XVA Metrics for CCP Optimisation

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Abstract

We study the cost of the clearance framework for a member of a CCP. We argue that two major inefficiencies related to CCPs, namely the costs for members of borrowing their initial margin and of their capital tied up in the default fund, could be significantly compressed by resorting to suitable initial margin funding scheme and default fund sizing, allocation and remuneration policies. In the context of XVA computations, which entail projections over decades, it might be interesting for a bank to compute the MVA and KVA corresponding to these alternative initial margin and default fund specifications even under the current regulatory environment, as a counterpart to the corresponding regulatory based XVA metrics.

Keywords: Central counterparty (CCP), initial margin, default fund, cost of funding initial margin (MVA), cost of capital (KVA).

JEL Classification: G13, G14, G28, G33, M41.

1 Introduction

In the aftermath of the financial crisis, the banking regulators undertook a number of initiatives to cope with counterparty credit risk. One major evolution is the generalization of central counterparties (CCPs), also known as clearing houses. A clearing house serves as an intermediary during the completion of the transactions between its

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clearing members. It organizes the collateralization of their transactions and takes care of the liquidation of the CCP portfolio of defaulted members. Non-members can have access to the services of a CCP through external accounts by the clearing members (see Figure 1).

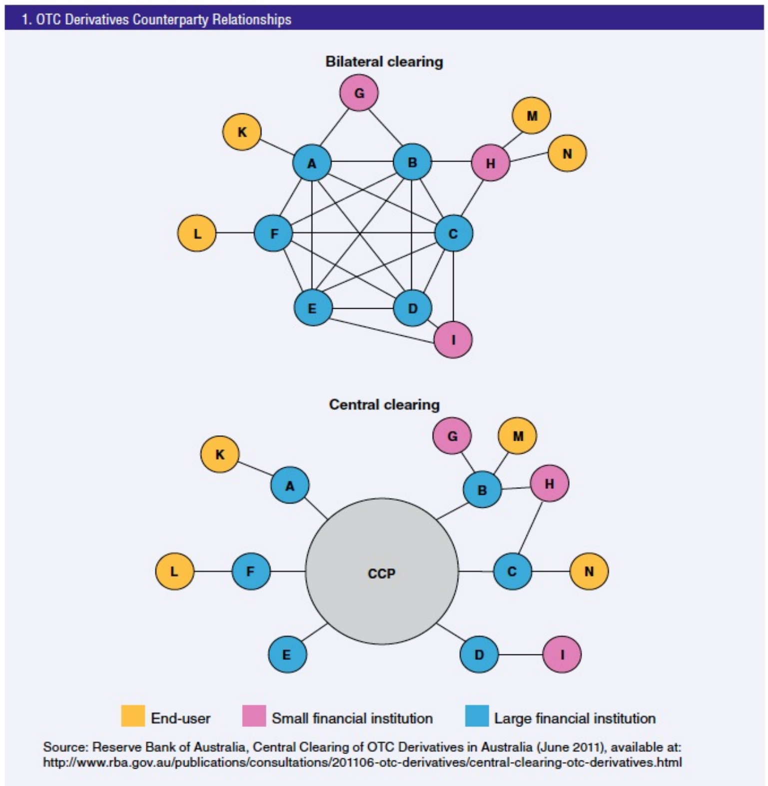

Figure 1: Bilateral vs. centrally cleared trading (*Source*: Reserve bank of Australia, 2011).

In order to mitigate counterparty risk, the CCP asks its clearing members to meet several collateralization requirements. Apart from the variation and initial margin (VM and IM) that are also required in bilateral trading (as gradually implemented since September 2016 regarding the IM), the clearing members contribute to a mutualized default fund (DF) set against extreme and systemic risk.

In the light of the literature, pros and cons of CCPs can be discussed as follows:

Counterparty credit risk and systemic risk: Reduced counterparty credit risk of the CCP itself and reduced default contagion effects between members, but concentration risk if a major CCP were to default, with 30 major CCPs today and only a few prominent ones. CCPs also pose joint membership and feedback liquidity issues. On these and related issues see Capponi, Cheng, and Rajan (2015), Glasserman, Moallemi, and Yuan (2014) and Barker, Dickinson, Lipton, and Virmani (2016).

Netting: Multilateral netting benefit versus loss of bilateral netting across asset classes. On these and related issues see Duffie and Zhu (2011), Cont and Kokholm (2014), Armenti and Crépey (2017) and Ghamami and Glasserman (2016).

Transparency: Portfolio wide information of the CCP and easier access to the data for the regulator versus opacity regarding the default fund for the clearing members and joint membership issues again. On related (and other) CCP issues see Gregory (2014).

Efficiency: Default resolution cheaper. Bilateral trading means a completely arbitrary network of transactions. An orderly default procedure cannot be done manually, it requires an IT network, whether it is CCP, blockchain, SIMM reconciliation appliance or whatever. However, the way CCPs are designed today entails two major inefficiencies for the clearing members, one related to the fact that default fund contributions are capital at risk not remunerated at a hurdle rate and another one related to the cost of borrowing their IM. See Albanese (2015) and Ghamami (2015).

1.1 Contents of the Paper

The margins and the default fund mitigate counterparty risk, but they generate substantial costs. Armenti and Crépey (2017) study the cost of the clearance framework for a member of a CCP, under standard regulatory assumptions on its default fund contributions and assuming unsecurely funded initial margin. Following up on the last item in the list above, the present paper challenges these assumptions in two directions.

First, we confront the current default fund Cover 2 EMIR sizing rule with a broader risk based approach, advocated in Ghamami (2015) and Albanese (2015), relying on a suitable notion of economic capital (EC) of a CCP. Regarding the allocation of the default fund between the clearing members, we compare a classical IM based allocation with the one based on member incremental economic capital (or the corresponding KVA).

Second, we assess the efficiency of an initial margin funding scheme, suggested in Albanese (2015), whereby a third party provides the IM in exchange of some service fee, as opposed to the standard procedure where clearing members unsecurely borrow their IM.

A detailed outline is as follows. Section 2 reviews the XVA principles of Albanese and Crépey (2016). Section 3 applies these principles to assess the cost of the clearance framework for a clearing member of a CCP. The critical components are the cost of funding their initial margin (MVA) and the cost of the capital (KVA) they have to put at risk as their default fund contribution. Section 4 studies ways of compressing the related market inefficiencies. A case study illustrates this numerically in Sections 5 through 7. Section 8 concludes.

