rules (see Remark 6.1).

Regarding ***objective 2***, we interpret the XVA add-on (1), meant incrementally at every new deal, as the cost of the possibility for the bank to go into run-off, i.e. lock its portfolio and let it amortize in the future, while staying in line with shareholder interest, from any point in time onward if wished. This "soft landing option" is key from a regulator point of view, as it guarantees that the bank should not be tempted to go into snowball or Ponzi kind of schemes, where always more trades are entered for the sole purpose of funding previously entered ones (cf., a contrario, the 2008 Ponzi scheme displayed in Figure 1). Thus we arrive to a sustainable strategy for profits retention, much in the spirit of the above-mentioned insurance regulation.

As also emphasized in Burgard and Kjaer (2011) and Andersen et al. (2017), the rise of the XVA metrics reflects a shift, regarding the pricing and risk management of financial derivatives, from a hedging paradigm to balance sheet optimization. The view that a bank cannot replicate jump-to-default related cash flows invalidates some of the conclusions of Modigliani-Miller theory but not all (see Section 5.5). The situation of market incompleteness that we consider in this paper is that of a bank subject to trading constraints preventing it from hedging jump-to-default risk. For other extensions of the Modigliani and Miller (1958) theorem in incomplete markets, see Gottardi (1995) and the references there.

1.3 Detailed Outline and Results

Section 2 sets a financial stage where a bank is split across several trading desks with cash flows detailed in Section 3. Consistency of valuation across the different trading desks is ensured by a suitable invariance assumption exposed in Section 4. However, in our setup, valuation is not price, which, in order for the bank to be able to remunerate shareholder capital at risk, entails an additional deduction in the form of

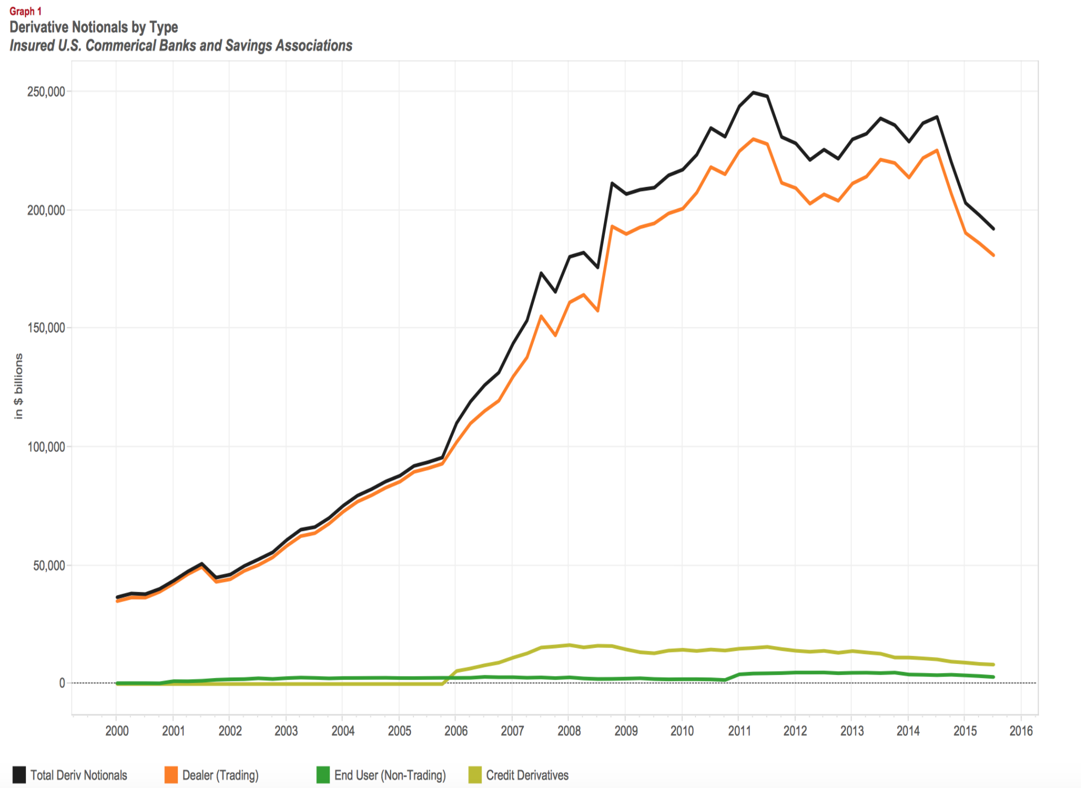

Figure 1: 2008 financial derivative Ponzi scheme (Source: Office of the comptroller of the currency, Q3 2015 quarterly bank trading revenue report).

a cost-of-capital risk premium. Based on these principles, Section 5 derives the XVA equations that apply to a static portfolio. Section 6 introduces the "soft landing" pricing, accounting, and dividend policy associated with the corresponding incremental XVA approach in the realistic case of a dynamic portfolio.

Sections 2 through 6 yield the conceptual backbone of our approach. Section 7 establishes various well-posedness and comparison results for the XVA equations of Section 5. Section 8 revisits the XVA equations when different choices are made on the fate of the residual reserve capital and risk margin in case of default of the bank. Section 9 discusses in the light of the outputs of this paper alternative XVA approaches that have been developed in the last years in what we therefore call the XVA benchmark model, namely a Black–Scholes model for the underlying market risk factor, combined with independent Poisson counterparties and bank defaults.

Section A details the balance sheet perspective on the XVA metrics. Section B yields explicit XVA formulas in a one-period static setup.

Three reference results of this paper are:

- Theorem 5.1, which identifies the contra-asset CA = CVA + FVA value process of the bank as the counterparty risk deduction to apply to the mark-to-market of the portfolio in order to account for the inability for the bank to hedge its own jump-to-default exposure;

- Theorem 7.1, which optimally solves the economic capital and KVA puzzle according to which the calculation of the KVA depends on economic capital projections in the future, while economic capital itself depends on the KVA, an apparently circular dependency;

- Theorem 6.1, which identifies the funds transfer price (FTP), i.e. the all-inclusive counterparty risk deduction aligning the counterparty-risk-free valuation of a new trade to the interest of bank shareholders, as the incremental (CVA+FVA+KVA) of the trade.

Three reference XVA formulas in the paper are the unilateral CVA, FVA, and KVA formulas (58), (59), and (46). The first two are specific to the illustrative setup of Section 7.4, but Lemma 7.4 makes it clear in general that, although we include the default of the bank itself in our modeling, the XVA formulas accounting for all the wealth transfers involved are unilateral, pricing the related cash flows until the final maturity T of the portfolio (as opposed to $\bar{\tau} = \tau \wedge T$ in most of the related XVA literature, where τ is the default time of the bank).

2 Financial Setup

This section sets the financial stage.

Assumption 2.1 We consider a bank, which is a market maker, engaged into derivative trading with clients. ∎

A market maker cannot decide on asset selection: trades are proposed by clients and the market maker needs to stand ready to bid for a trade at a suitable price no matter what the trade is and when it arrives.

Bank clients are price taker corporates willing to accept a loss in a trade for the sake of receiving benefits acounting for their base business line. These benefits become apparent only once one includes their real investment portfolio, which cannot be done explicitly in a pricing model.

Remark 2.1 In an asymmetric setup with a price maker and a price taker, the price maker passes his costs to the price taker. For transactions between dealers, it is possible that one is the price maker and the other one is the price taker. It is also possible that a transaction triggers gains for the shareholders of both entities. The detailed consideration of these dynamics would lead to an understanding of the drivers to economical equilibrium in a situation where multiple dealers are present (as opposed to our setup where only one bank is considered). ∎

2.1 From Hedging to Netting, Capital, and Collateralization

Counterparty risk is related to cash flows or valuations linked to either counterparty default or the default of the bank itself.

The risk of financial loss as a consequence of client default is hard to replicate, because single name CDS instruments that could in principle be used for that purpose are illiquid and are typically written on bonds, not on swaps with rapidly varying value ("contingent credit swap exposures" to which counterparty risk exposures are tantamount).

The possibility for the bank of hedging its own default is even more questionable since, in order to hedge it, a bank would need to be able to freely trade its own debt. But banks are special firms that are intrinsically leveraged and cannot be transformed into a pure equity entity. There is also an argument of scale. It has been estimated that, if all European banks were to be required to have capital equal to a third of liabilities, then the total capitalization of banks would be greater than the total capitalization of the entire equity market as we know it today. Last, even if a bank was free to redeem all its debt, bank shareholders could not effectively monetise the hedging benefit, which would be hampered by bankruptcy costs. See Castagna and Fede (2013, Section 10.7) for an extensive discussion regarding this impossibility for a bank to hedge its own jump-to-default exposure, and cf. also Burgard and Kjaer (2013, Sections 3.1 and 3.2).

As counterparty risk is hard to hedge in practice, the management of counterparty risk is more about netting and collateralization than hedging, the residual risk being handled on reserve and capital bases.

Netting means that counterparty default exposures are assessed at an aggregated portfolio (or netted sets) level, after accounting for value compensation between the components of each netting set. The exact specification of the netting sets depends on the trading setup of the bank (bilateral and/or centrally cleared, in particular). Collateral means cash or liquid assets that are posted to guarantee some transactions against the default of the posting party. Liquidation losses beyond the collateral posted by a defaulted party impact the capital at risk of the surviving party.

Assumption 2.2 There exists of a publically observable risk-free rate process r, called OIS rate.

We assume cash only collateral, which stays the property of the posting party unless a default in a related netting set happens. Posted collateral is remunerated at the OIS rate. ∎

OIS rate stands for overnight indexed swap rate, which, whenever available, is together the best market proxy for a risk-free rate and the reference rate for the remuneration of cash collateral. Collateral can be re-hypothecable, in the form of a variation margin that tracks the mark-to-market of a deal and can be re-used by the receiving party for its funding purposes. A variation-margined position is still subject to gap risk, which is the risk of slippage between the mark-to-market of a netting set and its variation margin during the liquidation period of a defaulted party. Hence, an additional layer of margin can be required, in the form of a (typically segregated and dynamically updated) "initial margin". Under the current regulation, both forms of collateralization are mandatory, with some variants specific to bilateral versus centrally cleared transactions (see e.g. Basel Committee on Banking Supervision (2015) and Official Journal of the European Union (2012)).

2.2 One Bank, Several Desks, Different Stakeholders

A bank is a defaultable entity and has two different classes of stakeholders: shareholders and creditors.

Assumption 2.3 Shareholders have the control of the bank and are solely responsible for investment decisions before bank default. At the bank default time τ, shareholders are wiped out.

The cash flows paid (or received) by the bank prior its default time τ affect the bank shareholders, whereas the cash flows received (or paid) by the bank during its default resolution period that starts at τ only affect creditors. ∎

The counterparty default losses and the (other than risk-free) funding expenditures of the bank can be viewed as synthetic derivative payoffs, which we call contra-assets, that emerge, due to counterparty risk, at the aggregated level of the portfolio of the bank. We call CA desk(s) a mathematical entity regrouping the CVA desk and the FVA desk (the latter sometimes known as the ALM or Treasury) of a bank. From a high level perspective to be detailed in Section 3.4, the CA desk values the contra-assets and charges the corresponding CVA and FVA to the clients of the bank, deposits the corresponding payment in a reserve capital account, and then is exposed to the corresponding payoffs. As time proceeds, counterparty default losses and funding expenditures occur and are covered by the CA desk with the reserve capital account.

After the contracts have thus been filtered out of their counterparty risk and risky funding implications by the CA desk, the other trading desks of the bank, which we call clean desks, are left with the the management of the market risk of the contracts in their respective business lines, ignoring counterparty risk. In particular, as we will see in more detail in Section 3.3, the re-hypothecable collateral provided by the Treasury of the bank to the clean desks funds their trading at the risk-free cost of collateral remuneration. Hence the clean desks do not need any cash account, the financing function of a cash account being already played by their OIS remunerated collateral account, dubbed clean margin account henceforth.

Observe that, in our setup, we, in fact, deal with two portfolios: the client portfolio, between the clients of the bank and the CA desk, and the cleaned portfolio, between the CA desk and the clean desks. The corresponding (cumulative streams of) contractually promised cash flows are the same. But, as should be intuitively clear from the above and seen in detail below, counterparty risk only impacts the client portfolio.

Each (CA or clean) trading desk may also setup a related hedge (typically dynamic and possibly imperfect, especially in the case of the CA desk), which generates a (cumulative) hedging loss process intended to control the fluctuations of their respective trading loss-and-profit.

Assumption 2.4 The activity of each trading desk, hence of the bank as a whole, is self-financing. ∎

At this stage this is only a broad axiom, to be made precise in concrete setups later (cf. (12), specialized in concrete setups as (79) and (55)), by which we mean that the wealth of a trading desk at a later time results from the development of its wealth at an earlier time as the sole consequence of its pricing, hedging, funding, and collateralization policy, and of the evolution of the underlying risk factors, without any further creation or annihilation of cash flows.

We emphasize that the CA desk is not in charge of the KVA, which under our approach is of a different nature and is treated separately by the management of the bank. Namely, on top of reserve capital, so-called risk margin is sourced by the bank from the clients at each deal, deposited into a risk margin account, and then gradually released as KVA payments into the shareholder dividend stream. The risk margin account also yields OIS interest payments to the shareholders.

Notation 2.1 We write CM, RC, and RM for the respective amounts on the clean margin, reserve capital, and risk margin accounts of the bank. ∎

Note that, from the point of view of the CA (respectively clean) desks of the bank, CM is actually an asset (respectively liability) if positive, a liability (respectively asset) if negative.

The rise of the XVA metrics reflects a shift, regarding the pricing and risk management of financial derivatives, from a hedging paradigm to balance sheet optimization. In fact, the afore-described setup can be viewed as an abstraction of the bank balance sheet (or, vice versa, the latter can be viewed as a metaphorical representation of the former), displayed in Figure 3 in Section A, where the horizontal subdivision (from bottom to top) embodies the functional distinction between clean desks, CA desk, and the management of the bank, whereas the vertical one has to do with the capitalistic distinction between shareholders and creditors.

2.3 Continuous Reset Assumption and Default Model of the Bank

In practice, losses-and-earnings realization times are typically quarter ends for bank profits, released as dividends, versus recapitalization managerial decision times for losses. For simplicity in our setup, we work under the so-called instantaneous (or continuous) reset assumption, according to which:

Assumption 2.5 Losses(-and-earnings) are marked to model and realized in real time. ∎

This means that, instead of viewing losses as depletions of the different bank accounts, we view losses as continuously realized and compensated by bank shareholders; instead of viewing losses as money flowing away from the balance sheet (cf. Section A), we view them as money flowing into it as refill, i.e. replenishment of the different accounts at their theoretical target levels (to be determined in the forthcoming sections), until the point of default of the bank where the shareholders cease willing to do so.

When this happens is modeled as a totally unpredictable event at some exogenous time τ calibrated to the bank CDS curve, which we view as the most reliable and informative credit data regarding anticipations of markets participants about future recapitalization, government intervention, bail-in (e.g. via contingent convertible bonds) and other bank failure resolution policies.

In particular, in line with Assumption 2.3:

Assumption 2.6 At the bank default time τ, the property of any residual value on the reserve capital and risk margin accounts is instantaneously transferred from the shareholders to the creditors of the bank.

Remark 2.2 The default of a bank means not exactly insolvency in the sense of negative equity, i.e. assets less than liabilities, but, more precisely, liquid assets less than short term liabilities: The legal definition of default is an unpaid coupon or cash flow, which is a liquidity event. Such liquidity events are indeed the typical failure scenario for large banks (or other regulated financial entities), including Bear Stearns, Lehman, and AIG in 2008 (see e.g. Duffie (2010)). Bear Stearns had billions of capital at the time of its default.

Actually, in our model where bank shareholders continuously pay for the losses until bank default, positive equity at bank default makes perfect sense: For shareholders, the bank not defaulting means that they continue to bear further losses. They should wish to give an end to it at some point, whatever the equity of the bank at that moment. See Section A.2 for more details about it. ∎

3 Cash Flows

In this section we study the impact on the wealth of the bank and the bank shareholders of the trading of the clean and CA desks.

Assumption 3.1 The derivative portfolio of the bank is static, i.e. set up at time 0 and such that no new trades ever enter the portfolio in the future. ∎

Notation 3.1 $\mathcal{P}$ is the cumulative stream of contractually promised cash flows of the (client or cleaned) portfolio;

T is an upper bound on the maturity of all claims in the portfolio, also accounting for the time of liquidating the position between the bank or any of its clients in case of default;

$\bar{\tau} = \tau \wedge T$. ∎

In practice, derivative portfolios are not static but incremental (unless the bank goes into run-off), and models are especially required for computing incremental XVA values at every new trade. Assumption 3.1 will be relaxed accordingly in Section 6.

3.1 Probabilistic Setup

We work in an abstract dynamic setup on a measurable space $(\Omega, \mathcal{A})$, meant to cover discrete and continuous time in a common formalism. This abstract setup is specialized to a one-period model in Section B and to a continuous-time model in Section 7.

Assumption 3.2 Clean desks price and hedge using some reference filtration $\mathbb{F} = (\mathfrak{F}_t)_{t\geq 0}$ such that the bank default time τ is not an $\mathbb{F}$ stopping time. The OIS rate r and the contractually promised cash flow stream $\mathcal{P}$ are $\mathbb{F}$ adapted.

The full model information used by the CA desk, as well as by the management of the bank in charge of the KVA payments, is a larger filtration $\mathbb{G} = (\mathfrak{G}_t)_{t\geq 0}$ such that τ is a $\mathbb{G}$ stopping time.