The following notation is used throughout the paper:

- $\mathbb{V}a\mathbb{R}^a$, $\mathbb{ES}^a$: Value at risk and expected shortfall of quantile level a.
- Φ, ϕ : Standard normal cdf and density functions.

Part I
Conceptual Framework

In this theoretical part of the paper we apply the XVA principles of Albanese and Crépey (2016) to the cost analysis of centrally cleared trading in general and to a sustainable design of the default fund and of the funding procedure for initial margin in particular.

2 XVA Principles

In this section we recall the XVA principles of Albanese and Crépey (2016), to which we refer the reader for more details. We consider a pricing stochastic basis $(\Omega, \mathbb{G}, \mathbb{Q})$, with model filtration $\mathbb{G} = (\mathcal{G}_t)_{t\in\mathbb{R}_+}$ and risk-neutral pricing measure $\mathbb{Q}$, such that all the processes of interest are $\mathbb{G}$ adapted and all the random times of interest are $\mathbb{G}$ stopping times. The corresponding expectation and conditional expectation are denoted by $\mathbb{E}$ and $\mathbb{E}_t$. We denote by r a $\mathbb{G}$ progressive OIS (overnight indexed swap) rate process, which is together the best market proxy for a risk-free rate and the reference rate for the remuneration of the collateral. We write $\beta = e^{-\int_0^{\cdot} r_s ds}$ for the corresponding risk-neutral discount factor.

By mark-to-market of a derivative portfolio, we mean the trade additive risk-neutral conditional expectation of its future discounted promised cash flows, ignoring the cost of counterparty risk and of the related market imperfections, i.e. without any XVAs.

We consider a bank trading with several risky counterparties. The bank is also default prone, with default time τ and survival indicator $J = \mathbb{1}_{[0,\tau)}$. We define $\bar{\tau} = \tau \wedge T$, where T is the final maturity of all claims.

2.1 Counterparty Credit Exposure

To mitigate counterparty risk, the bank and its counterparties post variation and initial margin as well as, in a centrally cleared setup, default fund contributions. The variation margin (VM) of each party tracks the mark-to-market of their portfolio on a daily (sometimes more frequent) basis, as long as they are alive. However, there is a liquidation period of positive length δ, usually a few days, between the default of a party and the liquidation of its portfolio. The gap risk of slippage of the mark-to-market of the portfolio and of unpaid contractual cash flows during the liquidation period is the motivation for initial margin (IM). Nowadays initial margin is also updated dynamically, at a frequency analogous to the one used for variation margin. Accounting for the VM, the IM of each party is set as a risk measure, such as value-at-risk of some quantile level a_{im}, of their loss-and-profit at the time horizon δ of the liquidation period. In case a party defaults, its IM provides an initial safety buffer to absorb the losses on its portfolio that may arise from adverse scenarios, exacerbated by wrong-way risk, during the liquidation period. In a centrally cleared setup, the default fund (DF) is used when these losses exceed the sum between the VM and the IM of the defaulted member. The default fund contribution (DFC) of the defaulted member is used first. If it does not suffice, the default fund contributions of the other clearing members are used in turn. The Cover 2 EMIR rule requires to size the default fund as, at least, the maximum of the largest exposure and of the sum of the second and third largest exposures of the CCP to its clearing members, updated on a periodic (e.g. monthly) basis based on "extreme but plausible" scenarios. The corresponding amount is allocated between the members, typically proportionally to their initial margins.

Remark 2.1 In this paper we ignore the equity or "skin-in-the-game" of the CCP, which is, as is well known and illustrated numerically in Armenti and Crépey (2017), typically small and therefore negligible from a loss-absorbing point of view (even if it may be an important CCP management incentive issue).

2.2 Funding and Hedging

Variation margin typically consists of cash that is re-hypothecable, meaning that received VM can be used for funding purposes and is remunerated OIS by the receiving party. Initial margin, as well as default fund contributions in a CCP setup, typically consist of liquid assets deposited in a segregated account, such as government bonds, which pay coupons or otherwise accrue in value. We assume that the bank can invest at the OIS rate r_t and obtain unsecured funding for borrowing VM, on the time interval $(t, t+dt]$, at the rate $(r_t + \lambda_t)$, where the unsecured funding spread λ is proxied by the bank instantaneous CDS spread. Initial margin is funded separately from variation margin, at a blended spread $\bar{\lambda}$ that depends on the IM funding policy of the bank. This is the topic of Sect. 4.2.

In order to focus on XVAs, we assume in the sequel that the bank sets up a fully collateralized, back-to-back hedge of its derivative portfolio. As a consequence, the market risk of the derivative portfolio is fully mitigated and all remains to be done is dealing with XVAs. In theory, a bank may also want to setup an XVA hedge. But, as (especially own) jump-to-default exposures are hard to hedge in pratice, such a hedge can only be very imperfect. For simplicity in this paper, we just suppose no XVA hedge.

2.3 Cost of Capital Pricing Approach in Incomplete Counterparty Credit Risk Markets

Again, jump-to-default exposures, own default ones in particular, are hardly hedgeable in practice. Consistent with this incompleteness of counterparty credit risk, we follow a cost of capital XVA pricing approach, in two steps. First, the so-called contra-assets are valued as the expected costs of the counterparty credit risk related expenses. Second, on top of these expected costs, a KVA risk premium (capital valuation adjustment) is computed as the cost of a sustainable remuneration of the shareholders capital at risk earmarked to absorb the exceptional (beyond expected) losses.

More precisely, the risk-neutral XVA of a bank (or contra-assets value process), which we denote by Θ, corresponds to its expected future counterparty default losses and funding expenditures, under some risk-neutral pricing measure $\mathbb{Q}$ calibrated to market quotes of fully collateralized derivative transactions. Incremental Θ amounts are charged by the bank to its clients at every new deal and put in a reserve capital account, which is then depleted by counterparty default losses and funding expenditures as they occur.