Any $\mathbb{G}$ stopping time η admits an $\mathbb{F}$ stopping time η' such that

$$\eta \wedge \tau = \eta' \wedge \tau; \tag{2}$$

Any $\mathbb{G}$ semimartingale Y admits a unique $\mathbb{F}$ semimartingale Y', called the $\mathbb{F}$ reduction of Y, that coincides with Y before τ. ∎

The different units within the bank use not only different filtrations, but also potentially different pricing measures measures $\mathbb{P}$ on $\mathfrak{F}_T$ and $\mathbb{Q}$ on $\mathfrak{G}_T$, equivalent on $\mathcal{F}_T$:

Notation 3.2 Conditional expectation with respect to $(\mathfrak{G}_t, \mathbb{Q})$ (respectively $(\mathfrak{F}_t, \mathbb{P})$) is denoted by $\mathbb{E}_t$ (respectively $\mathbb{E}'_t$), or simply $\mathbb{E}$ (respectively $\mathbb{E}'$) if $t = 0$. ∎

We also cover the (simpler but unrealistic) situation of a default-free bank as a separate case where $\tau = +\infty$ holds $\mathbb{Q}$ a.s. and $(\mathbb{F}, \mathbb{P}) = (\mathbb{G}, \mathbb{Q})$.

Notation 3.3 Given Y representing

(i) a process of cumulative cash flows, such as a hedging loss process,

(ii) respectively a spot quantity, such as a price or XVA, or the amount on a banking account,[2]

we denote by $\widetilde{Y}$ the corresponding process of (i) cumulative OIS discounted cash flows, (ii) respectively the corresponding OIS discounted process. ∎

[2]The latter is for consistency with the fact that, in our setup, each of the banking accounts yields OIS interest payments to its owner.

Example 3.1 In continuous time with OIS discount factor denoted by β, we have $\widetilde{\mathcal{P}} = \int_0^{\cdot} \beta_t d\mathcal{P}_t$, but $\widetilde{\mathrm{CVA}} = \beta\mathrm{CVA}$. ∎

Assuming the OIS based discount factor $\mathbb{F}$ predictable:

Corollary 3.1 *In the cumulative case* (i) *above, the process Y is a martingale (respectively constant) if and only if $\widetilde{Y}$ is a martingale (respectively constant).* ∎

Definition 3.1 Given a $\mathbb{G}$ (respectively $\mathbb{F}$) adapted cumulative cash flow stream $\mathcal{D}$, the OIS discounted $(\mathbb{G},\mathbb{Q})$ (respectively $(\mathbb{F},\mathbb{P})$) value process D of $\mathcal{D}$ is the $(\mathbb{G},\mathbb{Q})$ (respectively $(\mathbb{F},\mathbb{P})$) conditional expectation process of the future OIS discounted cash flows in $\mathcal{D}$, defined on the time interval $[0,\bar{\tau}]$ (respectively $[0,T]$). That is, using Notations 3.2 and 3.3:

$$\widetilde{D}_t = \mathbb{E}_t(\widetilde{\mathcal{D}}_T - \widetilde{\mathcal{D}}_t),\ t \in [0,\bar{\tau}],\ \text{respectively}\ \widetilde{D}_t = \mathbb{E}'_t(\widetilde{\mathcal{D}}_T - \widetilde{\mathcal{D}}_t),\ t \in [0,T]\,. ∎$$

3.2 Wealth and Loss Processes

Notation 3.4 $\mathcal{W}^{cl}$ is the wealth process of the clean desks, as it results from their trading by an application of the self-financing Assumption 2.4, defined with respect to the reference filtration $\mathbb{F}$ and as such ignoring the default of the bank;

$\mathcal{W}^{ca}$ is the wealth process of the CA desk, as it results from its trading by an application of the self-financing Assumption 2.4 with respect to the filtration $\mathbb{G}$;

$\mathcal{W}$ is the ensuing wealth process of the bank as a whole, i.e.

$$\mathcal{W} = \mathcal{W}^{cl} + \mathcal{W}^{ca}. ∎ \tag{3}$$

Notation 3.5 For any process Y, the corresponding process stopped before the bank default time τ is denoted by Y° (whenever well-defined). We also define $Y^{\bullet} = Y^{\circ} - Y$, so that

$$Y = Y^{\circ} - Y^{\bullet}. ∎ \tag{4}$$

Remark 3.1 In order to avoid the ambiguity between "a process Y stopped before τ" in the respective meanings of $Y = Y^{\circ}$ or to denote the stopped process Y°, we abuse the terminology by saying that a process Y is "without jump at τ" if $Y = Y^{\circ}$, i.e. $Y^{\bullet} = 0$. ∎

In view of the distinction between shareholders and creditors (cf. Assumptions 2.3 and 2.6), one should differentiate the impact of trading on the wealth of the bank as a whole (shareholders and creditors altogether), represented by the above wealth processes, from its impact on the wealth of the bank shareholders, corresponding in our terminology (assessed as loss after sign change) to loss processes. However, this is ignored by the clean desks, which do not incorporate the default of the bank in their modeling: the latter is only local, at the level of their particular business line. But this is then corrected by the CA desk, which, by nature, has a modeling view at the level of the bank as a whole:

Definition 3.2 By trading loss L^{cl} of the clean desks, we mean the negative of their wealth process $\mathcal{W}^{cl}$.

By trading loss L^{ca} of the CA desk, we mean the negative of its wealth process $\mathcal{W}^{ca}$, stopped before τ for alignment with shareholder interest, i.e.

$$L^{ca} = -(\mathcal{W}^{ca})^{\circ}. \tag{5}$$

By trading loss L of the bank, we mean the negative of its wealth process $\mathcal{W}$, stopped before τ, for alignment with shareholder interest, i.e. (cf. (3))

$$L = -(\mathcal{W}^{cl} + \mathcal{W}^{ca})^{\circ} = (L^{cl})^{\circ} + L^{ca}. \blacksquare \tag{6}$$

From the balance sheet perspective, the wealth process $\mathcal{W}$ of the bank and its trading loss process L correspond "morally" to the appreciation of accounting equity and to the depreciation of core equity tier I capital: See however Section A for more detail about this.

3.3 Clean Valuation, Mark-to-Market, and Clean Wealth Process

Definition 3.3 By clean valuation of a contract (or portfolio of contracts) with $\mathbb{F}$ adapted contractually promised cash flow stream $\mathcal{D}$, we mean the $(\mathbb{F}, \mathbb{P})$ value process of $\mathcal{D}$. $\blacksquare$

Note that, for simplicity, we only consider European derivatives.

Corollary 3.2 *Clean valuation is additive over contracts and intrinsic to the contracts themselves. In particular, it is independent of the involved parties and of their collateralization, funding, and hedging policies.* $\blacksquare$

Remark 3.2 Market quotes used for model calibration are typically prices provided by the clean desks of different banks at time 0. Hence clean valuation is the relevant notion of valuation at the stage of model calibration. $\blacksquare$

Notation 3.6 Given a contract labeled by c in the cleaned portfolio, we denote by $\mathcal{P}^c$, CM^c, $\mathcal{H}^c$, and P^c the related contractually promised cash flow stream, amount on the clean margin account, (clean) hedging loss, and clean valuation, all assumed $\mathbb{F}$ adapted. $\blacksquare$

In particular, we thus have

$$\mathcal{P} = \sum_c \mathcal{P}^c, \ \mathrm{CM} = \sum_c \mathrm{CM}^c \tag{7}$$

on $[0, T]$, where, here and henceforth, c in the summations ranges over all the contracts in the cleaned portfolio.

Ignoring the default of the bank (only taking client defaults into account), the liquidation time of each contract c is modeled as an $\mathbb{F}$ stopping time $\tau_c^{\delta} \leq T$.

Example 3.2 Assuming a contract c liquidated some time $\delta \geq 0$ after the default of the involved counterparty of the bank, labeled by say 1, with default time τ_1, then $\tau_c^\delta = (\tau_1 + \delta) \wedge T$.

If the contract is in fact novated (instead of liquidated strictly speaking) by a risk-free counterparty replacing the defaulted counterparty in its contractual obligations from time $(\tau_1 + \delta)$ onward, then $\tau_c^\delta = T$. ∎

Assumption 3.3 For each contract c in the cleaned portfolio, we set

$$\mathrm{CM}^c = P^c \mathbb{1}_{[0,\tau_c^\delta)}. \blacksquare \tag{8}$$

In the notion of mark-to-market of the (cleaned or client) portfolio, counterparty risk is ignored for the future, but only the contracts not yet liquidated are considered:

Definition 3.4 By mark-to-market (MtM) of the cleaned portfolio, we mean

$$\mathrm{MtM} = \sum_c \mathbb{1}_{[0,\tau_c^\delta)} P^c \tag{9}$$

on $[0, T]$. ∎

Of course, as long as no contracts have been liquidated (in particular, as long as none of the counterparties of the bank has defaulted), the clean valuation, say P, and the mark-to-market of the portfolio coincide. For instance, in the treatment of centrally cleared trading adopted in Armenti and Crépey (2017), we view contracts as novated rather than liquidated as the consequence of defaults (cf. Example 3.2), so that in this case $\mathrm{MtM} = P$ holds on $[0, T]$.

Combining (7), (8), and (9):

Corollary 3.3 *We have* $\mathrm{CM} = \mathrm{MtM}$ *on* $[0, T]$. ∎

See the dashed boxes at the bottom of the bank balance sheet in Figure 3.

Lemma 3.1 *We have*

$$\widetilde{W}^{cl} = \sum_c (\widetilde{P}^c + \widetilde{\mathcal{P}}^c - \widetilde{\mathcal{H}}^c)^{\tau_c^\delta}, \tag{10}$$

where $\cdot^{\tau_c^\delta}$ *means stopping at time* τ_c^δ.

Proof. The wealth equation (10) expresses that, ignoring the default of the bank, each cleaned contract c delivers its promised cash flows and the related hedging gains to the clean desk in charge of the contract until its liquidation time (after which it leaves the portfolio of the bank), also accounting, under Assumption 2.5, for the continuous reset of the clean margin account as per (8), and for the transfer of property of the collateral $P^c_{\tau_c^\delta}$ to the receiving party at time τ_c^δ. ∎

3.4 CA Desk Wealth Process

The contractually promised cash flows of the client and cleaned portfolios are both given by $\mathcal{P}$. But counterparty risk should only really impact the client portfolio. Toward this end, the standing market rule regarding the settlement of derivative transactions at defaults is that:

Assumption 3.4

- On the client portfolio side, at the time a party defaults, the property of the collateral posted on each involved netting set is transferred to the collateral receiver. Moreover, at each liquidation time:
  - any positive value due to a defaulted party is paid in full to its creditors (or the non payer is also declared in default),
  - any positive value due by a defaulted counterparty is only paid up to some recovery rate,

  where value is understood on a mark-to-market basis at the netted set level, and net of the corresponding (already transferred) collateral, but inclusive of all the promised contractual cash flows unpaid during the liquidation period;
- On the cleaned portfolio side, the cash flows corresponding to the contracts mirroring the defaulted ones are delivered as promised, until the contracts are settled at their clean value when liquidated. ∎

In concrete setups, taking the difference between the (setup dependent) client and cleaned portfolio cash flows as per the two bullet points in Assumption 3.4 yields the counterparty exposure (CVA and DVA originating) cash flows $\mathcal{C}$: See e.g. Lemmas B.1 and 7.6 below.

The funding cash flows of the CA desk (which is in charge of the risky funding of the bank) are of the form

$$(-\text{OIS accrual of the reserve capital account}) + \mathcal{F}, \tag{11}$$

where the first term is the negative of the funding benefit to which funding would boil down for the CA desk if risk-free funding was available to the bank. Hence, the process $\mathcal{F}$ is interpreted as the risky funding costs of the bank. In practice, the risky funding cash flows $\mathcal{F}$ are determined by Assumption 2.4 applied to the collateralization and funding policy of the bank: See e.g. Lemmas B.1 and 7.5 below.

Last, the (partial) hedging policy of the CA desk results in a cumulative CA desk hedging loss process $\mathcal{H}$.

Figure 2 gives a schematic representation of the different cash flows that affect the CA desk.

Remark 3.3 In the case of centrally cleared trading, the role of the clean desks is played by the CCP, which effectively "clears the delta" of the bank, providing a fully collateralized back-to-back hedge to its client portfolio (see Armenti and Crépey (2017, Figures 1 and 2)). ∎

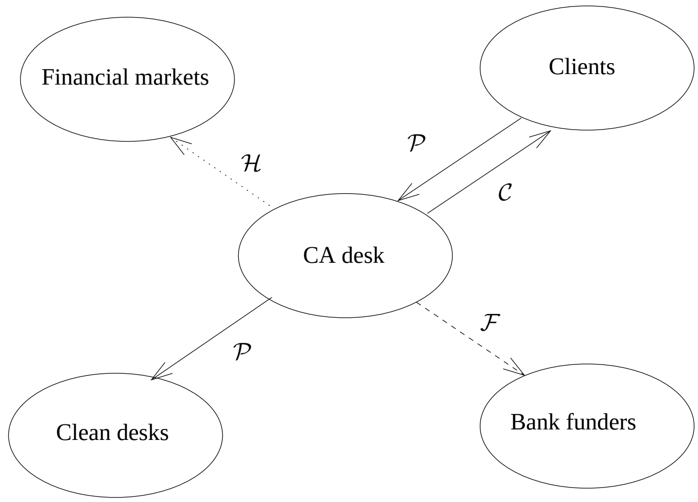

Figure 2: CA desk cash flow graph.

Lemma 3.2 *We have*

$$
\begin{aligned}
&\widetilde{\mathcal{W}}^{ca} = -(\widetilde{\mathrm{RC}} + \widetilde{\mathcal{C}} + \widetilde{\mathcal{F}} + \widetilde{\mathcal{H}}),\\
&\widetilde{\mathcal{W}} = -(\widetilde{\mathrm{RC}} + \widetilde{\mathcal{C}} + \widetilde{\mathcal{F}} + \widetilde{\mathcal{H}}) + \sum_c (\widetilde{P}^c + \widetilde{\mathcal{P}}^c - \widetilde{\mathcal{H}}^c)^{\tau_c^\delta}.
\end{aligned}
\tag{12}
$$

Proof. Under Assumption 2.4, the first line in (12) results from the CA desk cash flows structure summarized in Figure 2, also accounting, under Assumption 2.5, for the continuous reset of the reserve capital account. The second line follows in view of (3) and (10). ∎

See (79) and (55) for illustrations in concrete set-ups.

3.5 Loss Processes and Shareholder Dividends

By Definition 3.2 applied to the output of Lemma 3.2:

Corollary 3.4 *We have*

$$
\begin{aligned}
&\widetilde{L}^{ca} = \widetilde{\mathrm{RC}}^{\circ} + \widetilde{\mathcal{C}}^{\circ} + \widetilde{\mathcal{F}}^{\circ} + \widetilde{\mathcal{H}}^{\circ}\\
&\widetilde{L} = \widetilde{\mathrm{RC}}^{\circ} + \widetilde{\mathcal{C}}^{\circ} + \widetilde{\mathcal{F}}^{\circ} + \widetilde{\mathcal{H}}^{\circ} + (\widetilde{L}^{cl})^{\circ},
\end{aligned}
\tag{13}
$$

where

$$
\widetilde{L}^{cl} = -\sum_c (\widetilde{P}^c + \widetilde{\mathcal{P}}^c - \widetilde{\mathcal{H}}^c)^{\tau_c^\delta}. \ ∎
\tag{14}
$$

See (78) and (51) for illustrations in concrete set-ups.

Lemma 3.3 *Shareholder cumulative discounted dividends are given by*

$$-(\widetilde{\mathrm{RM}}^{\circ} + \widetilde{L}). \tag{15}$$

Proof. Under the continuous reset Assumption 2.5, the trading gains $-L = -L^{\circ}$ of the bank (cf. (5) and (6)) continuously flow into the shareholder dividend stream. Also accounting for the KVA payments from the risk margin account, for the OIS interest that is earned by the shareholders on this account, and for the default of the bank that stops the dividend stream before τ, cumulative discounted dividends net to (15). ∎

We emphasize that, in our model, negative dividends are possible (cf. Section 2.3). They are interpreted as recapitalisation, i.e. equity dilution.

4 Pricing Principles

This section states shareholder optimization cost-of-capital pricing principles for the cash flows of Section 3.

4.1 Valuation of the Funding and Hedging Assets

Assumption 4.1 The hedging loss $\mathcal{H}^c$ related to the hedging of each contract c by the clean desks, including the cost of setting the hedge, is an $(\mathbb{F}, \mathbb{P})$ martingale starting from zero. ∎

The rationale is that hedging gains arise in practice as the stochastic integral of predictable hedging ratios against wealth processes of individual hedging assets. Note that we are considering wealth processes inclusive of the associated funding costs here, which corresponds to the most common situation of hedges that are either swapped or traded through a repo market, without upfront payment (see Crépey, Bielecki, and Brigo (2014, Section 4.2.1) for more details). Under the cash flow $(\mathbb{F}, \mathbb{P})$ valuation rule in Definition 3.1, each (clean) hedging asset is valued as OIS discounted expectation of its future cash flows. Hence, the wealth processes related to long positions in any of the clean hedging assets are $(\mathbb{F}, \mathbb{P})$ martingales, as are stochastic integrals against them. As a consequence, $\mathcal{H}^c$ is an $(\mathbb{F}, \mathbb{P})$ martingale starting from zero.

Example 4.1 Assuming the hedge of a contract c of the cleaned portfolio implemented through a repo market on a Black-Scholes stock S with volatility σ (in a continuous-time setup), then, supposing no dividends and no repo basis on S:

$$d\mathcal{H}^c_t = -\zeta^c_t\big(dS - rS_t dt\big) = -\zeta^c_t \sigma S_t dW_t, \tag{16}$$

where W is the $(\mathbb{F}, \mathbb{P})$ Brownian motion driving S and ζ^c is the hedging ratio used in S.

The instantaneous cost of funding the hedge is $(-\zeta^c_t r S_t dt)$, which is included in (16). ∎

Assumption 4.2 The hedging loss $\mathcal{H}$ of the CA desk, including the costs of setting the hedge, is a $(\mathbb{G},\mathbb{Q})$ martingale starting from zero, without jump at τ. ∎

Same rationale as for Assumption 4.1, this time with respect to $(\mathbb{G},\mathbb{Q})$. Moreover, the assumption $\mathcal{H}=\mathcal{H}^{\circ}$ is made for consistency with our premise that a bank cannot hedge its own jump-to-default exposure (cf. Section 2.1).

Assumption 4.3 The risky funding cash flows $\mathcal{F}$ are a $(\mathbb{G},\mathbb{Q})$ martingale starting from 0, with nondecreasing $\mathcal{F}^{\circ}$ and $\mathcal{F}^{\bullet}$ components. ∎

The rationale underlying Assumption 4.3 is that risky funding is implemented in practice by the CA desk as the stochastic integral of predictable hedging ratios against funding assets. Under the cash flow $(\mathbb{G},\mathbb{Q})$ valuation rule in Definition 3.1, the value process of each of these assets is a martingale modulo risk-free accrual. Therefore the risky funding costs of the CA desk accumulate into a $(\mathbb{G},\mathbb{Q})$ martingale $\mathcal{F}$ in (11), coming on top of risk-free accrual (actual benefit, i.e. negative cost) of the reserve capital account: See e.g. Lemmas B.1 and 7.5 below.

The assumption that $\mathcal{F}^{\circ}$ is nondecreasing rules out models where the bank can invest (not only borrow) at his unsecured borrowing spread over OIS, because, as a consequence on $\mathcal{F}^{\bullet}$ through the martingale condition on the process $\mathcal{F}$ as a whole, this would imply that the bank can hedge its own jump-to-default exposure. That is, we assume only windfall at bank own default, no shortfall: See Section 9.1 below.

The last parts in Assumptions 4.2 and 4.3 also insure some kind of orthogonality between the risky funding and hedging loss martingales $\mathcal{F}$ and $\mathcal{H}$, so that $\mathcal{F}$ and $\mathcal{H}$ are nonsubstituable to each other (and the bank cannot manipulate by using one for the other, see again Section 9.1).

As immediate consequences of Assumptions 4.2–4.3:

Corollary 4.1 *The processes $\mathcal{H}=\mathcal{H}^{\circ}$ and $\mathcal{F}$ have zero $(\mathbb{G},\mathbb{Q})$ value.* ∎

4.2 Invariance Valuation Setup

Consistency of valuation across the perspectives of the different desks of the bank is ensured by the following:

Assumption 4.4 The process L^{cl} is an $(\mathbb{F},\mathbb{P})$ martingale on $[0,T]$. The process L^{ca} is a $(\mathbb{G},\mathbb{Q})$ martingale on $[0,\bar{\tau}]$, without jump at τ.

The pricing measures measures $\mathbb{P}$ and $\mathbb{Q}$ are equivalent (but possibly different) on $\mathfrak{F}_T$. Stopping before τ turns $(\mathbb{F},\mathbb{P})$ martingales on $[0,T]$ into $(\mathbb{G},\mathbb{Q})$ martingales on $[0,\bar{\tau}]$ without jump at τ. Conversely, the $\mathbb{F}$ reductions[3] of $(\mathbb{G},\mathbb{Q})$ martingales on $[0,\bar{\tau}]$ without jump at τ are $(\mathbb{F},\mathbb{P})$ martingales on $[0,T]$. ∎

[3] cf. Assumption 3.2.

The respective martingale assumptions on L^{cl} and L^{ca} in Assumption 4.4 should be viewed as broad shareholder no-arbitrage conditions, seen from the respective perspectives of the clean and CA desks. Note in particular that, as a consequence of Assumption 4.4 and (6):

Corollary 4.2 *The trading loss L of the bank is a $(\mathbb{G}, \mathbb{Q})$ martingale on $[0, \bar{\tau}]$ without jump at τ. Its $\mathbb{F}$ reduction L' is an $(\mathbb{F}, \mathbb{P})$ martingale on $[0, T]$.* ∎

In continuous time (see Section 7.1), the probabilistic setup of Assumptions 3.2 and 4.4 corresponds to the notions of invariance time τ and invariance probability measure $\mathbb{P}$ as per Crépey and Song (2017b).

Note that our choice (8) for the amount CM^c on the clean margin account related to each contract c, and the ensuing expression (14) for the clean desk trading loss L^{cl}, where P^c is the clean valuation of $\mathcal{P}^c$, plus Assumption 4.1 on $\mathcal{H}^c$, already ensure the $(\mathbb{F}, \mathbb{P})$ martingale condition on L^{cl} in Assumption 4.4.

Likewise, we will see in Section 5.1 that the $(\mathbb{G}, \mathbb{Q})$ martingale condition on L^{ca} in Assumption 4.4 embeds a theoretical target level for the level RC to be maintained by the bank on its reserve capital account.

4.3 Cost of Capital

Not only a bank cannot hedge its own jump-to-default: It cannot replicate its counterparty default losses either. Hence the trading of the bank generates a non-vanishing loss-and-profit process L. Then the regulator comes and requires that capital be set at risk by the shareholders, which therefore require a risk premium.

In practice, the level of compensation required by shareholders on their capital at risk in a firm is driven by market considerations. Typically, investors in banks expect a hurdle rate h of about 10% to 12%. In this paper:

Assumption 4.5 An exogenous and constant hurdle rate h prevails, in the sense that bank shareholders are constantly maintained by the KVA payments on an "efficient frontier" such that, at any time t,

$$\text{“Shareholder instantaneous average return}_t = \text{hurdle rate } \ h \ \times \ \mathrm{SCR}_t\text{,”} \tag{17}$$

where SCR denotes shareholder capital at risk. ∎

Of course there is some arbitrariness in the value of h to be used in practice, but no more than in analogous (risk aversion) parameters that would appear in other incomplete market pricing approaches, such as utility functions in indifference utility pricing schemes. Note that such parameters are not really intended to be calibrated or estimated from market data anyway, given that price quotations available from the market are mostly clean valuation, which do not contain any XVAs (see Remark 3.2), and are typically obtained from other banks anyway. The choice of h to be used in KVA computations should rather be seen as part of the managerial decisions of the bank itself, if not fixed by the regulator (as already done in the insurance context, cf. Remark 7.1). An endogenous and stochastic hurdle rate h could only arise in a model of competitive equilibrium, where different banks compete for clients (as opposed to our setup where only one bank is considered).

Valuation is risk-neutral with respect to the stochastic bases $(\mathbb{F}, \mathbb{P})$ or $(\mathbb{G}, \mathbb{Q})$. By contrast, economic capital and KVA assess risk and cost of capital, which refer to the historical probability measure. In our setup, the duality of perspective of the clean versus CA desks, on pricing as reflected by Assumptions 3.2 and 4.4, also applies to risk measurement. Capital calculations are always made "on a going concern", i.e. assuming that the bank is not in default, and therefore with respect to the reference filtration $\mathbb{F}$. Instead, cost of capital calculations are made by the management in a model including the default of the bank.

Moreover, in the context of XVA computations entailing projections over decades, the main source of information is market prices of liquid instruments, which allow the bank to calibrate the pricing measure (cf. Remark 3.2), and there is little of relevance that can be said about the historical probability measure. Hence, in our approach, we assume that:

Assumption 4.6 The estimates $\widehat{\mathbb{P}}$ and $\widehat{\mathbb{Q}}$ of the historical probability measure respectively used by the bank in its economic capital and cost of capital computations coincide with the pricing measures $\mathbb{P}$ and $\mathbb{Q}$. ∎

Any discrepancy between $\widehat{\mathbb{P}}$ and $\mathbb{P}$ or $\widehat{\mathbb{Q}}$ and $\mathbb{Q}$ is left to model risk, meant to be included in an AVA (additional valuation adjustment) in an FRTB terminology. Model risk in the XVA context is an important and widely open issue (see Glasserman and Yang (2016) for a possible treatment in the case of the CVA), which we leave for future research.

5 Derivation of the XVA Equations

In this section, based on the shareholder optimization (or indifference) principles embodied in Assumptions 4.4 and 4.5, we derive XVA equations that determine theoretical target XVA levels for the RC and RM amounts to be maintained by the bank on its reserve capital and risk margin accounts.

All XVA processes will be a suitable space $\mathcal{S}_2$ of $\mathbb{Q}$ (typically square) integrable $\mathbb{G}$ adapted processes defined until time $\bar{\tau} = \tau \wedge T$ (containing the null process). We denote by $\mathcal{S}_2^\circ$ the corresponding subspace of processes Y without jump at τ and such that $Y_T = 0$ holds on $\{T < \tau\}$.

5.1 Contra-assets, Contra-liabilities, and Wealth Transfer Analysis

The practical conclusion of this subsection will be that, in order for L^{ca} to be a $(\mathbb{G}, \mathbb{Q})$ martingale as required by Assumption 4.4, we should have RC = CA, instead of RC = CA − CL = CR if the bank could hedge its own jump-to-default risk, where:

Definition 5.1 We call CA (contra-asset value process), CVA (credit valuation adjustment), and FVA (funding valuation adjustment), the solutions to the following fixed-point problems, assumed well-posed in $\mathcal{S}_2^\circ$:[4] For $t \le \bar{\tau}$,

$$\widetilde{\mathrm{CA}} = \mathbb{E}_t\big(\widetilde{\mathcal{C}}_{\bar{\tau}}^\circ + \widetilde{\mathcal{F}}_{\bar{\tau}}^\circ - \widetilde{\mathcal{C}}_t^\circ - \widetilde{\mathcal{F}}_t^\circ + \mathbb{1}_{\{\tau<T\}}\widetilde{\mathrm{CA}}_\tau^\circ\big), \tag{18}$$

$$\widetilde{\mathrm{CVA}}_t = \mathbb{E}_t\big(\widetilde{\mathcal{C}}_{\bar{\tau}}^\circ - \widetilde{\mathcal{C}}_t^\circ + \mathbb{1}_{\{\tau<T\}}\widetilde{\mathrm{CVA}}_\tau^\circ\big), \tag{19}$$

$$\widetilde{\mathrm{FVA}}_t = \mathbb{E}_t\big(\widetilde{\mathcal{F}}_{\bar{\tau}}^\circ - \widetilde{\mathcal{F}}_t^\circ + \mathbb{1}_{\{\tau<T\}}\widetilde{\mathrm{FVA}}_\tau^\circ\big). \tag{20}$$

We define the contra-liabilities value process CL by

$$\mathrm{CL} = \mathrm{DVA} + \mathrm{FDA} + \mathrm{CVA}^{\mathrm{CL}} + \mathrm{FVA}^{\mathrm{CL}}, \tag{21}$$

where:

- DVA, the debt valuation adjustment, is the $(\mathbb{G},\mathbb{Q})$ value process of $\mathcal{C}^\bullet$;
- FDA, the funding debt adjustment, is the $(\mathbb{G},\mathbb{Q})$ value process of $\mathcal{F}^\bullet$;
- $\mathrm{CVA}^{\mathrm{CL}}$ and $\mathrm{FVA}^{\mathrm{CL}}$ are the respective $(\mathbb{G},\mathbb{Q})$ value processes of terminal cash flows $\mathbb{1}_{\{\tau<T\}}\mathrm{CVA}_\tau^\circ$ and $\mathbb{1}_{\{\tau<T\}}\mathrm{FVA}_\tau^\circ$ at time $\bar{\tau}$.

We call fair valuation of counterparty risk, denoted by CR, the $(\mathbb{G},\mathbb{Q})$ value of $\mathcal{C}$. ∎

Lemma 5.1 *We have*

$$\mathrm{CA} = \mathrm{CVA} + \mathrm{FVA} \tag{22}$$

and

$$\mathrm{CR} = \mathrm{CA} - \mathrm{CL}, \tag{23}$$

which is also the $(\mathbb{G},\mathbb{Q})$ *value process of* $(\mathcal{C} + \mathcal{F} + \mathcal{H})$.

Proof. The assumed uniqueness of $\mathcal{S}_2^\circ$ solutions to the CVA, FVA, and KVA fix-point equations yields (22). Since $\mathcal{F}$ and $\mathcal{H}$ are zero $(\mathbb{G},\mathbb{Q})$ valued (cf. Corollary 4.1), the other results directly follow from Definition 5.1. ∎

Contra-assets (contra-liabilities) draw their names from the fact that, from the point of view of the balance sheet of the bank developed in Section A, they are asset (liability) deductions. The contra-asset CVA and FVA components have been introduced before (cf. Section 2.2). Regarding the contra-liabilities:

- The DVA is the value that the bank clients lose due to the possible default of the bank in the future;

[4] See Section 7 for concrete well-posedness results.

- The FDA is the value of the amount of its funding debt that the bank fails to reimburse if it defaults;
- $\mathrm{CVA}^{\mathrm{CL}}$ and $\mathrm{FVA}^{\mathrm{CL}}$ are contra-liability components of the CVA and the FVA, valuing the residual amounts $\mathbb{1}_{\{\tau<T\}}\mathrm{CVA}^{\circ}_{\tau}$ and $\mathbb{1}_{\{\tau<T\}}\mathrm{FVA}^{\circ}_{\tau}$, summing up to $\mathbb{1}_{\{\tau<T\}}\mathrm{CA}^{\circ}_{\tau}=\mathbb{1}_{\{\tau<T\}}\mathrm{RC}^{\circ}_{\tau}$ (as will be seen in Theorem 5.1, unless the bank could hedge its own default), which are transferred from the reserve capital account to bank creditors at time τ (cf. Assumption 2.6).

Remark 5.1 The industry terminology tends to distinguish an FVA, in the specific sense of the cost of funding the cash collateral for variation margin, from an MVA defined as the cost of funding segregated collateral posted as initial margin (see Albanese, Caenazzo, and Crépey (2017)). In this paper, we merge the two in an overall FVA meant in the broad sense of the cost of funding the derivative business of the bank. ∎

Theorem 5.1 **(i)** *Assuming* RC *in* $\mathcal{S}_2^{\circ}$, *we have* $\mathrm{RC}=\mathrm{CA}$ *and*

$$\begin{aligned}
\widetilde{\mathcal{W}}^{ca} &= -(\widetilde{\mathrm{CA}}+\widetilde{\mathcal{C}}+\widetilde{\mathcal{F}}+\widetilde{\mathcal{H}}),\\
\widetilde{L}^{ca} &= \widetilde{\mathrm{CA}}+\widetilde{\mathcal{C}}^{\circ}+\widetilde{\mathcal{F}}^{\circ}+\widetilde{\mathcal{H}},\\
\widetilde{L} &= \widetilde{\mathrm{CA}}+\widetilde{\mathcal{C}}^{\circ}+\widetilde{\mathcal{F}}^{\circ}+\widetilde{\mathcal{H}}+(\widetilde{L}^{cl})^{\circ}.
\end{aligned} \tag{24}$$

(ii) *If, on top of everything else before, the bank could hedge its own jump-to-default exposure, in the sense that the bank could and would additionally sell on the financial markets a contract paying* CL_{τ} *at time* τ *(e.g. through repurchasing of its own bond as contemplated in Burgard and Kjaer (2011, 2013, 2017)), assuming further* RC *in* $\mathcal{S}_2^{\circ}$ *and* CL *in* $\mathcal{S}_2$, *then, before* τ, *we would have*

$$\mathrm{RC}=\mathrm{CA}-\mathrm{CL}=\mathrm{CR}. \tag{25}$$

(iii) *The contra-liability amount* $\mathrm{CL}=\mathrm{CA}-\mathrm{CR}$ *is interpreted as the wealth transfer triggered by the deals from shareholders to creditors, due to the inability of the bank to hedge its own jump-to-default exposure.*

Proof. **(i)** By Assumptions 4.2 and 4.4 (cf. also Corollary 3.1), the processes $\widetilde{\mathcal{H}}=\widetilde{\mathcal{H}}^{\circ}$ (cf. Assumption 4.2) and $\widetilde{L}^{ca}$ in the first line of (13) are $(\mathbb{G},\mathbb{Q})$ martingales. If RC is in $\mathcal{S}_2^{\circ}$ (which includes its $\mathbb{Q}$ integrability), then these $(\mathbb{G},\mathbb{Q})$ martingale properties imply that RC satisfies the CA equation (18). Hence $\mathrm{RC}=\mathrm{CA}$, by assumed uniqueness of an $\mathcal{S}_2^{\circ}$ solution to the CA equation. Then (24) is just a restatement of (12) and (13) (having assumed $\mathcal{H}=\mathcal{H}^{\circ}$).
(ii) Assuming the CA desk would additionally sell on the financial markets a contract paying CL_{τ} at time τ, contract valued by the CL process of Definition 5.1 as per the $(\mathbb{G},\mathbb{Q})$ valuation rule in Definition 3.1, the modified equation for the CA desk trading loss would become, instead of the second line in (13),

$$\widetilde{L}^{ca}=\widetilde{\mathrm{RC}}^{\circ}+\widetilde{\mathcal{C}}^{\circ}+\widetilde{\mathcal{F}}^{\circ}+\widetilde{\mathcal{H}}+\widetilde{\mathrm{CL}}^{\circ},$$

which should then be a $(\mathbb{G}, \mathbb{Q})$ martingale to comply with Assumption 4.4. Then, assuming RC in $\mathcal{S}_2^\circ$ and CL in $\mathcal{S}_2$, it is now the process $(\mathrm{RC} + \mathrm{CL}^\circ)$ that would solve the CA equation (18) in $\mathcal{S}_2^\circ$, so that $\mathrm{RC} + \mathrm{CL}^\circ = \mathrm{CA}$ would hold, by uniqueness.
(iii) Rephrasing (i) and (ii):

- If the bank could hedge its own jump-to-default exposure, then we would have $\mathrm{RC} = \mathrm{CR}$, the fair valuation of counterparty risk to the bank as a whole (cf. Lemma 5.1);
- But, for the reasons exposed in Section 2.1, a CL hedge cannot be achieved in practice by the bank, hence $\mathrm{RC} = \mathrm{CA}$.

Confronting these two points grounds the stated interpretation for $\mathrm{CL} = \mathrm{CA} - \mathrm{CR}$ (by (23)). ∎

Note that the addition of the CL hedge in Theorem 5.1(ii) also has funding implications, i.e. it modifies the risky funding $(\mathbb{G}, \mathbb{Q})$ martingale $\mathcal{F}$. However, as the conclusion (25) does not depend on the detailed specification of $\mathcal{F}$, this does not affect the end result and interpretation.