In addition, bank shareholders require a remuneration at some hurdle rate h, commonly estimated by financial economists in a range varying from 6% to 13%, for their capital at risk. Accordingly, an incremental risk margin (or KVA) is sourced from clients at every new trade in view of being gradually distributed to bank shareholders as remuneration for their capital at risk at rate h as time goes on. Cost of capital calculations involve projections over decades in the future. The historical probability measure $\mathbb{P}$ is hardly estimable on such time frames. As a consequence, we do all our price and risk computations under a risk-neutral measure $\mathbb{Q}$ calibrated to the market. In other words, we work under the modeling assumption that $\mathbb{P} = \mathbb{Q}$, leaving the residual uncertainty about $\mathbb{P}$ to model risk.

2.4 Contra-Assets Valuation

We work under the modeling assumption that every bank account is continuously reset to its theoretical target value, any discrepancy between the two being instantaneously realized as loss or earning.

In particular, the reserve capital account of the bank is continuously reset to its theoretical target Θ level so that, much like with futures, the trading position of the bank is reset to zero at all times, but it generates a trading loss-and-profit process L. As explained in Albanese and Crépey (2016, Remark 5.1), the equation for the contra-assets value process Θ can be derived from a risk-neutral martingale condition on the trading loss process L, along with a terminal condition $\Theta_{\bar{\tau}} = 0$.

2.5 Capital Valuation Adjustment

Likewise, the risk margin account is assumed to be continuously reset to its theoretical target KVA level. As a consequence, $(-d\mathrm{KVA}_t)$ amounts continuously flow from the risk margin account to the bank shareholder dividend stream, into which the bank also sends the $r_t\mathrm{KVA}_t dt$ OIS accrual payments generated as interest by the KVA account.

In line with the current trend of the financial regulation, the economic capital of the bank at time t, $\mathrm{EC} = \mathrm{EC}_t(L)$, is modeled as the conditional expected shortfall at some quantile level a of the one-year-ahead loss of the bank, i.e., also accounting for discounting:

$$\mathrm{EC}_t(L) = \beta_t^{-1}\mathbb{ES}_t^a(\int_t^{t+1}\beta_s dL_s). \tag{1}$$

The amount needed by the bank to remunerate its shareholders for their capital at risk in the future is

$$\mathrm{KVA}_t = h\mathbb{E}_t \int_t^{\bar{\tau}} e^{-\int_t^s (r_u+h)du}\mathrm{EC}_s(L)ds,\ t \in [0,\bar{\tau}]. \tag{2}$$

This formula yields the size of a risk margin (or KVA) account such that, if the bank gradually releases from this account to its shareholders an average amount

$$h(\mathrm{EC}_t - \mathrm{KVA}_t)dt \tag{3}$$

at any time $t \in [0,\bar{\tau}]$, there is nothing left on the account at time $\bar{\tau}$ (see Albanese and Crépey (2016, Section 6.1)). The "$-\mathrm{KVA}_t$" in (3) or the "$+h$" in the discount factor in (2) reflect the fact that the KVA is itself loss-absorbing and as such it is part of economic capital. Hence, shareholder capital at risk, which needs be remunerated at the hurdle rate h, only corresponds to the difference (EC $-$ KVA). For simplicity we are skipping here the constraint that the economic capital must be greater than the ensuing KVA in order to ensure a nonnegative shareholder equity SCR=EC$-$KVA (cf. Albanese and Crépey (2016, Section 6.2)).

2.6 Funds Transfer Price

The total (or risk-adjusted) XVA is the sum between the risk-neutral XVA Θ and the KVA risk premium. In the context of XVA computations derivative portfolios are typically modeled on a run-off basis, i.e. assuming that no new trades will enter the portfolio in the future. This is intended to ensure that the ensuing XVA amounts are sufficient to allow the bank safely stopping its business, if this should happen. Otherwise the bank could be led into snowball Ponzi schemes, whereby always more deals are entered for the sole purpose of funding previously entered ones. Moreover the trade-flow of a bank, which is a price-maker, does not have a stationarity property that could allow the bank forecasting future trades.

Of course in reality a bank deals with incremental portfolios, where trades are added or removed as time goes on. Accordingly, incremental XVAs must be computed at every new (or newly considered) trade, as the differences between the portfolio XVAs with and without the new trade, the portfolio being assumed held on a run-off basis in both cases.

The incremental risk-adjusted XVA of a new deal (or set of deals), called funds transfer price (FTP), corresponds for the bank to the "fabrication cost" of the deal, computed on an incremental run-off basis given the endowment (pre-deal portfolio) of the bank.

3 Clearing Member XVA Analysis

In Albanese, Caenazzo, and Crépey (2016), the guiding principles of Sect. 2 are applied to the XVA analysis of a bank engaged into bilateral transactions with n counterparties. In this paper we consider the situation of a bank trading as a member of a clearing house with n other clearing members. Our view is a clearing house effectively eliminating counterparty risk (the default of the clearing house is outside the scope of XVA analysis), but at a certain cost for a reference clearing member bank, cost that we analyze in this section.

We consider a CCP with $(n+1)$ risky members, labeled by $i = 0, 1, 2, \ldots, n$. We denote by

- τ_i: The default time of the member i, with survival indicator process $J^i = \mathbb{1}_{[0,\tau_i)}$.
- D^i_t: The cumulative contractual cash flow process of the CCP portfolio of the member i. The cash flows are counted positively when they flow from the clearing member to the CCP.
- $\mathrm{MtM}^i_t = \mathbb{E}_t[\int_t^T \beta_t^{-1}\beta_s dD^i_s]$: The mark-to-market of the CCP portfolio of the member i, with final maturity time T of the overall CCP portfolio.
- $\Delta^i_t = \int_{[t-\delta,t]} \beta_t^{-1}\beta_s dD^i_s$: The cumulative contractual cash flows of the member i, accrued at the OIS rate, over a past period of length δ.
- $\mathrm{VM}^i_t, \mathrm{IM}^i_t \geq 0, \mathrm{DFC}^i_t \geq 0$: VM, IM and DFC posted by the member i.