In Hull and White (2016) (cf. also Hull and White (2012)), the authors propose that FVA should not be passed to clients, not because the bank could hedge its own jump-to-default exposure (which they know is impossible), but on the ground that creditors to banks should recognize the wealth transfers from equity holders to themselves which will occur as a consequence of future trading. Instead, we consider that, in the case of a bank, which is a market maker and as such cannot anticipate future trades (cf. Assumption 2.1), it is not acceptable to make any assumption about future trade flows and putative profits deriving from them. Accordingly (cf. also the last paragraph of Section 2 in Burgard and Kjaer (2013)):

Assumption 5.1 The positive impact of future trades on the realized recovery of the bank is not reflected in the bank funding spreads. ∎

5.2 Economic Capital

The economic capital (EC) of the bank is its resource devoted to cope with losses beyond their expected levels, the latter being taken care of by reserve capital (RC). Capital requirements are focused on the solvency issue, because it is when a regulated firm becomes insolvent that the regulator may choose to intervene, take over or restructure the firm. Specifically, Basel II Pillar II defines economic capital as the 99% value-at-risk of the negative of the variation over a one-year period of core equity tier I capital (CET1), the regulatory metric that represents the wealth of the shareholders within the bank (see Definition A.2). In our setup, equity depletions correspond to the trading loss process L of the bank (see Section A.2 for more detail about it). Moreover, the FRTB required a shift from 99% value-at-risk to 97.5% expected shortfall as the reference risk measure in capital calculations.

Accordingly, under Assumption 4.6 and also accounting for discounting (we recall that L' is the $\mathbb{F}$ reduction of L):

Definition 5.2 Our reference definition for the discounted economic capital of the bank at time t is the $(\mathfrak{F}_t, \widehat{\mathbb{P}} = \mathbb{P})$ conditional 97.5% expected shortfall of $(\widetilde{L}'_{t+1} - \widetilde{L}'_t)$, which we denote by $\widetilde{\mathbb{ES}}'_t(L)$. ∎

Solvency II introduces a further modification of economic capital, which is required to be in excess of the risk margin (RM). This modification is considered in Sections 5.4 and 7.2–7.3.

Note that in practice, the equity capital of the bank (economic capital EC and uninvested capital UC beyond it, if any) can be used by the bank for its funding purposes. As developed in Crépey, Élie, and Sabbagh (2017), this induces an intertwining of economic capital and the FVA, which in turn implies a feedback of the KVA into the FVA and, as a consequence, into the trading loss process L of the bank. Instead, for simplicity in this paper:

Assumption 5.2 The bank does not use its equity capital (EC + UC) for funding purposes. ∎

Hence, for the purpose of economic capital and cost of capital computations in this paper, the trading loss process L of the bank can be considered as an exogenous process. In what follows, we write $\widetilde{\mathrm{ES}}_t$ for $\widetilde{\mathbb{ES}}'_t(L)$, and ES_t for $\mathbb{ES}'_t(L)$, the undiscounted version of $\widetilde{\mathbb{ES}}'_t(L)$.

Lemma 5.2 ES *is nonnegative.*

Proof. By Corollaries 4.2 and 3.1, the process $\widetilde{L}'$ is an $(\mathbb{F}, \mathbb{P})$ martingale. The result then follows from the fact that the 97.5% expected shortfall of a centered random variable is nonnegative. ∎

5.3 Unconstrained Linear KVA Equation

Counterparty default losses, as also funding payments, are materialities for default if not paid. In contrast, KVA payments are at the discretion of the bank management. Accordingly:

Assumption 5.3 The risk margin is loss-absorbing, hence part of economic capital. ∎

Corollary 5.1 *Shareholder capital at risk (*SCR*) is the difference between the economic capital (*EC*) of the bank and the risk margin (*RM*), i.e.*

$$\mathrm{SCR} = \mathrm{EC} - \mathrm{RM}. \blacksquare \tag{26}$$

In view of Lemma 3.3 and Corollary 5.1, and since $\widetilde{L}$ in (13) is a $(\mathbb{G}, \mathbb{Q} = \widehat{\mathbb{Q}})$ martingale (cf. Corollary 4.2, Corollary 3.1, and Assumption 4.6), also accounting for the OIS remuneration to shareholders of the risk margin account, the informal statement (17) translates in mathematical terms into the following KVA equation in $\mathcal{S}_2^\circ$ for the theoretical target RM level:

$$(-\widetilde{\mathrm{KVA}}) \text{ has the } (\mathbb{G}, \mathbb{Q}) \text{ drift coefficient } h\,(\widetilde{\mathrm{EC}} - \widetilde{\mathrm{KVA}}). \tag{27}$$

Some actuarial literature dwells with the puzzle according to which the calculation of RM depends on economic capital projections in the future, while economic capital EC = RM + SCR depends on the risk margin, an apparently circular dependency (see e.g. Salzmann and Wüthrich (2010, Sect. 4.4) and Robert (2013)). This circular dependency, which is captured by the fix-point equation (27) for the theoretical target RM = KVA level, is therefore solved by our approach provided this fix-point equation is well-posed.

However, the equation (27), even if well-posed, is only preliminary if EC there is just meant as ES, which would then be forgetful of a consistency condition SCR ≥ 0. This is fixed in the next section by pushing EC above ES until the constraint is satisfied.

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5.4 The KVA Constrained Optimization Problem

Assume that, for any tentative economic capital process C in a suitable space $\mathcal{L}_2$ of $\mathbb{Q}$ (typically square) integrable processes containing both $\mathcal{S}_2$ and the process ES, the equation (cf. (27))

$$(-\widetilde{K}) \text{ has the } (\mathbb{G}, \mathbb{Q}) \text{ drift coefficient } h(\widetilde{C} - \widetilde{K}) \tag{28}$$

defines a unique process $K = K(C)$ in $\mathcal{S}_2^\circ$. Then:

Definition 5.3 The set of admissible economic capital processes is defined as

$$\mathcal{C} = \{C \in \mathcal{L}_2; C \geq \max\big(K(C), \mathrm{ES}\big)\}, \tag{29}$$

where (b) $C \geq$ ES is the risk acceptability condition and (a) $C \geq K(C)$ corresponds to the self-consistency condition that shareholder capital at risk be nonnegative. ∎

In view of (28) and (29), the natural guess for the minimal and cheapest admissible economic capital process is

$$\mathrm{EC} = \max(\mathrm{ES}, \mathrm{KVA}), \tag{30}$$

for a theoretical target RM = KVA level in $\mathcal{S}_2^\circ$ such that

$$(-\widetilde{\mathrm{KVA}}) \text{ has the } (\mathbb{G}, \mathbb{Q}) \text{ drift coefficient } h\big(\max(\widetilde{\mathrm{ES}}, \widetilde{\mathrm{KVA}}) - \widetilde{\mathrm{KVA}}\big). \tag{31}$$

In particular, the discounted KVA is a $(\mathbb{G}, \mathbb{Q} = \widehat{\mathbb{Q}})$ supermartingale.

Theorem 7.1 establishes the corresponding conjecture, including the well-posedness of (31), in a continuous-time setup.

Remark 5.2 In the case of perfect clean and CA hedges where the process L (hence L' and $\widetilde{L}'$) would be constant, i.e., in view of (14) and of the second line in (24), for

$$\widetilde{\mathcal{H}}^c = \widetilde{P}^c + \widetilde{\mathcal{P}}^c, \text{ for each contract c, and } \widetilde{\mathcal{H}} = -\big(\widetilde{\mathrm{CA}} + \widetilde{\mathcal{C}}^\circ + \widetilde{\mathcal{F}}^\circ\big)$$

(assuming these would be achievable hedging loss processes), then $\mathrm{ES}_t = \mathbb{ES}'_t(L)$ (see before Lemma 5.2) would vanish and KVA $= 0$ would obviously solve (31) in $\mathcal{S}_2^\circ$. Hence there would be no KVA risk premium in this case. ∎

5.5 Connections with the Modigliani-Miller Theorem

The Modigliani and Miller (1958) theorem includes two key assumptions. One is that, as a consequence of trading, total wealth is conserved. The second assumption is that markets are complete.

In our setup we keep the wealth conservation hypothesis, in the form of Assumption 2.4 (cf. (12) and (24), as well as (79) and (55) in concrete setups below). Assuming wealth conservation ensures the validity of the part of the Modigliani and Miller (1958) theorem stating that the wealth of a firm does not depend on its funding policy: The value of counterparty risk to the bank as a whole (shareholders and creditors altogether), i.e. the $(\mathbb{G},\mathbb{Q})$ value of $(\mathcal{C}+\mathcal{F}+\mathcal{H})$, reduces to the value CR of $\mathcal{C}$ (cf. Corollary 4.1).

But, as follows from Theorem 5.1, the part of the Modigliani-Miller theorem stating that the interests of shareholders and creditors are aligned with each other would only be valid if the bank could hedge its own default. Otherwise the derivative portfolio of the bank triggers a wealth transfer from the shareholders to the creditors by the amount of CL.

This misalignment of interest is enhanced by the risk incurred by shareholders on their capital due to the inability of the bank of perfectly hedging its trading losses (counterparty default losses in particular).

For related considerations see also Andersen et al. (2017, Section VII.A).

6 Incremental XVA Approach, Funds Transfer Price, and Cost of Run-Off Interpretation

So far we worked under Assumption 3.1 that the derivative portfolio of the bank is static, i.e. set up at time 0 and assuming that no new trades will ever enter the portfolio in the future.

In practice derivative portfolios are incremental. Yet, given the derivative porfolio of a bank at time 0, our XVA equations (and the MtM formula (9) to begin with) can be applied to the version of the portfolio that ***would be*** run-off by the bank from time 0 onward, i.e. locked and let amortizing until its final maturity. The ensuing numbers are then interpreted as the amounts

$$\mathrm{CM} = \mathrm{MtM},\ \mathrm{RC} = \mathrm{CVA} + \mathrm{FVA},\ \mathrm{RM} = \mathrm{KVA} \tag{32}$$

to maintain on the clean margin, reserve capital, and risk margin accounts, so that the bank ***could go*** into run-off, while staying in line with shareholder interest, if wished (cf., a contrario, Figure 1). Moreover, since we relied on a dynamic analysis, this possibility, for a bank respecting the balance conditions (32), of going run-off in line with shareholder interest, is granted not only from time 0 onward, but from any future time onward, as long as there are no new deals in the portfolio.

Now, when a new deal shows up, since shareholders have the control of the bank before bank default (cf. Assumption 2.3), it is natural to postulate the following:

Assumption 6.1 In the context of a real-life incremental portfolio, for a new deal to occur (before bank default), shareholders should be (at least) indifferent to it, i.e. the entry price of the deal should be set at a level allowing the shareholder balance conditions (32) to be preserved, without the shareholders having to pay out of their own pockets, throughout the deal. ∎

Remark 6.1 From an abstract principle point of view, Assumption 6.1 belongs to the family of indifference pricing rules, like the KVA principles postulated in a stylized setup in Brigo et al. (2017). ∎

Theorem 6.1 *Replacing Assumption 3.1 by Assumption 6.1 in the context of an incremental portfolio, in case a new contract c (not already involved in a liquidation procedure) shows up at time t, then, denoting by $\Delta\cdot$, for any of our* MtM *or* XVA *metrics, the difference between the time t values of the metric for the run-off version of the portfolio with and without the new deal:*

- *The client sells the deal for $(P^c - \Delta\mathrm{CA})$ to the CA desk, which adds $\Delta\mathrm{CA}$ on the reserve capital account and resells the (cleaned deal) for its clean valuation P^c to the clean desk, while adding the amount P^c on the clean margin account;*
- *The management of the bank charges the amount $\Delta\mathrm{KVA}$ to the client and adds it on the risk margin account;*
- *The all-inclusive XVA add-on passed by the bank to the client of the deal, called funds transfer price (*FTP*), is*

$$\begin{aligned}\mathrm{FTP} &= \Delta\mathrm{CA} + \Delta\mathrm{KVA} = \Delta\mathrm{CVA} + \Delta\mathrm{FVA} + \Delta\mathrm{KVA}\\ &= \Delta\mathrm{CR} + \Delta\mathrm{CL} + \Delta\mathrm{KVA},\end{aligned} \tag{33}$$

which is interpreted as the rebate to deduct from the clean valuation P^c of the deal in order to realign the portfolio including the new deal to the interest of the bank shareholders, accounting for counterparty risk and the inability of the bank to hedge jump-to-default cash flows.

Proof. If the new trade occurs, it has three impacts: it modifies the mark-to-market of the portfolio (cf. Section 3.3), triggers a wealth transfer from shareholders to creditors (cf. Section 5.1), and alters the risk profile of the portfolio (cf. Section 5.3–5.4). Hence the shareholder balance conditions (32) and the associated soft landing option of the bank are impaired by the deal, unless a third party, which in the case of a market maker can only be the client of the deal (cf. Assumptions 6.1 and 2.1), fills in the missing amounts on the reserve capital and risk margin accounts of the bank in order to restore the second and third conditions in (32).

In parallel, in order to restore the first condition in (32) so that clean desks stay immunized against counterparty risk and funded by the OIS remuneration of the clean margin account, the amount CM on the latter needs be updated by an amount $\Delta\mathrm{MtM} = P^c$ (as it follows from (9), having assumed the new deal not already involved in a liquidation procedure at inception time). ∎

Note that the value on the clean margin account is only "lent" by the clean margin poster to the clean margin receiver, without transfer of property between them (unless a default happens and some contracts are liquidated). Hence, the net output of the first bullet point in Theorem 6.1 is that:

- The CA desk charges to the client an amount $\Delta\mathrm{CA}$, corresponding to the additional liability (contra-asset) triggered by the new deal, and adds it to the reserve capital account;
- The clean desks pay P^c to the client in exchange of future cash flows $\mathcal{P}$ guaranteed by the action of the CA desk.

In view of the FTP formula (33), the only XVAs that ultimately matter in entry prices are the CVA, the FVA, and the KVA. Obviously, the endowment (legacy portfolio of the bank) has a key impact on the FTP of a new trade. It may very well happen that a new deal is risk-reducing with respect to the endowment, in which case $\mathrm{FTP} < 0$, i.e. the XVA-inclusive price paid by the bank to the client is $P^c - \mathrm{FTP} > P^c$.

The application of the FTP (33) as a deduction from the clean valuation of every new deal, associated with the clean collateralization, accounting, and dividend policies preserving the balance conditions (32) between deals, ensure to the bank the possibility of "soft landing" its portfolio (going into run-off in line with shareholder interest) from any point in time onward if wished. Thus we arrive to a sustainable strategy for profits retention, which is already the key principle behind Solvency II.

In view of Theorem 6.1, XVA computations, whether they regard a new trade, which is then treated on an incremental run-off basis, or targeting the preservation of the shareholder balance conditions (32) between deals, only entail static portfolios. Hence, **in the following we may and do restrict attention to the case of a static portfolio**.

Moreover, it is natural to perform the XVA computations under the following assumption, in line with a run-off procedure where market risk is first hedged out by the bank, but we conservatively assume no XVA hedge, and the portfolio is then let to amortize until its final maturity:

Assumption 6.2 We assume a perfect clean hedge by the clean traders, i.e. L^{cl} constant, which can be taken as zero as loss processes only matter through their fluctuations, and no CA hedge, i.e. $\mathcal{H} = 0$. ∎

As it then immediately follows from the last line in (24):

Corollary 6.1 *We have*

$$\widetilde{L} = \widetilde{L}^{ca} = \widetilde{\mathrm{CA}} + \widetilde{\mathcal{C}}^{\circ} + \widetilde{\mathcal{F}}^{\circ}. \blacksquare \tag{34}$$

Hence, the process L that is used as input to capital and KVA computations[5] is the output of the CA = CVA + FVA computations. Upstream of this, as visible in Lemmas B.1, 7.5, and 7.6 below, a key input data to the counterparty exposure and risky funding data $\mathcal{C}$ and $\mathcal{F}$ is the MtM process, i.e. the clean valuation of the contracts. These connections make the MtM, CA, and KVA equations, hence the pricing problem as a whole, a self-contained problem.

Clean valuation that is involved in mark-to-market (MtM) computations is of course standard (or, at least, not the focus of this paper). Hence we focus on the XVA computations in the sequel.

7 XVA Equations Well-Posedness and Comparison Results

In Section 5, we formulated the CVA, FVA, and KVA equations as fix-point problems (19), (20), and (31) in an abstract space $\mathcal{S}_2^{\circ}$ of integrable processes without jump at the bank default time τ. In this section, we establish concrete well-posedness and comparison results for these problems in a continuous-time setup, as necessary for calibration of the whole approach to a real-life banking portfolio. See Section B for a stylized XVA approach in a one-period static setup.

7.1 Continuous-Time Setup

In continuous time, Assumptions 3.2 and 4.4, with filtrations $\mathbb{F}$ and $\mathbb{G}$ satisfying the usual conditions, martingale understood as local martingale, and existence of $\mathbb{F}$ semimartingale reductions reinforced[6] into existence of an "$\mathbb{F}$ predictable reduction" on $[0, T]$ coinciding until τ included with any $\mathbb{G}$ predictable process, mean that τ is an invariance time as per Crépey and Song (2017b). This covers the mainstream immersion setup (but not only, see e.g. Crépey and Song (2017a)), where $(\mathbb{F}, \mathbb{Q})$ local martingales are $(\mathbb{G}, \mathbb{Q})$ local martingales without jump at τ, in which case τ is an invariance time with $\mathbb{P} = \mathbb{Q}$. More generally, denoting by $J = \mathbb{1}_{[0,\tau)}$ the survival indicator process of τ:

[5] cf. Definition 5.2 and (31), where $\widetilde{\mathrm{ES}} = \widetilde{\mathbb{ES}}'(L)$.

[6] As shown in Song (2016).