We do not exclude simultaneous defaults, but we suppose that all the default times are positive and endowed with an intensity (in particular, defaults at any constant or $\mathbb{G}$ predictable time have zero probability). Regarding the liquidation procedures, for ease of analysis, we assume the existence of a risk-free (hence, non IM or DFC posting) "buffer" replacing defaulted members in their transactions with the surviving members, after a liquidation period of length δ. In the interim the positions of the defaulted members are carried by the clearing house. For every time $t \geq 0$, we write

$$\bar{t} = t \wedge T,\ t^\delta = t + \delta,\ \bar{t}^\delta = \mathbb{1}_{t<T} t^\delta + \mathbb{1}_{t \geq T} T. \tag{4}$$

Accounting for the OIS accrued value $\Delta^i_{\tau_i^\delta} = \int_{[\tau_i,\tau_i^\delta]} \beta^{-1}_{\tau_i^\delta}\beta_s dD^i_s$ of the cash flows contractually due by the member i to the other clearing members from time τ_i onward (cash flows unpaid due to the default of the member i at τ_i), the loss triggered by the liquidation of the member i at time τ_i^δ is then

$$\Big(\mathrm{MtM}^i_{\tau_i^\delta} + \Delta^i_{\tau_i^\delta} - \beta^{-1}_{\tau_i^\delta}\beta_{\tau_i}(\mathrm{VM}^i_{\tau_i} + \mathrm{IM}^i_{\tau_i} + \mathrm{DFC}^i_{\tau_i})\Big)^+ \tag{5}$$

(considering the cash flows from the perspective of the CCP, assuming that margin and DFC accounts accrue at rate r and noting that there is no recovery to expect from the liquidation of the CCP portfolio of a defaulted member).

In the sequel the bank plays the role of the reference member 0. For notational simplicity we remove any index 0 referring to it. Since the CCP is simply an interface between the clearing members, the overall CCP portfolio clears, i.e.

$$\mathrm{MtM} = \mathrm{MtM}^0 = -\sum_{i\neq 0} \mathrm{MtM}^i \tag{6}$$

and we assume likewise

$$\mathrm{VM} = \mathrm{VM}^0 = -\sum_{i\neq 0} \mathrm{VM}^i.$$

Recall we do not exclude simultaneous defaults. For any $Z \subseteq \{1, 2, \dots, n\}$ let τ_Z denote the time when members in Z and only in Z default (or $+\infty$ if this never happens). At each $t = \tau_Z^\delta < \bar{\tau}$, the loss of the bank, assumed instantaneously realized as refill to its default fund contribution, is (also accounting for the unwinding of the back-to-back market hedges of the defaulted members)

$$\epsilon_{\tau_Z^\delta} = w_{\tau_Z^\delta} \sum_{i\in Z} \big(\mathrm{MtM}^i_{\tau_Z^\delta} + \Delta^i_{\tau_Z^\delta} - \beta^{-1}_{\tau_Z^\delta}\beta_{\tau_Z}(\mathrm{VM}^i_{\tau_Z} + \mathrm{IM}^i_{\tau_Z} + \mathrm{DFC}^i_{\tau_Z})\big)^+, \tag{7}$$

for some refill allocation key w_t. For instance, a typical specification proportional to the default fund contributions of the surviving members corresponds to

$$w_t = \frac{\mathrm{DFC}_t}{\mathrm{DFC}_t + \sum_{i\neq 0} J^i_t \mathrm{DFC}^i_t}. \tag{8}$$

Note that the sum in (8) conservatively ignores the impact of netting in the context of the joint liquidation of several defaulted members (and recall that in this paper we ignore the equity or "skin-in-the-game" of the CCP, see the remark 2.1).

In what follows we apply and revisit the approach of Sect. 2 in the context of a reference clearing member bank.

3.1 Contra-Assets Valuation

We denote by $\boldsymbol{\delta}_t$ a Dirac measure at time t.

Lemma 3.1 *In the case of a centrally cleared portfolio of trades between a reference clearing member bank and n other clearing members $i = 1, \dots, n$, given a putative risk-neutral XVA process Θ, the trading loss (and profit) process L of the bank satisfies the following forward SDE:*

$$\begin{aligned} &L_0 = z \text{ (the initial trading loss of the bank) and, for } t \in (0, \bar{\tau}^\delta],\\ &dL_t = d\Theta_t - r_t\Theta_t dt + J_t \sum_Z \epsilon_{\tau_Z^\delta} \boldsymbol{\delta}_{\tau_Z^\delta}(dt) \\ &\qquad + J_t\Big(\lambda_t\big(\mathrm{VM}_t - \mathrm{MtM}_t - \Theta_t\big)^+ + \bar{\lambda}_t \mathrm{IM}_t\Big)dt \end{aligned} \tag{9}$$

(and L is constant from time $\bar{\tau}^\delta$ onward).

***Proof*.** Collecting all the cash flows in the above description, we obtain

$$
\begin{aligned}
&L_0 = z \text{ and, for } t \in (0, \bar{\tau}^{\delta}],\\
&dL_t = \underbrace{J_t \sum_Z \epsilon_{\tau_Z^{\delta}} \boldsymbol{\delta}_{\tau_Z^{\delta}}(dt)}_{\text{Counterparty default losses of the bank}}\\
&+ \underbrace{J_t\Big((r_t + \lambda_t)\big(\mathrm{VM}_t - \mathrm{MtM}_t - \Theta_t\big)^+ - r_t\big(\mathrm{VM}_t - \mathrm{MtM}_t - \Theta_t\big)^-\Big)dt}_{\substack{\text{Bank costs/benefits of funding the VM posted on its derivative portfolio,}\\ \text{net of MtM received as VM on its back-to-back hedge and of the amount } \Theta_t\\ \text{available as a funding source in its reserve capital account}}}\\
&+ \underbrace{J_t(r_t + \bar{\lambda}_t)\mathrm{IM}_t dt}_{\text{Bank IM funding costs}}\\
&- \underbrace{J_t r_t\big(\mathrm{VM}_t - \mathrm{MtM}_t + \mathrm{IM}_t\big)dt}_{\substack{\text{Posted VM is remunerated OIS by the receiving party}\\ \text{and IM accrues at the OIS rate}}}\\
&- \underbrace{(1 - J_t) r_t \Theta_t dt}_{\substack{\text{Risk-free funding of the bank position taken over by the CCP}\\ \text{during the bank liquidation period}}}\\
&- \underbrace{(-d\Theta_t)}_{\text{Depreciation of the liability } \Theta \text{ of the bank}},
\end{aligned}
$$

which gives (9). ∎

Proposition 3.1 In the case of a centrally cleared portfolio of trades between a reference clearing member bank and n other clearing members $i = 1, \ldots, n$:
(i) The risk-neutral XVA value process Θ of the bank satisfies the following backward SDE (BSDE) on $[0, \bar{\tau}^{\delta}]$:

$$
\begin{aligned}
&\Theta_{\bar{\tau}^{\delta}} = 0 \text{ and, for } t \in (0, \bar{\tau}^{\delta}],\\
&d\Theta_t = -J_t \sum_Z w_{\tau_Z^{\delta}} \epsilon_{\tau_Z^{\delta}} \boldsymbol{\delta}_{\tau_Z^{\delta}}(dt)\\
&\qquad - J_t\Big(\lambda_t\big(\mathrm{VM}_t - \mathrm{MtM}_t - \Theta_t\big)^+ + \bar{\lambda}_t \mathrm{IM}_t\Big)dt + r_t\Theta_t dt + dL_t,
\end{aligned}
\tag{10}
$$

for some risk-neutral local martingale L corresponding to the trading loss process of the bank.

(ii) Assuming integrability, it holds that

$$\Theta_t = \underbrace{\mathbb{E}_t \sum_{t<\tau_Z^\delta<\bar{\tau}} \beta_t^{-1}\beta_{\tau_Z^\delta} w_{\tau_Z^\delta} \sum_{i\in Z} \Big(\mathrm{MtM}^i_{\tau_Z^\delta} + \Delta^i_{\tau_Z^\delta} - \beta^{-1}_{\tau_Z^\delta}\beta_{\tau_Z}(\mathrm{VM}^i_{\tau_Z} + \mathrm{IM}^i_{\tau_Z} + \mathrm{DFC}^i_{\tau_Z})\Big)^+}_{\mathrm{CVA}_t}$$
$$+ \underbrace{\mathbb{E}_t \int_t^{\bar{\tau}} \beta_t^{-1}\beta_s\lambda_s\Big(\mathrm{VM}_s - \mathrm{MtM}_s - \Theta_s\Big)^+ ds}_{\mathrm{FVA}_t} + \underbrace{\mathbb{E}_t \int_t^{\bar{\tau}} \beta_t^{-1}\beta_s\bar{\lambda}_s \mathrm{IM}_s ds}_{\mathrm{MVA}_t},\ 0 \le t \le \tau^\delta. \quad (11)$$

Proof. As recalled in Sect. 2, the trading loss process L must be a risk-neutral local martingale. Therefore, also accounting for a terminal condition $\Theta_{\bar{\tau}^\delta} = 0$, the SDE (9) in Lemma 3.1 implies (i) and (assuming integrability) (ii). ∎

3.2 Capital Valuation Adjustment

As default fund contributions are loss-absorbing and survivor-pay (beyond the level of losses covered by the margins and the DFC of the defaulted members), they are capital at risk of the clearing members. In fact, the capital at risk of a bank operating as clearing member of a CCP takes the form of its default fund contribution. In principle, capital is also required from the bank for dealing with the potential losses of the bank beyond its margin and default fund contribution. But, given the high levels of IM (such as 99% $\mathbb{V}\mathrm{a}\mathbb{R}$ or more) that are used in practice and the DFC, such regulatory capital is typically negligible: see Armenti and Crépey (2017) for numerical illustration.

As a result, in a centrally cleared trading setup, the KVA formula (2) needs to be amended as

$$\mathrm{KVA}_t = h\mathbb{E}_t \int_t^{\bar{\tau}} e^{-\int_t^s (r_u+h)du} \mathrm{DFC}_s ds,\ t \in [0, \bar{\tau}]. \quad (12)$$

Recalling the discussion following (2), this perspective opens the door to an organization of the clearance framework, whereby a CCP could remunerate the clearing members for their default fund contributions. This would make the clearing members much less reluctant to put capital at risk in the default fund. In fact, if it was remunerated at a hurdle rate h, the default fund of a CCP could even become attractive to external investors.

3.3 Funds Transfer Price

In case of a new deal (or package of deals) through the CCP at time 0, the FTP of the reference clearing member bank appears as

$$\mathrm{FTP} = \Delta\Theta + \Delta\mathrm{KVA} = \Delta\mathrm{CVA} + \Delta\mathrm{FVA} + \Delta\mathrm{MVA} + \Delta\mathrm{KVA}, \quad (13)$$

computed on an incremental run-off basis relatively to the portfolios with and without the new deal as explained in Sect. 2.6.

Given the high level of collateralization that applies in the context of centrally cleared trading, the credit valuation adjustment (CVA) of a clearing member, i.e. its expected loss on the default fund due to other members defaults, is typically quite small. Moreover, for daily (or even more frequent) remargining on the derivative portfolio, the variation margins of a clearing member on its derivative portfolio and on its back-to-back hedge tend to match each other. Hence the funding variation adjustment (FVA), or expected cost for a member of funding its VM, is also quite small and much smaller than its MVA. As a consequence, in a centrally cleared setup with daily remargining, the prominent XVA numbers of a clearing member are its MVA and its KVA (the latter being magnified further by model risk, as it is related to tail events).

Accordingly, in case a new trade (or set of offsetting trades, given the clearing constraint (6), typically a pair of opposite trades) is considered in the CCP, each of the clearing members should compute their FTP (or at least its MVA and KVA components) as of the corresponding formula (13). That is, one should alternately consider each clearing member as the reference clearing member bank in the above, performing incremental XVA computations from the perspective of each of them. The parties at the origin of the trade (whether this is an external client trade in the first place or a hedge trade between the clearing members) should then be required an overall entry add-on for the deal equal to the sum of the individual FTPs of the other clearing members, so that the reserve capital (resp. risk margin) account of each of them can be incremented by their respective incremental risk-neutral XVA (resp. KVA). The corresponding costs should be shared among the parties at the origin of the trades according to some allocation key.