Theorem 3.5 in Crépey and Song (2017b) *Assuming that an $\mathbb{F}$ predictable reduction of any $\mathbb{G}$ predictable process exists and that $\mathbb{Q}(\tau > T|\mathfrak{F}_T) > 0$, then any $\mathbb{G}$ optional process admits an unique $\mathbb{F}$ optional reduction coinciding with it before τ on $[0,T]$.*

If, moreover, τ has a $(\mathbb{G},\mathbb{Q})$ intensity process γJ_- such that $e^{\int_0^T \gamma_s ds}$ is $\mathbb{Q}$ integrable, then existence on $\mathcal{A}$ and uniqueness on $\mathfrak{F}_T$ of an invariance probability measure $\mathbb{P}$ hold. ■

We work in the sequel under the corresponding specialization of Assumptions 3.2 and 4.4 and we assume that γ is $\mathbb{F}$ predictable, without loss of generality by $\mathbb{F}$ reduction.

As can be established by section theorem, for any $\mathbb{G}$ progressive Lebesgue integrand X such that the $\mathbb{G}$ predictable projection ${}^pX$ exists,[7] the indistinguishable equality $\int_0^\cdot {}^pX_s ds = \int_0^\cdot X_s ds$ holds. As a consequence, one can actually consider the $\mathbb{F}$ reduction X' of any $\mathbb{G}$ progressive Lebesgue integrand X (even if this means replacing X by ${}^pX$).

We assume an $\mathbb{F}$ progressive risk-free (OIS) short interest rate $r = (r_t)_{t\in\mathbb{R}_+}$ and we write $\beta_t = e^{-\int_0^t r_s ds}$ for the corresponding discount factor (cf. Example 3.1). All the value, XVA, and price processes are now continuous-time processes modeled as $\mathbb{G}$ (or "clean" $\mathbb{F}$) semimartingales (in a càdlàg version).

In continuous time, the process Y° in (4) is defined, for any left-limited process Y, by

$$Y^\circ = Y^{\tau-} = JY + (1-J)Y_{\tau-}.$$

The CVA, FVA, and KVA fix-point problems (19), (20), and (31) are addressed in a mathematical formalism of backward stochastic differential equations (BSDEs), involving the following spaces of processes (recall $\mathbb{E}$ and $\mathbb{E}'$ denote $\mathbb{Q}$ and $\mathbb{P}$ expectations):

- $\mathcal{S}_2$, the space of càdlàg $\mathbb{G}$ adapted processes Y over $[0,\bar\tau]$ such that, denoting $Y^*_t = \sup_{s\in[0,t]}|Y_s|$:

$$\mathbb{E}\Big[Y_0^2 + \int_0^T e^{\int_0^s \gamma_u du}\mathbb{1}_{\{s<\tau\}} d\big(|Y^*_s|^2\big)\Big] < \infty; \tag{35}$$

- $\mathcal{S}_2^\circ$, the subspace of the processes in $\mathcal{S}_2$ such that Y is without jump at τ and $Y_T = 0$ holds on $\{T < \tau\}$;

- $\mathcal{L}_p$, for $p > 1$, the space of $\mathbb{G}$ progressive processes X over $[0,T]$ such that

$$\mathbb{E}\big[\int_0^T e^{\int_0^s \gamma_u du}\mathbb{1}_{\{s<\tau\}}|X_s|^p ds\big] < +\infty; \tag{36}$$

- $\mathcal{S}_2'$, the space of càdlàg $\mathbb{F}$ adapted processes Y over $[0,T]$ such that

$$\mathbb{E}'\Big[\sup_{t\in[0,T]}|Y_t|^2\Big] < \infty \tag{37}$$

  and $Y_T = 0$;

[7] For which σ integrability of X valued at any stopping time, e.g. X bounded or càdlàg, is enough.

- $\mathcal{L}'_p$, for $p > 1$, the space of $\mathbb{F}$ progressive processes X over $[0, T]$ such that

$$\mathbb{E}'\big[\int_0^T |X_t|^p dt\big] < +\infty. \tag{38}$$

By the results of Crépey and Song (2017c), we have the following isometry in terms of the respective (squared) norms (37) and (35):

$$\mathcal{S}'_2 \ \underset{\cdot'}{\overset{\cdot^{\tau-}}{\rightleftarrows}} \ \mathcal{S}^\circ_2, \tag{39}$$

as well as the one induced on their respective subspaces of local martingales (cf. Assumption 4.4).

Sections 7.2–7.3 establish, in this continuous-time setup, the KVA result announced in the end of Section 5.4. Note that the primary raison d'etre for the KVA is the default of bank clients, as opposed to the default of the bank itself (which on the other hand is the key of the contra-liabilities wealth transfer issue of Section 5.1). In Section 7.2 we suppose the bank default free, i.e. (see after Notation 3.2)

$$\tau = +\infty,\ (\mathbb{F}, \mathbb{P}) = (\mathbb{G}, \mathbb{Q}).$$

The results are then extended to the case of a defaultable bank in Section 7.3.

7.2 KVA in the Case of a Default-Free Bank

At that stage we use the "$\cdot'$" notation, not in the sense of $\mathbb{F}$ reduction (as $\mathbb{F} = \mathbb{G}$), but simply in order to distinguish the equations in this subsection, where $\mathbb{F} = \mathbb{G}$, from the ones in Section 7.3, where $\mathbb{F} \neq \mathbb{G}$ (the present data will then be interpreted a posteriori as the $\mathbb{F}$ reductions of the corresponding data in Section 7.3).

Given $C' \geq \mathrm{ES}$ representing a putative economic capital process for the bank, we consider the following BSDEs (cf. (28) and (31) with τ set to ∞):

$$K'_t = \mathbb{E}'_t \int_t^T \big(hC'_s - (r_s + h)K'_s\big)ds,\ t \in [0, T], \tag{40}$$

$$\mathrm{KVA}'_t = \mathbb{E}'_t \int_t^T \big(h \max(\mathrm{ES}_s, \mathrm{KVA}'_s) - (r_s + h)\mathrm{KVA}'_s\big)ds,\ t \in [0, T] \tag{41}$$

to be solved for respective processes K' and KVA'.

Lemma 7.1 *Assuming that r is bounded from below and in $\mathcal{L}'_2$:*

- *If C' is in $\mathcal{L}'_2$, then the BSDE* (40) *is well posed in $\mathcal{S}'_2$, with $\mathcal{S}'_2$ solution*

$$K'_t = h\mathbb{E}'_t \int_t^T e^{-\int_t^s (r_u + h)du} C'_s ds,\ t \in [0, T], \tag{42}$$

- *If* ES *is in* $\mathcal{L}'_2$, *then the BSDE* (41) *is well posed in* $\mathcal{S}'_2$,

where well-posedness here and later includes existence, uniqueness and comparison.

Proof. In terms of the coefficient

$$k_t(y) = h\big(\max(\mathrm{ES}_t, y) - y\big) - r_t y = h \max\big(\mathrm{ES}_t, y\big) - (r_t + h)y,\ y \in \mathbb{R}, \tag{43}$$

the KVA$'$ BSDE (41) is written as

$$\mathrm{KVA}'_t = \mathbb{E}'_t \int_t^T k_s(\mathrm{KVA}'_s)ds,\ t \in [0, T]. \tag{44}$$

For any real $y, y' \in \mathbb{R}$ and $t \in [0, T]$, we have

$$\begin{aligned}\big(k_t(y) - k_t(y')\big)(y - y') &= -(r_t + h)(y - y')^2 + h\big(\max(\mathrm{ES}_t, y) - \max(\mathrm{ES}_t, y')\big)(y - y')) \\ &\leq -r_t(y - y')^2 \leq C(y - y')^2,\end{aligned}$$

for some constant C (having assumed r bounded from below), so that the BSDE coefficient k satisfies the so-called monotonicity condition. Moreover, for $|y| \leq \bar{y}$, we have (recalling ES ≥ 0 by Lemma 5.2):

$$|k_\cdot(y) - k_\cdot(0)| \leq h \max\big(\mathrm{ES}, \bar{y}\big) + |h + r|\bar{y} + h\mathrm{ES}.$$

Hence, assuming that ES and r are in $\mathcal{L}'_2$, the following integrability conditions hold:

$$\sup_{|y| \leq \bar{y}} |k_\cdot(y) - k_\cdot(0)| \in \mathcal{L}'_1,\ \text{for any } \bar{y} > 0, \qquad \text{and} \qquad k_\cdot(0) \in \mathcal{L}'_2.$$

Therefore, by application of the general filtration BSDE results of Kruse and Popier (2016, Sect. 4),[8] the BSDE (41) is well-posed in $\mathcal{S}'_2$, including existence, uniqueness and comparison. Analogous (even simpler) considerations prove the statements regarding the linear BSDE (40). Finally, (42) obviously solves (40). ∎

To emphasize its dependence on C', we henceforth denote by $K' = K'(C')$ the solution (42) to the linear BSDE (40). Assuming that r is bounded from below and that r and ES are in $\mathcal{L}'_2$, we define the set of admissible economic capital processes as (cf. (29))

$$\mathcal{C}' = \{C' \in \mathcal{L}'_2; C' \geq \max\big(\mathrm{ES}, K'(C')\big)\}$$

and we set (cf. (30))

$$\mathrm{EC}' = \max(\mathrm{ES}, \mathrm{KVA}'), \tag{45}$$

where KVA$'$ is the $\mathcal{S}'_2$ solution to the BSDE (41).

[8] cf. also Kruse and Popier (2017), noting that we only use their results regarding square integrable solutions.

Lemma 7.2 *Assuming that r is bounded from below and that r and* ES *are in $\mathcal{L}'_2$, the $\mathcal{S}'_2$ solution* KVA′ *to* (41) *solves the linear BSDE* (40) *for the implicit data $C' = \mathrm{EC}'$ in* (45)*, i.e. we have* $\mathrm{KVA}' = K'(\mathrm{EC}')$*, that is,*

$$\mathrm{KVA}'_t = h\mathbb{E}'_t \int_t^T e^{-\int_t^s (r_u+h)du} \mathrm{EC}'_s ds,\ t \in [0,T]. \tag{46}$$

Proof. The process KVA′ is in $\mathcal{S}'_2$ and, by virtue of (41) and (45), we have, for $t \in [0,T]$,

$$\begin{aligned} \mathrm{KVA}'_t &= \mathbb{E}'_t \int_t^T \Big(h \max\big(\mathrm{ES}_s, \mathrm{KVA}'_s\big) - (r_s+h)\mathrm{KVA}'_s\Big) ds \\ &= \mathbb{E}'_t \int_t^T \Big(h\mathrm{EC}'_s - (r_s+h)\mathrm{KVA}'_s\Big) ds. \end{aligned} \tag{47}$$

Hence, the process KVA′ solves the linear BSDE (40) for $C' = \mathrm{EC}'$ in $\mathcal{L}'_2$. The identity $\mathrm{KVA}' = K'(\mathrm{EC}')$ follows by uniqueness of an $\mathcal{S}'_2$ solution to the linear BSDE (40) established in Lemma 7.1. Equation (46) then follows by an application of (42). ∎

Remark 7.1 The formula (46) can be seen as a continuous-time analog of the risk margin formula in the Solvency II eurozone insurance regulation (itself adapted from the Swiss Solvency Test (2004)), in which h is set as 6%. ∎

Proposition 7.1 *Under the assumptions of Lemma 7.2, we have:*
(i) $\mathrm{EC}' = \min \mathcal{C}', \mathrm{KVA}' = \min_{C' \in \mathcal{C}'} K'(C')$;
(ii) *The process* KVA′ *is nonnegative and it is nondecreasing in h.*

Proof. **(i)** We saw in Lemma 7.2 that $\mathrm{KVA}' = K'(\mathrm{EC}')$, hence

$$\mathrm{EC}' = \max(\mathrm{ES}, \mathrm{KVA}') = \max\big(\mathrm{ES}, K'(\mathrm{EC}')\big),$$

therefore $\mathrm{EC}' \in \mathcal{C}'$. Moreover, for any $C' \in \mathcal{C}'$, we have (cf. (43)):

$$k_t(K'_t(C')) = h \max\big(\mathrm{ES}_t, K'_t(C')\big) - (r_t+h)K'_t(C') \le hC'_t - (h+r_t)K'_t(C').$$

Hence, the coefficient k_t of the KVA BSDE (41) never exceeds the coefficient of the linear BSDE (40) when both coefficients are evaluated at the solution $K'_t(C')$ of (40). Since these are BSDEs with equal (null) terminal condition, the BSDE comparison Proposition 4 in Kruse and Popier (2016) applied to the BSDEs (40) and (41) yields $\mathrm{KVA}' \le K'(C')$. Consequently, $\mathrm{KVA}' = \min_{C' \in \mathcal{C}'} K(C')$ and, for any $C' \in \mathcal{C}'$,

$$C' \ge \max(\mathrm{ES}, K'(C')) \ge \max(\mathrm{ES}, \mathrm{KVA}') = \mathrm{EC}'.$$

Hence $\mathrm{EC}' = \min \mathcal{C}'$.
(ii) By Lemma 5.2, ES is nonnegative. Then so is KVA′, by (46) and (45); Moreover, as follows from (43), the coefficient $k_t(y)$ of the KVA′ BSDE (41) is nondecreasing in the hurdle rate parameter h. So is therefore the $\mathcal{S}'_2$ solution KVA′ to (41), by the BSDE comparison theorem of Kruse and Popier (2016, Proposition 4) applied to the BSDE (41) for different values of h. ∎

7.3 KVA in the Case of a Defaultable Bank

In the case of a defaultable bank, specializing to the continuous-time setup of Section 7.1 the abstract discussion of Section 5.4, "$\cdot'$" now denoting $\mathbb{F}$ reduction (predictable, optional, or progressive, as applicable), then, by the results of Crépey and Song (2017c):

- For any $C \in \mathcal{L}_2$, we have $C' \in \mathcal{L}'_2$, and the $(\mathbb{G}, \mathbb{Q})$ BSDE (28) for a process $K = K(C)$ in $\mathcal{S}_2^\circ$ is equivalent, through the isometry (39), to the $(\mathbb{F}, \mathbb{P})$ BSDE (40) for a process $K'(C')$ in $\mathcal{S}'_2$;
- Assuming ES in $\mathcal{L}'_2$, the $(\mathbb{G}, \mathbb{Q})$ KVA BSDE (31) in $\mathcal{S}_2^\circ$ is equivalent, through the isometry (39), to the $(\mathbb{F}, \mathbb{P})$ KVA$'$ BSDE (41) in $\mathcal{S}'_2$.

As a result:

Lemma 7.3 *Assuming that r is bounded from below and in $\mathcal{L}'_2$:*

- *If $C \in \mathcal{L}_2$, hence $C' \in \mathcal{L}'_2$ (cf. the first bullet point above), then the $(\mathbb{G}, \mathbb{Q})$ linear BSDE* (28) *for $K = K(C)$ is well posed in $\mathcal{S}_2^\circ$ and the $\mathbb{F}$ optional reduction K' of its $\mathcal{S}_2^\circ$ solution K is the $\mathcal{S}'_2$ solution to the continuous-time version* (40) *of* (28)*;*
- *If* ES *is in $\mathcal{L}'_2$, then the $(\mathbb{G}, \mathbb{Q})$* KVA *BSDE* (31) *is well posed in $\mathcal{S}_2^\circ$ and the $\mathbb{F}$ optional reduction* KVA$'$ *of its $\mathcal{S}_2^\circ$ solution* KVA *is the $\mathcal{S}'_2$ solution to the continuous-time version* (41) *of* (31)*.*

Proof. This follows from Lemma 7.1 through the above equivalences. ∎

Next we prove the result announced at the end of Section 5.4, according to which the consistency condition SCR ≥ 0 is optimally handled by defining RM = KVA as per the BSDE (31) and the ensuing EC process as (30).

Theorem 7.1 *Assuming that r is bounded from below and that r and* ES *are in $\mathcal{L}'_2$, using the notation of Lemma 7.3, the admissible set $\mathcal{C}$ and the* EC *process being defined by* (29) *and* (30)*, for the* KVA *process from Lemma 7.3:*
(i) *We have* $\mathrm{EC} = \min \mathcal{C}, \mathrm{KVA} = \min_{C \in \mathcal{C}} K(C)$;
(ii) *The process* KVA *is nonnegative and it is nondecreasing in h.*

Proof. This follows by application of Proposition 7.1 through Lemma 7.3. ∎

7.4 CVA and FVA

In a continuous-time setup, the counterparty exposure and funding cash flows $\mathcal{C}$ and $\mathcal{F}$ (recall Assumption 6.2 set $\mathcal{H} = 0$) are given as $\mathbb{G}$ finite variation processes (cf., regarding $\mathcal{F}$, the last part in Assumption 4.3). Moreover, $\mathcal{C}^\circ$ and $\mathcal{F}^\circ$ can be assumed to be $\mathbb{F}$ finite variation processes, without loss of generality by $\mathbb{F}$ reduction. Regarding the funding cash flows, whenever the CVA process is already well-defined in $\mathcal{S}_2^\circ$ so that the quest for a theoretical target RM = CVA + FVA level (cf. (32)) reduces to the one for the FVA component, we postulate a more specific:

$$d\mathcal{F}_t^\circ = f_t(\mathrm{FVA}_t)dt \text{ until } \tau, \tag{48}$$

for some random function $f = f_t(y)$ measurable with respect to the product of the $\mathbb{F}$ predictable σ field by the Borel σ field on $\mathbb{R}$.

Lemma 7.4 *For $\mathcal{C}$ and $\mathcal{F}$ specialized as above, the* CVA *and* FVA *equations* (19) *and* (20) *in $\mathcal{S}_2^\circ$ are equivalent, through the isometry* (39)*, to the following equations in $\mathcal{S}_2'$:*

$$\mathrm{CVA}_t' = \mathbb{E}_t' \int_t^T \beta_t^{-1}\beta_s d\mathcal{C}_s^\circ,\; t \in [0,T]\,, \tag{49}$$

and (provided CVA' *defined by* (58) *is in $\mathcal{S}_2'$)*

$$\mathrm{FVA}_t' = \mathbb{E}_t' \int_t^T \beta_t^{-1}\beta_s f_s(\mathrm{FVA}_s')ds,\; t \in [0,T]\,. \tag{50}$$

Proof. This follows by application of the results of Crépey and Song (2017c). ∎

Note that a structure (48) for $\mathcal{F}$ is a slight departure from the abstract setup postulated since Section 3.4, where, for simplicity of presentation, $\mathcal{F}$ was introduced as an exogenous process. But, in practice, the dependence (48) of $\mathcal{F}$ on the FVA is only semi-linear (cf. (86) in the one-period setup). Provided the ensuing FVA fixed-point problem is well-posed, for which sufficient conditions will be given in Proposition 7.2, one can readily check, by revisiting all the above, that this dependence does not affect any of the qualitative conclusions drawn in the previous sections of the paper.