However, even though it would be desirable that clearing member banks are in a position to pass their costs to their clients (or at least are aware of the actual level of these costs), the evaluation of these costs is difficult for the clearing members themselves. This is because the default fund depends on the overall CCP portfolio, whereas clearing members only know their own positions. Members do not know either which IM and DF models will be used by the CCP in future. Hence the above approach would require a collaborative behavior, where the CCP itself would compute the FTPs of a new (or tentative new) deal for each of the clearing members (the counterparties in the deal, mainly, but also the members outside the deal, due to the mutualization of the system) and communicate their FTP to each of them. If the deal takes place (which may also depend on this cost analysis), then these costs could be passed to the members at the origin of the deal and in turn, if there are external clients at the starting point, to the latter. Or not, depending on the commercial relationship, but at least these costs would be known transparently.

On the one hand, such an XVA pricing approach in a CCP setup is particularly heavy, because the coupling of the system through the default fund implies that, in principle, the XVA metrics relative to each of the clearing members must be recomputed at every tentative new trade of any of the clearing members. On the other hand, since CCP portfolios are mostly composed of standardized trades, much of the calculations can be precomputed.

4 From Transient to Equilibrium XVA Analysis

Whatever the prevailing regulation and market practice in terms of capital and funding policies, for XVA computations that entail projections of these over decades, an economical equilibrium view is more appropriate than the ad-hoc and ever-changing regulatory specifications supposed to approximate it. Two important considerations in this regard are the specification of the default fund and of the funding policy for initial margins.

4.1 Economic Capital Based Default Fund

As explained in Sect. 3.2, through their default fund contributions, the clearing members are also the shareholders of the CCP (ignoring the skin-in-the-game of the CCP, which is negligible from a loss-absorbing point of view). However, as of today, default fund contributions, even though effectively corresponding to clearing member capital put at risk in the CCP, are not remunerated at a hurdle rate. As a matter of fact, they are in fact subject to margin fees that we ignore for simplicity in this paper (see Armenti and Crépey (2017)).

The economical capital and KVA methodology of Albanese and Crépey (2016) can be used for designing an economically sound and sustainable specification of the default fund and of its allocation between the clearing members, also opening the door to a possible remuneration of default fund contributions as capital at risk. Beyond the theoretical interest and message to the regulator for the future, this approach yields a valuable specification even today for the default fund and its allocation that intervene as data in the risk-neutral XVA BSDE (10) and the KVA formula (12).

Recall that we assume the CCP default free in our setup. Accordingly, we assume that the CCP can obtain unsecured funding at the OIS rate. Hence, if there was no default fund, the risk-neutral XVA of the CCP would reduce to a CVA given as

$$\mathrm{CVA}_t^{ccp} = \mathbb{E}_t \sum_{t<\tau_i^\delta<T^\delta} \beta_t^{-1}\big(\beta_{\tau_i^\delta}(\mathrm{MtM}^i_{\tau_i^\delta} + \Delta^i_{\tau_i^\delta}) - \beta_{\tau_i}(\mathrm{VM}^i_{\tau_i} + \mathrm{IM}^i_{\tau_i})\big)^+,\ 0 \le t \le T^\delta. \quad (14)$$

The corresponding loss process L^{ccp} of the CCP is given as

$$\begin{aligned} &L_0^{ccp} = z^{ccp} \text{ (the initial loss of the CCP) and, for } t \in (0, T^\delta],\\ &\beta_t dL_t^{ccp} = \beta_t(d\mathrm{CVA}_t^{ccp} - r_t \mathrm{CVA}_t^{ccp})\\ &\qquad + \sum_i \big(\beta_{\tau_i^\delta}(\mathrm{MtM}^i_{\tau_i^\delta} + \Delta^i_{\tau_i^\delta}) - \beta_{\tau_i}(\mathrm{VM}^i_{\tau_i} + \mathrm{IM}^i_{\tau_i})\big)^+ \boldsymbol{\delta}_{\tau_i^\delta}(dt)dt \end{aligned} \quad (15)$$

(and L^{ccp} constant from time T^δ onward). The ensuing economic capital process

$$\mathrm{EC}_t(L^{ccp}) = \mathbb{ES}_t^{a_{df}}\Big(\int_t^{t+1} \beta_t^{-1}\beta_s dL_s^{ccp}\Big) \quad (16)$$

(cf. (1)), where

$$\int_t^{t+1} \beta_s dL_s^{ccp} = \sum_{t<\tau_i^\delta \le t+1} \left(\beta_{\tau_i^\delta}(\mathrm{MtM}^i_{\tau_i^\delta} + \Delta^i_{\tau_i^\delta}) - \beta_{\tau_i}(\mathrm{VM}^i_{\tau_i} + \mathrm{IM}^i_{\tau_i})\right)^+ \\ - (\beta_t \mathrm{CVA}_t^{ccp} - \beta_{t+1}\mathrm{CVA}_{t+1}^{ccp}), \tag{17}$$

yields the size of an overall risk based default fund at the quantile level a_{df}. The random variable under $\mathbb{ES}_t^{a_{df}}$ in (16) represents the one year ahead loss and profit of the CCP if there was no default fund. The Cover 2 EMIR rule is purely based on market risk considerations. By contrast, the sizing rule (16) reflects a broader notion of risk of the CCP, in the form of a risk measure of its one-year ahead loss-and-profit if there was no default fund, as it results from the combination of the credit risk of the clearing members and of the market risk of their portfolios.

Moreover a member incremental EC or KVA allocation of $\mathrm{EC}(L^{ccp})$ between the $(n+1)$ clearing members could be used as an alternative to the usual IM based allocation of the default fund (see Sect. 6.2 for detailed specifications).

4.2 Specialist Lending of Initial Margin

Let $\lambda = \gamma(1-R)$ denote the instantaneous CDS spread of the bank, where γ is its risk-neutral default intensity and R its recovery rate as implicit in CDS spread quotations (typically $R = 40\%$).

The time-0 MVA of the bank when its IM is funded through unsecured borrowing is given by

$$\mathrm{MVA}_0^{ub} \quad = \mathbb{E}[\textstyle\int_0^{\bar\tau} \beta_s \lambda_s \mathrm{IM}_s ds]. \tag{18}$$

However, instead of assuming its IM borrowed by the bank on an unsecured basis, we can consider a more efficient scheme whereby IM is provided by a liquidity supplier, dubbed “specialist lender”, which lends IM in exchange of some fee and, in case of default of the bank, receives back from the CCP the portion of IM unused to cover losses.