The remainder of this section is only an illustrative example. We refer the reader to Albanese et al. (2017) and Armenti and Crépey (2017) for more developed applications, involving in particular initial margin. Let

$$\begin{aligned} dB_t &= r_t B_t dt \\ dD_t &= (r_t + \lambda_t)D_t dt + (1-R)D_{t-}dJ_t = r_t D_t dt + D_{t-}\big(\lambda_t dt + (1-R)\,dJ_t\big) \end{aligned}$$

represent the risk-free OIS deposit asset and a risky bond issued by the bank, which are supposed to be used by the bank for its respective investing and unsecured borrowing purposes.

The $(\mathbb{G},\mathbb{Q})$ martingale condition that applies to (βD) under the $(\mathbb{G},\mathbb{Q})$ valuation rule in Definition 3.1 implies that $\lambda = (1-R)\gamma$, where γ is the $(\mathbb{G},\mathbb{Q})$ default intensity of the bank and R its assumed recovery rate (also consistent with Assumption 5.1). Hence

$$\lambda_t dt + (1-R)\,dJ_t = (1-R)d\mu_t,$$

where $d\mu_t = \gamma dt + dJ_t$ is the $(\mathbb{G},\mathbb{Q})$ compensated jump-to-default martingale of the bank.

We assume all re-hypothecable collateral and we denote by Q a $\mathbb{G}$ optional process representing the difference between the collateral MtM posted by the CA desk to the clean desks (cf. Corollary 3.3) and the collateral received by the CA desk from the clients.

Lemma 7.5 *For* $0 \le t \le \bar{\tau}$*:*

$$\begin{aligned} d\mathcal{F}_t &= (1-R)(Q_{t-} - \mathrm{RC}_{t-})^+ d\mu_t, \\ d\mathcal{W}^{ca}_t &= -(d\mathrm{RC}_t + d\mathcal{C}_t) + r_t \mathrm{RC}_t dt - (1-R)(Q_{t-} - \mathrm{RC}_{t-})^+ d\mu_t, \\ dL_t &= dL^{ca}_t = d\mathrm{RC}^\circ_t - r_t \mathrm{RC}_t dt + d\mathcal{C}^\circ_t + \lambda_t (Q_t - \mathrm{RC}_t)^+ dt. \end{aligned} \tag{51}$$

Assuming the CVA *equation* (19) *already well-posed in* $\mathcal{S}^\circ_2$*, we have*

$$\begin{aligned} d\mathcal{F}^\circ_t &= \lambda_t (Q_t - \mathrm{CVA}'_t - \mathrm{FVA}_t)^+ dt, \\ d\mathcal{F}^\bullet_t &= (1-R)(Q_{t-} - \mathrm{CVA}_{t-} - \mathrm{FVA}_{t-})^+ (-dJ_t). \end{aligned} \tag{52}$$

In particular, $\mathcal{F}^\circ$ *is of the form* (48) *for* $f_t(y) = \lambda_t (Q_t - \mathrm{CVA}'_t - y)^+$.

Proof. Under Assumption 5.2 that equity capital is not used for funding purposes by the bank, the funding policy of the CA desk reduces to a splitting of the amount RC_t on the reserve capital account as

$$\begin{aligned} \mathrm{RC}_t = & \underbrace{Q_t}_{\text{Posted collateral remunerated OIS}} \\ & + \underbrace{(\mathrm{RC}_t - Q_t)^+}_{\text{Cash invested at the risk-free rate}} \\ & - \underbrace{(\mathrm{RC}_t - Q_t)^-}_{\text{Cash unsecurely funded}} \\ = & \underbrace{\big(Q_t + (\mathrm{RC}_t - Q_t)^+\big)}_{\text{Invested at the risk-free rate as } \nu_t B_t} - \underbrace{(\mathrm{RC}_t - Q_t)^-}_{\text{Unsecurely funded as } \eta_t D_t}. \end{aligned} \tag{53}$$

In the line of Assumption 2.4, a standard continuous-time self-financing equation expressing the conservation of cash flows at the level of the CA desk yields[9]

$$\begin{aligned} d\left(\nu_t B_t - \eta_t D_t\right) &= \nu_t dB_t - \eta_{t-} dD_t \\ &= \nu_t r_t B_t dt - \eta_t (r_t + \lambda_t) D_t dt - (1-R)\eta_{\tau-} D_{\tau-} dJ_t \\ &= r_t \mathrm{RC}_t dt - (1-R)\eta_{t-} D_{t-} d\mu_t, \; 0 \le t \le \bar{\tau}. \end{aligned} \tag{54}$$

By identification with (11), we obtain the first line in (51), which, through the to-be-satisfied second balance condition $\mathrm{RC} = \mathrm{CVA} + \mathrm{FVA}$ in (32), is equivalent to (52) (assuming the CVA already well-defined in $\mathcal{S}^\circ_2$). Also accounting for the resets of the

[9] A left-limit in time is required in η because D jumps at time τ, so that the process η, which is defined through (53) as $\frac{(\mathrm{RC}-Q)^-}{D}$, is not predictable.

RC account and for the counterparty exposure of the CA desk (and recalling that Assumption 6.2 set $\mathcal{H}=0$), we conclude that

$$\begin{aligned} d\mathcal{W}^{ca}_t &= -(d\mathrm{RC}_t + d\mathcal{C}_t) + d\left(\nu_t B_t - \eta_t D_t\right) \\ &= -(d\mathrm{RC}_t + d\mathcal{C}_t) + r_t \mathrm{RC}_t dt - (1-R)\eta_{t-} D_{t-} d\mu_t, \end{aligned} \tag{55}$$

which yields the second and third liness in (51) (also consistent with the first line in (12), since Assumption 6.2 set $L^{cl}=0$). ∎

In what follows we further assume that the bank portfolio involves a single client with default time denoted by τ_1, that $\mathbb{Q}(\tau_1=\tau)=0$, that the liquidation of a defaulted party is instantaneous, and that no contractual cash flows are promised at the exact times τ and τ_1. We refer the reader to Albanese et al. (2017) and Armenti and Crépey (2017) for relaxation of all these assumptions, in respective bilateral and centrally cleared setups.

Let J and J^1, respectively R and R_1, denote the survival indicator processes and the assumed recovery rates (cf. Definition 3.4) of the bank and its client. In this case, Q is of the form J^1Q^1, where Q^1 is the difference between the clean valuation P^1 of the client portfolio and the amount Γ^1 of variation margin to be transferred[10] between the client and the CA desk in case of a default.

Lemma 7.6 *For* $0 \le t \le \bar{\tau}$,

$$\begin{aligned} d\mathcal{C}^{\circ}_t &= (1-R_1)(Q^1_{\tau_1})^+(-dJ^1_t), \\ d\mathcal{C}^{\bullet}_t &= \mathbb{1}_{\{\tau\le\tau_1\}}(1-R)(Q^1_{\tau})^-(-dJ_t). \end{aligned} \tag{56}$$

Proof. Before the defaults of the bank or its client, the contractual cash flows from the client to the CA desk exactly compensate the contractual cash flows from the clean desk to the CA desk, so that there are no contributions to the process $\mathcal{C}$. Because of this, and since liquidations are instantaneous, it is enough to focus on the contributions to $\mathcal{C}$ at time $\tau\wedge\tau_1$. By symmetry, it is enough to prove the first line in (56). Let $\epsilon=(Q^1_{\tau_1})^+$. By Assumption 3.4, if the counterparty defaults at $\tau_1<\tau$, then (recalling $Q^1=P^1-\Gamma^1$ and having excluded the possibility of contractual cash flows at times τ or τ_1):

- On the client portfolio side, the CA desk receives

$$\Gamma^1_{\tau_1} + R_1(Q^1_{\tau_1})^+ - (Q^1_{\tau_1})^- = \mathbb{1}_{\epsilon=0}P^1_{\tau_1} + \mathbb{1}_{\epsilon>0}(\Gamma^1_{\tau_1}+R_1\epsilon);$$

- The cleaned portfolio between the CA desk and the clean desk of the bank is unwound, resulting in a settlement of $P^1_{\tau_1}$ from the CA desk to the clean desks.

[10] Property-wise, having already been posted as a loan by the client to the CA desk (if positive, or by the CA desk to the client otherwise).

Combining both cash flows, the CA desk loss amounts to

$$P^1_{\tau_1} - \big(\mathbb{1}_{\epsilon=0}P^1_{\tau_1} + \mathbb{1}_{\epsilon>0}(\Gamma^1_{\tau_1} + R_1\epsilon)\big) = \mathbb{1}_{\epsilon>0}(P^1_{\tau_1} - \Gamma^1_{\tau_1} - R_1\epsilon) = (1-R_1)\epsilon,$$

which shows the first line in (56). ∎

Note that, by $\mathbb{F}$ reduction, we may and do assume that τ_1 is an $\mathbb{F}$ stopping time and that Q^1 is an $\mathbb{F}$ optional process.

Proposition 7.2 *Assuming that r is bounded from below, that* CVA$'$ *defined by* (58) *is in $\mathcal{S}'_2$, and that the processes r, λ, and $\lambda(J^1Q^1 - \mathrm{CVA}')^+$ are in $\mathcal{L}'_2$, then the* FVA$'$ *equation* (50) *is well-posed in $\mathcal{S}'_2$, the* CA, CVA, *and* FVA *equations* (18), (19), *and* (20) *are well-posed in $\mathcal{S}^\circ_2$, and we have, for $0 \le t \le \bar{\tau}$:*

$$\mathrm{CVA}_t = (\mathrm{CVA}')^{\tau-}_t \text{ and } \mathrm{FVA}_t = (\mathrm{FVA}')^{\tau-}_t, \text{ where, for } 0 \le t \le T: \tag{57}$$

$$\mathrm{CVA}'_t = \mathbb{E}'_t\big[\mathbb{1}_{\{t<\tau_1<T\}}\beta_t^{-1}\beta_{\tau_1}(1-R_1)(Q^1_{\tau_1})^+\big], \tag{58}$$

$$\mathrm{FVA}'_t = \mathbb{E}'_t\int_t^T \beta_t^{-1}\beta_s\lambda_s(J^1_sQ^1_s - \mathrm{CVA}'_s - \mathrm{FVA}'_s)^+ds; \tag{59}$$

$$\begin{aligned}\mathrm{CL}_t = {} & \underbrace{\mathbb{E}_t\big[\mathbb{1}_{\{t<\tau\le\tau_1\wedge T\}}\beta_t^{-1}\beta_{\tau_1}(1-R)(Q^1_\tau)^-\big]}_{\mathrm{FTDDVA}_t}\\ & + \underbrace{\mathbb{E}_t\big[\beta_t^{-1}\beta_\tau\mathbb{1}_{\{t<\tau<T\}}(J^1_{\tau-}Q^1_{\tau-} - \mathrm{CA}_{\tau-})^+\big]}_{\mathrm{FDA}_t}\\ & + \underbrace{\mathbb{E}_t\big[\beta_t^{-1}\beta_\tau\mathbb{1}_{\{t<\tau<T\}}\mathrm{CVA}_\tau\big]}_{\mathrm{CVA}^{\mathrm{CL}}_t} + \underbrace{\mathbb{E}_t\big[\beta_t^{-1}\beta_\tau\mathbb{1}_{\{t<\tau<T\}}\mathrm{FVA}_\tau\big]}_{\mathrm{FVA}^{\mathrm{CL}}_t};\end{aligned} \tag{60}$$

$$\begin{aligned}\mathrm{CR}_t = {} & \underbrace{\mathbb{E}_t\big[\mathbb{1}_{\{t<\tau_1\le\tau\wedge T\}}\beta_t^{-1}\beta_{\tau_1}(1-R_1)(Q^1_{\tau_1})^+\big]}_{\mathrm{FTDCVA}_t}\\ & - \underbrace{\mathbb{E}_t\big[\mathbb{1}_{\{t<\tau\le\tau_1\wedge T\}}\beta_t^{-1}\beta_{\tau_1}(1-R)(Q^1_\tau)^-\big]}_{\mathrm{FTDDVA}_t};\end{aligned} \tag{61}$$

$$dL_t = (1-R_1)(Q^1_{\tau_1})^+(-dJ^1_t) + d\mathrm{CVA}_t - r_t\mathrm{CVA}_tdt \tag{62}$$

$$+\lambda_t(J^1_tQ^1_t - \mathrm{CVA}_t - \mathrm{FVA}_t)^+dt + d\mathrm{FVA}_t - r_t\mathrm{FVA}_tdt. \tag{63}$$

Proof. Under the assumptions of the proposition, the $(\mathbb{F},\mathbb{P})$ FVA$'$ BSDE (50) is a monotonous coefficient BSDE well-posed in $\mathcal{S}'_2$, based on the results of Kruse and Popier (2016, Sect. 4) by similar computations as in the proof of Lemma 7.2. In view of Lemma 7.4, this proves the CVA and FVA, hence CA = CVA + FVA, related statements. The CL and CR formulas (60) and (61) readily follow from (21), (52) and (56), for CL, and by (56) and the last sentence in Definition 5.1, for CR. The dynamics (63) for L are obtained by plugging, into the last line in (51), the first line in (56) and the second shareholder balance condition RC = CVA + FVA in (32) (also recalling that CA does not jump at τ). ∎

In the fair valuation or contra-liability formulas (61) and (60), "FTD" in FTDCVA or FTDDVA stands for "first to-default" CVA and DVA, which only value the related cash flows until the bank default τ.

By contrast, even though our setup includes the default of the bank itself (which is the essence of the contra-liabilities wealth transfer issue), we end up with unilateral CVA, FVA, and KVA formulas (58), (59), and (46) pricing the related cash flows until the final maturity T of the portfolio (as opposed to $\bar{\tau} = \tau \wedge T$)! And these equations only involve the original discount factor β, without any credit spread.

Recall from (33) that the only XVAs that ultimately matter in entry prices are the CVA, the FVA, and the KVA. Unilateral XVA costs to be accounted for in entry prices is indeed what follows from a proper accounting of the wealth transfers involved (cf. Assumption 2.6 and the last bullet point before Remark 5.1) and from the mathematical analysis of the resulting fix-point equations. However, this also makes the corresponding XVAs more expensive than the bilateral XVAs that appear in most of the related literature (cf. Burgard and Kjaer (2017, Section 4.4)).

8 Unilateral Versus Bilateral XVAs

Following up on the above, a unilateral CVA is actually required for being in line with the regulatory requirement that reserve capital should not diminish as an effect of the sole deterioration of the bank credit spread (see Albanese and Andersen (2014, Section 3.1)). But a bilateral FVA already satisfies this regulatory monotonicity requirement (essentially, as, when the bank credit spread deteriorates, the shortest duration of a bilateral FVA is compensated by the higher funding spread). And the KVA is not concerned by this requirement.

In this section we revisit the XVA equations when different choices are made on the fate of the residual reserve capital and risk margin in case of default of the bank (cf. Assumption 2.6 and the last bullet point before Remark 5.1). We denote by $\mathcal{S}_2^\bullet$ the subspace of the processes in $\mathcal{S}_2$ with a null terminal condition at $\bar{\tau}$.

8.1 From Unilateral to Bilateral KVA

A unilateral KVA as per (46) might arguably be unjustified, with regard to the fact that bank insolvency means depletion of the whole economic capital of the bank, which includes the risk margin. Hence, the notion of transfer of the residual risk margin to creditors at bank default (cf. Assumption 2.6) would be pointless. However, we recall from Remark 2.2 that default of a bank does not mean insolvency, but illiquidity mainly.

Yet, let us assume here, for the sake of the argument, all the risk margin already gone at time $\bar{\tau}$ through some additional model feature, such as an operational loss that occurs at τ and triggers instantaneous depletion of economic capital. Revisiting the KVA derivations of Sections 4.3 and 5.2–5.4 in the thus-modified setup, we obtain the following modified KVA equation in $\mathcal{S}_2^{\bullet}$:

$$(-\widetilde{\mathrm{KVA}}^{\circ}) \text{ has the } (\mathbb{G},\mathbb{Q}) \text{ drift coefficient } h\big(\max(\widetilde{\mathrm{ES}},\widetilde{\mathrm{KVA}})-\widetilde{\mathrm{KVA}}\big), \tag{64}$$

instead of our original KVA equation (31) in $\mathcal{S}_2^{\circ}$. That is, in the continuous-time setup of Sections 7.2–7.3:

$$\mathrm{KVA}_t = h\mathbb{E}_t \int_t^{\bar{\tau}} e^{-\int_t^s (r_u+h)du} \max(\mathrm{ES}_s, \mathrm{KVA}_s)ds,\ t\in[0,\bar{\tau}]. \tag{65}$$

Or, in an an equivalent $(\mathbb{F},\mathbb{P})$ formulation: $\mathrm{KVA}=(\mathrm{KVA}')^{\tau}$ on $[0,\bar{\tau}]$, where

$$\mathrm{KVA}'_t = h\mathbb{E}'_t \int_t^{T} e^{-\int_t^s (r_u+h+\gamma_u)du} \max(\mathrm{ES}_s, \mathrm{KVA}'_s)ds,\ t\in[0,T]. \tag{66}$$

By comparison with our original KVA' equations (45)–(46), note in particular the "$+\gamma_u$" in the discount factor in (66).

8.2 From Unilateral to Bilateral FVA

A bilateral FVA, which already satisfies the regulatory monotonicity requirement on the related reserve capital, might be advocated as follows (similar developments would apply to the MVA if we had it explicitly in our model, see Remark 5.1).

Assume, for the sake of the argument, that the portfolio of the defaulted bank with clients is unwounded with risk-free counterparties, called novators. The residual amount of CVA reserve capital is required by the novators to deal with the residual counteparty risk on the deals. But the residual amount of FVA reserve capital is useless to the novators. In view of this, one could disentangle the CA desk into a CVA desk and an FVA desk, each endowed with their own reserve capital account, and decide that, upon bank default, as an exception to Assumptions 2.3 and 2.6, the residual FVA capital reserve flows back into equity capital and not to creditors.

Revisiting the CVA and FVA derivations of Sections 5.1 and 7.4 in the thus-modified setup, the CVA equations do not change, but we obtain the following modified FVA equation in $\mathcal{S}_2^{\bullet}$:

$$\widetilde{\mathrm{FVA}}_t = \mathbb{E}_t\big(\widetilde{\mathcal{F}}^{\circ}_{\bar{\tau}} - \widetilde{\mathcal{F}}^{\circ}_{t}\big),\ t\le\bar{\tau}, \tag{67}$$

instead of the FVA equation (20) in $\mathcal{S}_2^{\circ}$. That is, in the continuous-time setup of Section 7.4:

$$\mathrm{FVA}_t = \mathbb{E}_t \int_t^{\bar{\tau}} \beta_t^{-1}\beta_s f_s(\mathrm{FVA}_s)ds,\ t\in[0,\bar{\tau}]. \tag{68}$$

Or, equivalently: $\mathrm{FVA}=(\mathrm{FVA}')^{\tau}$ on $[0,\bar{\tau}]$, where

$$\mathrm{FVA}'_t = \mathbb{E}'_t \int_t^{T} e^{-\int_t^s (r_u+\gamma_u)du} f_s(\mathrm{FVA}'_s)ds,\ t\in[0,T]. \tag{69}$$

Note again the blended discount factor in (69), as opposed to the risk-free discount factor β in our original FVA' formula (50).

8.3 A Bank with Four Floors

The considerations of Section 8.2 point out to the vision of a bank with four floors:[11] clean desks are the bottom floor. On top of it sits the CVA floor, which obtains risk-free funding from the FVA floor above it, just like the clean desks, and is only in charge of filtering out counterparty risk from client deals. Conversely, the FVA (third) floor is just in charge of the funding of the bank, obtaining risky funding from the outside of the bank and distributing risk-free funding, in the form of collateral provided at an OIS cost, to the CVA and clean desks for funding their trading.

The CVA and FVA desks are now on different floors: They have their own and separate reserve capital account, hedge, and trading loss process, whereas they were sharing each of these in earlier sections of the paper. Clean desks continue to be served by the floors above them with a cleaned portfolio worth MtM (in the sense of Definition 3.4). Each of the three trading floors generates a martingale trading loss process:

- For the clean floor, an $(\mathbb{F}, \mathbb{P})$ martingale that, stopped before τ, is a $(\mathbb{G}, \mathbb{Q})$ martingale;
- For the CVA floor, a $(\mathbb{G}, \mathbb{Q})$ martingale without jump at τ;
- For the FVA floor, a $(\mathbb{G}, \mathbb{Q})$ martingale jumping to 0 at time τ, this jump to 0 reflecting the deviation from Assumption 2.3 considered in Section 8.2.

The upper (fourth) floor is the management in charge of the KVA payments, i.e. of the dividend distribution policy of the bank.

This view is applied to bilateral portfolios in Albanese et al. (2017). A more simplistic take made in Armenti and Crépey (2017), where the focus is less mathematical and more on the CCP feature of the setup there, is to view all the XVA equations as endowed with a null terminal condition at $\bar{\tau}$. This allows avoiding any enlargement of filtration technicality, as all equations can then be solved directly in $\mathbb{G}$. However, it results in a bilateral CVA that misses the $\mathrm{CVA}^{\mathrm{CL}}$ wealth transfer from shareholders to creditors at the bank default time (cf. Assumption 2.6 and the last bullet point before Remark 5.1). In a CCP setup where the CVA is very small (and of a nonstandard nature anyway) this may be acceptable, but in a bilateral setup it violates the regulatory monotonicity requirement on reserve capital.

In the earlier sections of the paper, rather than having to invoke an externality in the form of "an exceptional loss at τ" in order to justify a bilateral KVA as in Section 8.1, or having to make specific assumptions regarding the novation of the defaulted bank portfolio for obtaining a bilateral FVA as in Section 8.2, we stick to internally consistent (cf. Remark 2.2), all unilateral XVAs. If found overly conservative, the absolute size of such unilateral valuation adjustments (it is the relative consistency between them that matters most) can be mitigated at the calibration stage.

[11] It would even be five if the MVA was disintangled from a FVA then meant in the strict sense of the cost of funding variation margin, cf. Remark 5.1.

9 Comparison with Other Approaches in the XVA Benchmark Model

To conclude the main body of this paper, we compare our findings with alternative approaches in the literature that have been developed in the last years in what we therefore call the XVA benchmark model, namely a Black–Scholes model S for the underlying market risk factor, in conjunction with independent Poisson counterparties and bank defaults: See, not exhaustively, the Burgard and Kjaer (2011, 2013, 2017) CVA and FVA approach, referred to as the BK approach below, the Green et al. (2014) and Green and Kenyon (2015) KVA approach, referred to as the GK approach, Bichuch et al. (2017), or Crépey et al. (2014, Section 4.6).[12]

A general comment in this regard is that using a Black–Scholes replication framework as an XVA toy model is misleading. The view developed in the present paper is that XVAs are mainly about market incompleteness, and therefore fall under a logic orthogonal to Black–Scholes. Promoting a Black–Scholes replication approach in the XVA context is a bit comparable to the mispractice developed during the credit derivative pre-crisis era, when the notion of "Gaussian copula implied correlation" of a CDO tranche was presented as a relative of the Black–Scholes implied volatility of an option, whereas the Gaussian copula model is a purely static device not supported by a sound hedging basis.

Unsurprisingly, this leads to some fuzziness, such as Burgard and Kjaer (2011, 2013, 2017) availing themselves of a replication pricing framework and blaming risk-neutral approaches outside the realm of replication (see the first paragraph in their 2013 paper). However, BK papers themselves end-up doing what they call semi-replication, which is nothing but a form of risk-neutral pricing without (exact) replication. Things becomes sort of a contradiction in terms when KVA itself tries and be included in such framework, as done under the GK approach.

In fact, in the XVA field, even the restriction to a Markov setup (beyond Black–Scholes) is not necessarily innocuous, as we will see in the next-to-last paragraph of Section 9.1. What follows provides more details and comparison with the findings of the present paper.

9.1 CVA and FVA: Comparison with the BK Approach

Burgard and Kjaer (2011, 2013, 2017) repeatedly (and rightfully) say that only pre-default cash-flows matter to shareholders. For instance, quoting the first paragraph in the second reference:

> "Some authors have considered cases where the post-default cash flows on the funding leg are disregarded but not the ones on the derivative. But it is not clear why some post default cashflows should be disregarded but not others",

[12] In journal form Crépey (2015, Part II, Section 5).

to which we subscribe fully.

The introduction of their now classical "(funding) strategy I : semi-replication with no shortfall at own default" (see e.g. (Burgard and Kjaer 2013, Section 3.2)) seems to be in line with the idea, which we also agree with (see Assumption 4.3 and the comment following it), that a shortfall of the bank at its own default does not make much sense and should be excluded from a model.

Problems come with their implementation of these premises.

First, being rigorous with the first principle above implies that the valuation jump of the portfolio at the own default of the bank should be disregarded in the shareholder cash flow stream, as included in our Assumption 2.6 (cf. also the last bullet point before Remark 5.1). However, their computations, stated in terms of $(d\widehat{V} + d\bar{\Pi})$ in Burgard and Kjaer (2013, equation (9)) or $(d\widehat{V}^{\alpha} + d\Pi)$ in Burgard and Kjaer (2017, equation (3.5)), include this cash flow.

In order to be able to restrict attention on the CVA for simplicity, let us thus assume, for the sake of the argument, that the bank, although risky, can both fund itself and invest at the risk-free rate $r = 0$.[13] This corresponds to the limiting case where $\mathcal{F} = 0$ in Assumption 4.3. We put ourselves in the framework of Section 7.4 where Proposition 7.2 was derived, specialized further to a BK setup with volatility σ of a stock S underlying a (single) contract with payoff $\phi(S_T)$ sold by the client to the bank. Taking the difference between the CVA$'$ PDE (in the present BK setup, cf. (57)) and the Black–Scholes PDE for the clean valuation of the contract shows that the bank pre-default CVA-deducted value $\widehat{V}'_t := P^1_t - \mathrm{CVA}'_t$ can be represented in functional form as $\widehat{V}'(t, S_t, J^1_t)$, where the function $\Pi(t,S) = \widehat{V}'(t, S, J^1 = 1)$ satisfies the following pricing equation:

$$\begin{aligned} &\Pi(T,S) = \phi(S) \text{ and, for } t < T, \\ &(\partial_t + \frac{1}{2}\sigma^2 S^2 \partial^2_{S^2})\Pi + \gamma_1\big(R_1(P^1)^+ - (P^1)^-\big) - \gamma_1\Pi = 0. \end{aligned} \tag{70}$$

Here γ_1 is the default intensity of the counterparty, assumed constant in BK. Now, (70) is nothing that the equation (10) for $\widehat{V}$ in Burgard and Kjaer (2013) or (3.8) for $\widehat{V}^{\alpha}$ in Burgard and Kjaer (2017), but for a default intensity γ of the bank, denoted by λ_B there, formally set equal to 0.

Hence, in this BK setup, the CVA-deducted value of the option truly dis-regarding all cash flows from time τ onward, including the jump in valuation at time τ, is not given by the solution $\widehat{V}$ to equation (10) in Burgard and Kjaer (2013) or $\widehat{V}^{\alpha}$ to equation (3.8) in Burgard and Kjaer (2017), but by $\widehat{V} = (\widehat{V}')^{\tau-}$, where $\widehat{V}'$ satisfies the formal analog of these equations with intensity of the bank set equal to 0.

Incidentally, the Itô derivation (9) of the portfolio XVA-deducted value process in Burgard and Kjaer (2013) relies on the implicit assumption that this value process $\widehat{V}$ is in the first place a (regular enough) function of the postulated risk factors, which is not justified in their paper. For instance, in the simplified (CVA only) BK setup

[13] For instance because the bank is highly capitalized and, notwithstanding Assumption 5.2, can in fact use its capital for funding its trading.

above, it is only the process CVA' that is obviously Markov in t, S, and the nondefault indicator process J^1 of the bank client, and then it only holds that $\text{CVA} = (\text{CVA}')^{\tau-}$.

As a consequence, reviewing the funding strategies in Burgard and Kjaer (2017, Section 4), in the special case with $s_B = 0$ there of a pure CVA setup:

- Strategy III, claimed to imply a unilateral CVA as per Albanese and Andersen (2014) (i.e. (57)–(58) as made more precise in the present paper), does in fact not: Duly accounting for the transfer of the residual reserve capital from shareholders to creditors at the bank default time τ (cf. Assumption 2.6 and the last bullet point before Remark 5.1), the funding strategy that does so is simply funding and investing at $r = 0$ (having assumed $s_B = 0$);

- Their respective strategies I and II, not only do not imply the claimed XVA formulas (as all the BK equations forget the above-mentioned wealth transfer), but directly violate the second part in Assumption 4.3 (unless in their notation $V \geq 0$, respectively $\widehat{V} \geq 0$, i.e. in our notation $P^1 \geq 0$, respectively $\Pi \geq 0$);

- Their replication strategy is even less viable, as admitted at the end of Burgard and Kjaer (2013, Section 3.1), which is the motivation for their other strategies.

9.2 KVA: Comparison with the GK Approach

As the risk margin is loss-absorbing, the KVA is not a liability. In fact, despite what the "valuation adjustment" terminology fallaciously induces one to believe, in our view, the KVA is not part of the value of the derivative portfolio, but a risk premium.

In Green et al. (2014) and as also discussed in some theoretical actuarial literature (see Salzmann and Wüthrich (2010, Section 4.4)), the KVA is instead treated as a liability. Viewing the KVA as a liability, hence non loss-absorbing, results in $\text{EC} = \text{SCR} = \text{ES}$ (as opposed to (26) in our setup), and therefore $h\widetilde{\text{ES}}$ instead of $h(\widetilde{\text{EC}} - \widetilde{\text{KVA}})$ in the KVA equation (27). This implies r instead of $(r + h)$ as discount rate in the KVA formula (46) (where KVA' and KVA coincide before τ), and therefore a "too high" KVA (especially if unilateral, cf. Section 8.1).

Moreover, if the KVA is viewed as a liability, forward starting one-year-ahead fluctuations of the KVA must be simulated for economic capital calculation. This makes it intractable numerically, unless one switches from economic capital to regulatory capital in the KVA equation. Using regulatory instead of economic capital is then motivated by practical considerations but is less self-consistent. It loses the connection, established from shareholder-optimization principles in the above, whereby the correct KVA input is the CA desk loss process $L = L^{ca}$ as per (34).

It also leads to unachievement of the corresponding XVA theory: Viewing the KVA as a liability means that KVA payments contribute to the trading profit-and-loss of the bank. But dividends are meant to remunerate risk, i.e. unhedged losses. Hence the XVA loop does not close.

In addition, Green et al. (2014) derive their KVA equation in a replication framework, whereas the main motivation for capital requirements, such as CVA reserve capital to be held against counterparty default losses, is that credit markets are incomplete and hedging is not possible. A KVA equation similar to the one in Green et al. (2014) is derived in an expectation setup in Elouerkhaoui (2016).

Remark 9.1 We believe that, if a bank chooses not to hedge some risk even if the hedging instruments are present and markets are complete, then, assuming at least that hedges are fairly priced, they are behaving sub-optimally for a market maker. Whenever possible, hedging leads to a reduction of risk (and cost of capital) without affecting returns. Having said this, we note that our conclusions in this paper hold as soon as the bank does not hedge its jump-to-default exposures, irrespectively of the fact that the bank could potentially hedge these or not. ∎

A XVAs Accounting Interpretation

With XVAs, the pricing and risk management of financial derivatives evolves from a hedging paradigm to balance sheet optimization. This section provides the balance sheet perspective on the XVA metrics. In particular, it allows understanding in more depth the use of L instead of CET1 in Definition 5.2. This substitution is important as it underlies the conclusion, drawn after Corollary 6.1, that the correct KVA input is a CA desk loss process $L = L^{ca}$ as per (34), which makes the XVA problem as a whole self-contained.

A.1 Accounting Equity

The accounting equity represents the wealth of the bank as a whole:

Definition A.1 The accounting equity (AE) is given by

$$\mathrm{AE} = \mathrm{Assets} - \mathrm{Liabilities} + \mathrm{CL} - \mathrm{CA}, \tag{71}$$

where CA and CL have been formally introduced in Definition 5.1, and:

- In the Assets (respectively Liabilities), one places the mark-to-market MtM^+ (respectively MtM^-) of the portfolio receivables (respectively payables) and the collateral CM^- posted (respectively CM^+ received) by the clean desks: cf. Definition 3.4 and Corollary 3.3.
- Assets also include reserve capital (RC), risk margin (RM), shareholder capital at risk (SCR), and an additional (typically unknown) amount of uninvested capital (UC). ∎

The above decompositions are reflected in the bank balance sheet Figure 3, where:

- The mark-to-market of the portfolio payables and receivables, as well as the corresponding collateral (dubbed clean margin in the paper), are shown in dashed boxes at the bottom, because their role vanishes under Assumption 6.2, as the clean desks then generate no trading losses ($L^{cl} = 0$);

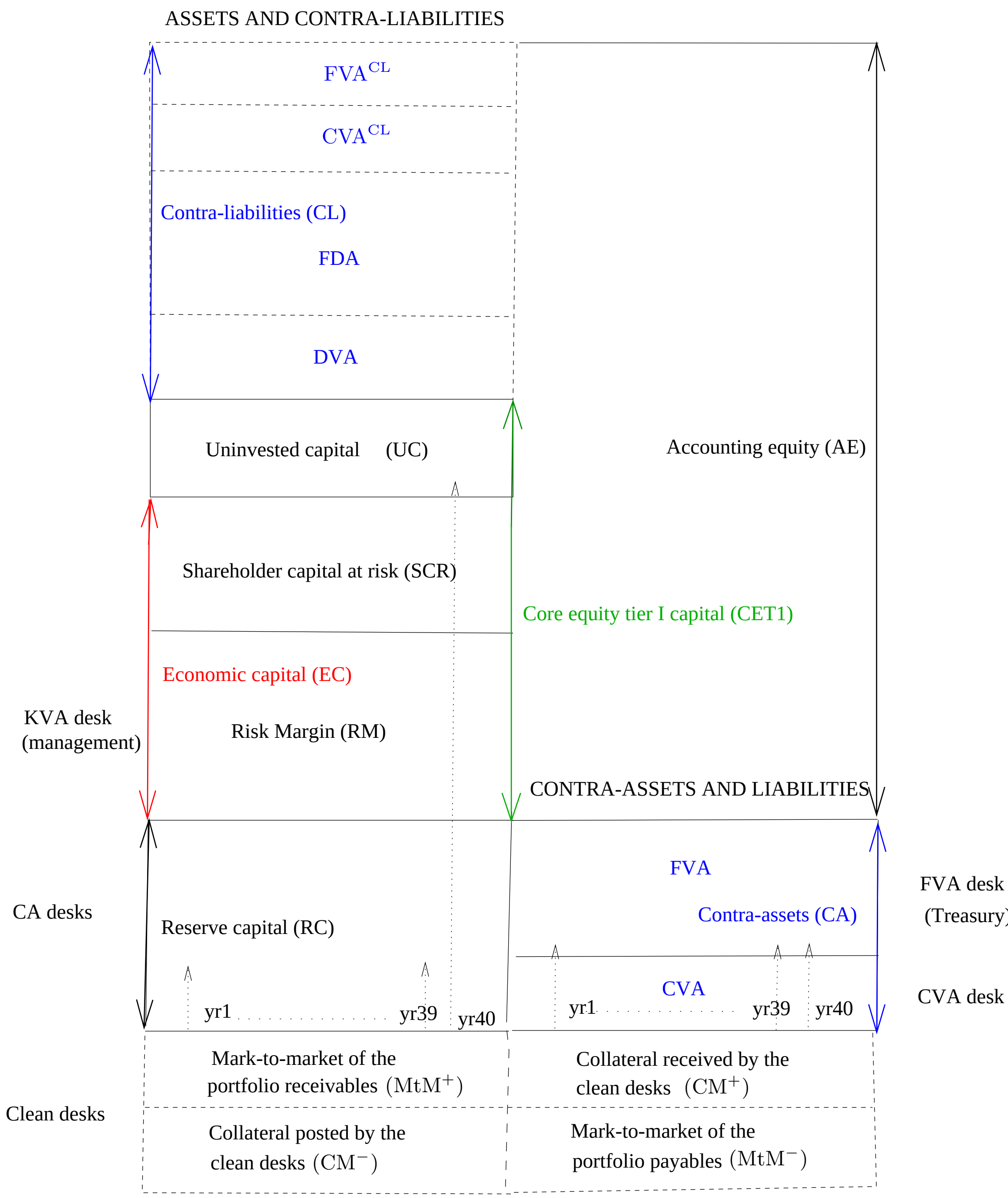

Figure 3: Balance sheet of the bank.