Assuming as standard that IM is subordinated to own DFC, i.e. that the first levels of losses are absorbed by IM, the exposure of the specialist lender to the default of the bank is

$$(1-R)(G^+_{\tau^\delta} \wedge \beta^{-1}_{\tau^\delta}\beta_\tau \mathrm{IM}_\tau),$$

for a time-t gap

$$G_t = \mathrm{MtM}_t + \Delta_t - \beta_t^{-1}\beta_{t-\delta}\mathrm{VM}_{t-\delta}. \tag{19}$$

The time-0 MVA of the bank under such a third party arrangement follows as

$$\mathrm{MVA}_0^{sl} \quad = \mathbb{E}\Big[\beta_{\tau^\delta}\mathbb{1}_{\tau<T}(1-R)\big(G^+_{\tau^\delta} \wedge \beta^{-1}_{\tau^\delta}\beta_\tau \mathrm{IM}_\tau\big)\Big] = \mathbb{E}[\textstyle\int_0^{\bar\tau} \beta_s \lambda_s \xi_s ds], \tag{20}$$

where ξ is a $\mathbb{G}$ predictable process, which exists by Corollary 3.23 2) in He, Wang, and Yan (1992), such that $\mathbb{E}_{\tau-}\big[(\beta_{\tau^\delta} G^+_{\tau^\delta} \wedge \beta_\tau \mathrm{IM}_\tau)\big] = \beta_\tau \xi_\tau$.

By identification with the generic expression $\bar{\lambda}_s \mathrm{IM}_s$ in (9), the formula (20) corresponds to a blended IM funding spread $\bar{\lambda} = \frac{\xi}{\mathrm{IM}}\lambda$. Under a common specification where $\beta_s \mathrm{IM}_s$ is set as a high quantile (value-at-risk) of $\beta_{s^\delta} G_{s^\delta}$ (cf. (24) below, assuming there for simplicity continuous-time variation margining VM = MtM in (19)), the blending factor $\frac{\xi}{\mathrm{IM}}$ is typically much smaller than one. Hence $\bar{\lambda}$ is much smaller than λ and MVA_0^{sl} is much smaller than MVA_0^{ub}. Subordinating own DFC to IM would result in even more efficient specialist lender IM funding schemes.

Part II
Case Study

In the ensuing case study we test the methodology of part one in the CCP toy model of Armenti and Crépey (2017, Section 7). In particular, CVA^{ccp} is analytic in this model, which avoids the numerical burden of nested Monte Carlo that is required otherwise for simulating the loss and profit processes involved in capital computations.

5 CCP Toy Model

In this section we briefly recap the CCP setup of Armenti and Crépey (2017, Section 7), to which we refer the reader for more details.

5.1 Market Model

As common asset driving all our clearing member portfolios, we consider a stylized swap with strike rate $\bar{S}$ and maturity T on an underlying interest rate process S. At discrete time points T_l such that $0 < T_1 < T_2 < ... < T_d = T$, the swap pays an amount $h_l(\bar{S} - S_{T_{l-1}})$, where $h_l = T_l - T_{l-1}$. The underlying rate process S is supposed to follow a standard Black-Scholes dynamics with risk-neutral drift κ and volatility σ, so that the process $\widehat{S}_t = e^{-\kappa t} S_t$ is a Black martingale with volatility σ. For $t \in [T_0 = 0, T_d = T]$, we denote by l the index such that $T_{l_t - 1} \le t < T_{l_t}$. The mark-to-market of a short position in the swap is given by

$$\begin{aligned}\mathrm{MtM}_t &= \mathbb{E}_t\big[\beta_t^{-1}\beta_{T_{l_t}} h_{l_t}(\bar{S} - S_{T_{l_t-1}}) + \sum_{l=l_t+1}^{d} \beta_t^{-1}\beta_{T_l} h_l(\bar{S} - S_{T_{l-1}})\big] \\ &= \beta_t^{-1}\beta_{T_{l_t}} h_{l_t}(\bar{S} - S_{T_{l_t-1}}) + \beta_t^{-1} \sum_{l=l_t+1}^{d} \beta_{T_l} h_l\big(\bar{S} - e^{\kappa T_{l-1}} \widehat{S}_t\big),\end{aligned} \quad (21)$$

by the martingale property of the process $\widehat{S}$.

The following numerical parameters are used:

$$r = 2\%,\ S_0 = 100,\ \kappa = 12\%,\ \sigma = 20\%,\ h_l = 3 \text{ months},\ T = 5 \text{ years},$$

and the nominal (Nom) of the swap is set so that each leg has a mark-to-market of 1 at time 0. Figure 2 shows the resulting mark-to-market process viewed from the

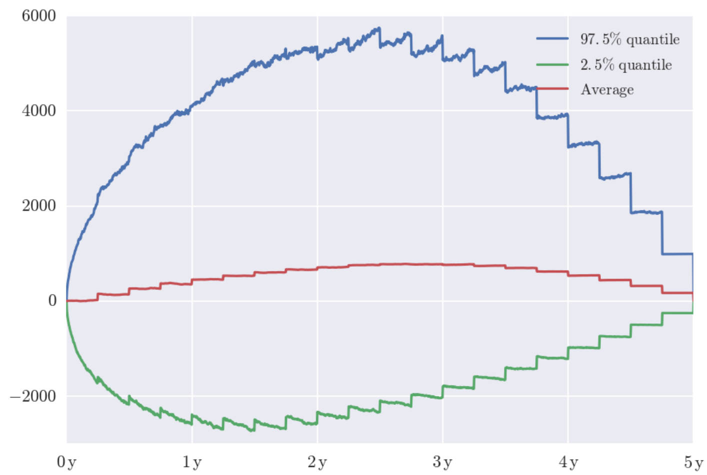

Figure 2: Mean and 2.5% and 97.5% quantiles, in basis points as a function of time, of the process (−MtM), calculated by Monte Carlo simulation of 5000 Euler paths of the process S.

perspective of a party with a long unit position in the swap, i.e. the process (−MtM).