- Contra-liabilities at the top of the figure are shown in dotted boxes because, as shown in Theorem 5.1, they are only an actual benefit to bank creditors, whereas, under Assumption 2.3 (cf. also Assumption 6.1), only the interest of shareholders matters regarding the determination of derivative entry prices and the related accounting and dividend policies of the bank;

- The arrows in the left column represent trading losses of the CA desk in "normal years 1 to 39" and in an "exceptional year 40" with full depletion (i.e. refill, under our continuous reset Assumption 2.5) of RC, RM, and SCR (the numberings yr1 to yr40 are fictitious yearly scenarios in line with the 97.5% expected shortfall of the losses that underlies economic capital);

- The arrows in the right column symbolize the average depreciation in time of contra-assets between deals.

Note that we are representing here the balance sheet of a "three floor" bank, as considered in most of the paper. To render the situation of a "four floor" bank as per Sections 8.2–8.3, the CA floor needs to be disentangled into a CVA floor and an FVA floor, each endowed with their own reserve capital account (and hedge). In any case, we emphasize that, since our risk margin is loss-absorbing (part of economic capital), we do not put its theoretical KVA target value as a liability (or contra-asset) on the balance sheet.

A.2 Core Equity Tier I Capital

Core equity tier I capital is the regulatory metric that represents the core financial strength of a bank. Since contra-liabilities do not benefit to the shareholders:

Definition A.2 (CET1) Core equity tier I capital (CET1) is given by

$$\mathrm{CET1} = \mathrm{AE} - \mathrm{CL}. \blacksquare \tag{72}$$

In the spirit of structural models of the default time of a firm, such as the Merton (1974) model, CET1 corresponds to "distance to default" and the regulation accordingly defines economic capital in terms of a risk measure of its (negative) variation over a one-year period (see Section 5.2).

However, in our setup where the trading losses L of the bank are instantaneously realized and compensated by shareholders, we never have structural default, but only totally unpredictable one (see Section 2.3): The last sentence in Section 3.2, according to which the wealth process $\mathcal{W}$ of the bank and its trading loss process L in (3) and (6) correspond to the appreciation of accounting equity and the depreciation of core equity tier I capital, is "morally" true, but it needs be mitigated accounting for our continuous reset Assumption 2.5.

Namely, as the mark-to-market MtM of the portfolio constantly matches the corresponding collateral CM (cf. Corollary 3.3), the difference between Assets and Liabilities reduces, in view of Definition A.1, to

$$\text{Assets} - \text{Liabilities} = \text{RC} + \text{EC} + \text{UC} \tag{73}$$

(recalling RM + SCR = EC). Hence, by (71) and (72),

$$\text{CET1} = (\text{RC} - \text{CA}) + (\text{EC} + \text{UC}). \tag{74}$$

But, under our continuous reset Assumption 2.5, the first parenthesis in (74) vanishes (this is the first shareholder balance condition in (32), recalling from (22) that CVA + FVA = CA). Moreover, the impacts of the derivative portfolio on the different entries in

$$\text{CET1} = \text{EC} + \text{UC} \tag{75}$$

are interconnected.

Example A.1 If the risk of the bank decreases, i.e. if EC decreases (for instance because the portfolio simply amortizes, or because of a favorable evolution of the underlying risk factors, or because clients post collateral to the bank), then this can be compensated by a corresponding increase in uninvested capital (see Figure 4 for an illustrative scenario). Hence a continuously reset CET1 as per (75) does not need to change. If the default of a counterparty occurs, then the counterparty default loss is

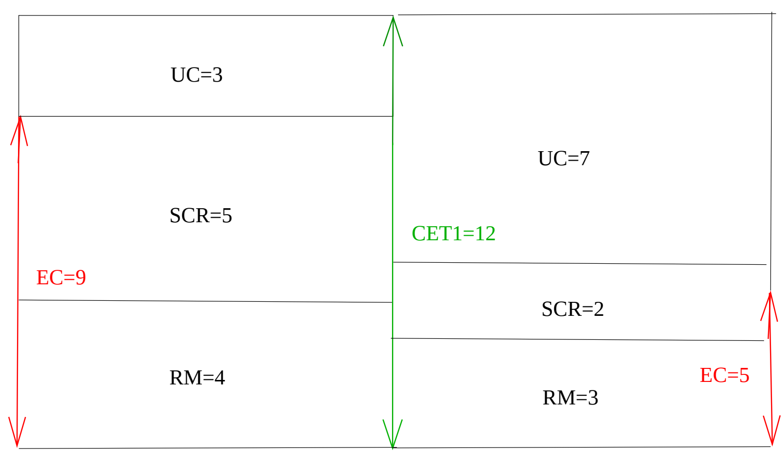

Figure 4: Illustrative impacts of a risk decrease on the different entries in (73), where EC = RM + SCR.

instantaneously realized as a loss by the CA desk, and CET1 in (75) does not need to change. ∎

In fact, we could even exacerbate the continuous reset Assumption 2.5 by postulating on top of it that

$$\text{CET1} = \text{EC} + \text{UC} = y, \tag{76}$$

a positive constant (assuming, as done for the risk margin account in Section 2.2, that OIS interest payments on SCR and UC go to the shareholders directly, as opposed to accruing the SCR and UC amounts themselves).

But, under the continuous reset Assumption 2.5, by core equity tier I capital in Section 5.2, we must understand not only the continuously reset (possibly even constant) formal CET1 process as per (76). We also need to account for the amounts $(-L)$ that are gained as trading profits or absorbed as trading losses by the shareholders.

In fact, under Assumption 2.5 with a formal CET1 process given as the constant y in (76), discounted equity depletions correspond exactly to the process $\widetilde{L} = \int_0^{\cdot} \beta_t dL_t$, or L' instead of L in this formula understood "on a going concern basis" (see Section 5.2), which is the rationale for our use of $\widetilde{L}'$ in Definition 5.2.

B XVA Formulas in a One-Period Setup

In this section, we apply our XVA approach to a portfolio made of a single deal between a client and the bank without prior endowment, in an elementary one-period (one year) setup.

A one-period setup is too narrow for practical XVA purposes. However, the key concept of XVAs as pricing adjustments aligning the mark-to-market of a deal (or portfolio) to the interest of bank shareholders remains (cf. the last sentence in Theorem 6.1). Moreover, this setup gives useful insights, as it is possible to derive stylized formulas for all the quantities at hand.

B.1 One-Period XVA Setup

In a one-period setup, there are no filtrations involved and it is enough to work with, instead of processes throughout the paper, random variables, which correspond to the increment of these processes over the single time step in the model. Accordingly, in this part, all the processes $Y = \mathcal{P}, \mathcal{C}, \mathcal{F}$ (recall Assumption 6.2 set $\mathcal{H} = 0$) and the related processes Y° and $Y^{\bullet}$ in Sections 4–5, as also all the trading loss processes, are identified with random variables on the (one and only, in this context) probability space $(\Omega, \mathcal{A}, \mathbb{Q})$. In addition, we set r equal to 0. Hence, we may and do dismiss the $\cdot'$ and $\widetilde{\cdot}$ notations everywhere.

Since values, prices, and XVAs only matter at time 0 in a one-period setup, we identify all the XVA processes, as well as the clean valuation process P of the portfolio, with their values at time 0. We assume that all the prices that are due are instantaneously paid at time 0. We assume likewise that the KVA paid by the client at time 0 is immediately transferred by the bank management to the shareholders. Hence there is no need and purpose for RC and RM bank accounts, so that we may and do assume these to be zero. The balance conditions (32) are irrelevant in this context.

There is no residual value on the (non-existing) reserve capital account when the bank defaults. Hence $\mathrm{CVA}^{\mathrm{CL}}$ and $\mathrm{FVA}^{\mathrm{CL}}$ are redefined as zero (cf., a contrario, Assumption 2.6 and the last bullet point before Remark 5.1), so that CL in (21) reduces to $(\mathrm{DVA} + \mathrm{FDA})$.

As there is no risk margin account, hence $\mathrm{EC} = \mathrm{SCR}$. By Lemma 5.2, we have $\mathrm{ES} \geq 0$. Hence the consistency condition $\mathrm{SCR} \geq 0$ (cf. Section 5.4) is naturally achieved by simply taking $\mathrm{EC} = \mathrm{SCR} = \mathrm{ES}$.

The bank and client are both default prone with zero recovery. We denote by J and J_1 the survival indicators (random variables) of the bank and client at time 1, with default probability of the bank $\mathbb{Q}(J = 0) = \gamma$ and no joint default for simplicity, i.e $\mathbb{Q}(J = J_1 = 0) = 0$. The terms Y° and $Y^\bullet$ in the decomposition (4) are meant as

$$\text{Random variables } Y^\circ\text{, independent from } J\text{, and } Y^\bullet\text{, of the form } (1-J)Z. \tag{77}$$

To fit this framework, we assume $\mathcal{P}$ independent from J.

We assume that the client portfolio is uncollateralized.

B.2 One-Period XVA Analysis

Lemma B.1 *We have*

$$\begin{aligned}
&\mathcal{C}^\circ = (1-J_1)\mathcal{P}^+,\ \mathcal{F}^\circ = \gamma(P - \mathrm{CA})^+\\
&\mathcal{C}^\bullet = (1-J)\mathcal{P}^-,\ \mathcal{F}^\bullet = (1-J)(P - \mathrm{CA})^+\\
&\mathcal{W}^{ca} = -\big(\mathcal{C}^\circ + \mathcal{F}^\circ - \mathrm{CA}\big) + \mathcal{C}^\bullet + \mathcal{F}^\bullet\\
&L = L^{ca} = \mathcal{C}^\circ + \mathcal{F}^\circ - \mathrm{CA}.
\end{aligned} \tag{78}$$

Proof. In order to buy the client portfolio for $(P - \mathrm{CA})$ while lending P to the clean desks so that these can buy the cleaned portfolio from the CA desk itself for P (cf. the first bullet point in Theorem 6.1), the CA desk needs to borrow $(P - \mathrm{CA})^+$ unsecured or invest $(P - \mathrm{CA})^-$ risk-free. Under Assumption 5.1, having assumed zero recoveries, unsecured borrowing is priced $\gamma \times$ the amount borrowed by the bank, so that at time 0 the bank must pay

$$\gamma(P - \mathrm{CA})^+$$

for its funding.

At time 1, by application of Assumption 3.4:

- On the client portfolio side:
  - If the bank is not in default (i.e. $J = 1$), then the CA desk closes the position with the client while receiving $\mathcal{P}$ from its client if the latter is not in default (i.e. $J_1 = 1$), whereas the bank pays $\mathcal{P}^-$ to its client if the latter is in default (i.e. $J_1 = 0$). In addition, the CA desk reimburses its funding debt $(P - \mathrm{CA})^+$ or receives back the amount $(P - \mathrm{CA})^-$ it had lent at time 0.
  - If the bank is in default (i.e. $J = 0$), then the CA desk receives back $\mathcal{P}^+$ on the derivative as well as the amount $(P - \mathrm{CA})^-$ it had lent at time 0.

- On the cleaned portfolio side, the CA desk delivers a cash flow $\mathcal{P}$ to the clean desks at time 1, whatever the default status of the parties at hand.

Collecting all cash flows under Assumption 2.4, the result $\mathcal{W}^{ca}$ of the CA desk over the year is:

$$\begin{aligned}\mathcal{W}^{ca} &= -\gamma(P-\mathrm{CA})^+\\ &\quad + J\big(J_1\mathcal{P}-(1-J_1)\mathcal{P}^- -(P-\mathrm{CA})^+ +(P-\mathrm{CA})^-\big)\\ &\quad +(1-J)\big(\mathcal{P}^+ +(P-\mathrm{CA})^-\big)-(\mathcal{P}-P)\\ &= -\big(\underbrace{(1-J_1)\mathcal{P}^+}_{\mathcal{C}^\circ}+\underbrace{\gamma(P-\mathrm{CA})^+}_{\mathcal{F}^\circ}-\mathrm{CA}\big)\\ &\quad +\underbrace{(1-J)\mathcal{P}^-}_{\mathcal{C}^\bullet}+\underbrace{(1-J)(P-\mathrm{CA})^+}_{\mathcal{F}^\bullet}\end{aligned} \tag{79}$$

(as easily checked for each of the three possible values of the pair (J, J_1)), where the identification of the different terms follows from their financial interpretation. In view of (5) and of the interpretation (77) of the decomposition (4) in a one-period setup, (79) implies the last two lines in (78) (recall Assumption 6.2 set $L^{cl}=0$), which are also consistent with (24) (remembering that, in the present one-period setup, CA corresponds to the time 0 value or, equivalently, to "the negative of the first increment", of the CA process in (24)). ∎

Observe that $\mathcal{F}^\circ=\gamma(P-\mathrm{CA})^+$ is deterministic in our one-period setup.

Also note that, in case of default of the bank, because of the deal, the realized recovery of bank creditors is

$$\mathcal{P}^- +(P-\mathrm{CA})^+ \tag{80}$$

(cf. the last line in (79)), instead of the zero recovery rate of the bank that was anticipated at the time when the bank issued the debt. But, in line with Assumption 5.1, this realized recovery was not anticipated and not reflected in the price of borrowing for the bank, which was assumed to be its risk-neutral default probability (i.e. credit spread) γ.

Remark B.1 Dropping Assumption 5.2 while conservatively assuming $\mathrm{UC}=0$ would lead to the same expressions for the various quantities in (78), except for $(P-\mathrm{CA})$ replaced by $(P-\mathrm{CA}-\mathrm{EC})$ in the funding cash flows. ∎

Remark B.2 The derivation (79) implicitly allows for negative equity (CET1 as per Definition A.2), which is interpreted as recapitalization. In a variant of the model excluding recapitalization, where the default of the bank would be modeled in a structural fashion as $L>\mathrm{EC}$ and negative equity is excluded, we would get instead of (79), recalling Assumption 6.2 set $L^{cl}=0$:

$$\mathcal{W}=\mathcal{W}^{ca}=(\mathrm{EC}-L)^+ +\mathbb{1}_{\{\mathrm{EC}<L\}}(\mathcal{P}^- +(P-\mathrm{CA})^+)-\mathrm{EC}, \tag{81}$$

where

$$L = L^{ca} = (\mathcal{C}^\circ + \mathcal{F}^\circ - \mathrm{CA}) \wedge \mathrm{EC}$$

(compare with the last line in (78)). In this paper we consider a model with recapitalization for the reasons explained in Section 2.3. ∎

Proposition B.1 *We have:*

$$\mathrm{CVA} = \mathbb{E}[(1 - J_1)\mathcal{P}^+],\ \mathrm{DVA} = \mathbb{E}[(1 - J)\mathcal{P}^-] \tag{82}$$

$$\mathrm{FVA} = \mathrm{FDA} = \frac{\gamma}{1+\gamma}(P - \mathrm{CVA})^+ \tag{83}$$

$$\mathrm{EC} = \mathrm{ES} = \mathbb{ES}(L) = \mathbb{ES}(\mathcal{C}^\circ) - \mathrm{CVA} \tag{84}$$

$$\mathrm{KVA} = h\mathrm{EC}, \tag{85}$$

where $\mathbb{ES}(\cdot)$ *denotes the 97.5% expected shortfall.*

Proof. In our one-period setup, there is no risk margin account and we have EC = SCR = ES, so that (17) reduces to (85), whereas (84) corresponds to Definition 5.2 through the last line in (78), constancy of $\mathcal{F}^\circ$, and cash-invariance of the expected shortfall risk measure.

In line with Definition 5.1, but for $\mathrm{CVA}^{\mathrm{CL}}$ and $\mathrm{FVA}^{\mathrm{CL}}$ redefined as zero in a one-period setup, we have

$$\mathrm{CVA} = \mathbb{E}\mathcal{C}^\circ,\ \mathrm{DVA} = \mathbb{E}\mathcal{C}^\bullet$$
$$\mathrm{FVA} = \mathrm{FDA} = \mathbb{E}\mathcal{F}^\circ = \mathbb{E}\mathcal{F}^\bullet,$$

where the involved cash flows are given by Lemma B.1. However, since $\mathcal{F}^\circ = \gamma(P - \mathrm{CA})^+$ where CA = CVA + FVA (cf. (22)), the formula $\mathrm{FVA} = \mathbb{E}\mathcal{F}^\circ$ in the above is in fact a semi-linear equation

$$\mathrm{FVA} = \gamma(P - \mathrm{CVA} - \mathrm{FVA})^+, \tag{86}$$

which, as γ (a probability) is nonnegative, has the unique solution given by the right-hand side in (83). ∎

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