5.2 Credit Model

For the default times τ_i of the clearing members, we use the "common shock" or dynamic Marshall-Olkin copula (DMO) model of Crépey, Bielecki, and Brigo (2014, Chapt. 8–10) and Crépey and Song (2016). The idea of the static Marshall-Olkin copula is that defaults can happen simultaneously with positive probabilities. The model is then made dynamic, as required for XVA computations, by the introduction of the filtration of the indicator processes of the τ_i.

First we define shocks as pre-specified subsets of the clearing members, i.e. the singletons $\{0\}, \{1\}, \{2\}, ..., \{n\}$, for single defaults, and a small number of groups representing members susceptible to default simultaneously.

Example 5.1 A shock $\{1, 2, 4, 5\}$ represents the event that whoever among the members 1, 2, 4 and 5 is still alive defaults at that time. ∎

As shown numerically in Crépey, Bielecki, and Brigo (2014, Section 8.4), a few common shocks are typically enough to ensure a good calibration of the model to market data regarding the credit risk of the clearing members and their default dependence (or to expert views about these).

Given a family $\mathcal{Y}$ of shocks, the times τ_Y of the shocks $Y \in \mathcal{Y}$ are modeled as independent time-inhomogeneous exponential random variables with intensity functions

γ_Y. For each clearing member $i = 0, \ldots, n$, we then set

$$\tau_i = \min_{\{Y \in \mathcal{Y}; i \in Y\}} \tau_Y \tag{22}$$

(and we recall that the defaut time τ of the reference clearing member bank corresponds to τ_0). The specification (22) means that the default time of the member i is the first time of a shock Y that contains i. As a consequence, the intensity function γ_i of τ_i is given by

$$\gamma_i = \sum_{\{Y \in \mathcal{Y};\ i \in Y\}} \gamma_Y$$

and we also define the instantaneous CDS spread term structure $\lambda_i = (1 - R_i)\gamma_i$, where $R_i = 40\%$ is taken as recovery rate implicit in CDS spread market quotations.

Example 5.2 Consider a family of shocks

$$\mathcal{Y} = \{\{0\}, \{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{1,3\}, \{2,3\}, \{0,1,2,4,5\}\}$$

(with $n = 5$). The following illustrates a possible default path in the model.

$$\begin{array}{llcccccccl} t = 0.9: & \{3\} & 0 & 1 & 2 & \textcircled{3} & 4 & 5 & & \tau_3 = 0.9 \\ t = 1.4: & \{5\} & 0 & 1 & 2 & 3 & 4 & \textcircled{5} & & \tau_5 = 1.4 \\ t = 2.6: & \{1,3\} & 0 & \textcircled{1} & 2 & 3 & 4 & 5 & & \tau_1 = 2.6 \\ t = 5.5: & \{0,1,2,4,5\} & \textcircled{0} & 1 & \textcircled{2} & 3 & \textcircled{4} & 5 & & \tau_2 = \tau_4 = 5.5. \end{array}$$

At time $t = 0.9$, shock $\{3\}$ occurs. This is the first time that a shock involving member 3 appears, hence the default time of member 3 is 0.9. At $t = 1.4$, member 5 defaults as the consequence of the shock $\{5\}$. At time 2.6, the shock $\{1,3\}$ triggers the default of member 1 alone as member 3 has already defaulted. Finally, only members 0, 2 and 4 default simultaneously at $t = 5.5$ since members 1 and 5 have already defaulted before. ∎

In the sequel we consider a CCP with $n + 1 = 9$ members, chosen among the 125 names of the CDX index as of 17 December 2007, at the beginning of the subprime crisis. The default times of the 125 names of the index are jointly modeled by a DMO model with 5 common shocks, with piecewise-constant shock intensity functions γ_Y calibrated to the corresponding CDS and CDO market data of that day (see Crépey, Bielecki, and Brigo (2014, Sect. 8.4.3)). Table 1 shows the credit spread $\sum_i$ and the

ν_i	9.20	(1.80)	(4.60)	1.00	(6.80)	0.80	(13.80)	8.80	7.20
$\sum_i$	45	52	56	61	73	108	176	367	1053

Table 1: (*Top*) Swap position ν_i of each member, where parentheses mean negative numbers (i.e. short positions). (*Bottom*) Average 3 and 5 year CDS spread Σ_i of each member at time 0 (17 December 2017), in basis points.

swap position ν_i of each of our nine clearing members. Hence

$$\mathrm{MtM}^i = \nu_i \times \mathrm{MtM} \tag{23}$$

(recalling that the mark-to-market processes MtM^i are considered from the point of view of the CCP). We write $\mathrm{Nom}_i = \mathrm{Nom} \times |\nu_i|$.

5.3 Initial Margin

For simplicity we assume that the margins and default fund contribution of a clearing member are updated in continuous time[1] while the member is alive and are stopped at its default time, until the liquidation of its portfolio occurs after a period of length δ. Accordingly, we assume that $\beta_s \mathrm{IM}^i_s$ is given as

$$\beta_t \mathrm{IM}^i_t = \mathbb{V}\mathrm{a}\mathbb{R}^{a_{im}}_t \Big(\beta_{t^\delta}(\mathrm{MtM}^i_{t^\delta} + \Delta^i_{t^\delta}) - \beta_t \mathrm{MtM}^i_t\Big), \tag{24}$$

for some IM quantile level a_{im}. Hence (21) and (23) yield

$$\beta_{t^\delta}(\mathrm{MtM}^i_{t^\delta} + \Delta^i_{t^\delta}) - \beta_t \mathrm{MtM}^i_t = \mathrm{Nom} \times \nu_i \times f(t) \times (\widehat{S}_t - \widehat{S}_{t^\delta}), \tag{25}$$

where $f(t) = \sum_{l=l_{t^\delta}+1}^{d} \beta_{T_l} h_l e^{\kappa T_{l-1}}$.

Remark 5.1 At least (25) holds whenever there is no coupon date between t and t^δ. Otherwise, i.e. whenever $l_{t^\delta} = l_t + 1$, the term $\beta_{T_{l_t}} h_{l_t} (\bar{S} - S_{T_{l_t - 1}})$ in (21) induces a small and centered difference

$$\mathrm{Nom} \times \nu_i \times h_{l_{t^\delta}} \beta_{T_{l_{t^\delta}}} \left(e^{\kappa T_{l_t}} \widehat{S}_t - S_{T_{l_t}} \right) \tag{26}$$

between the left hand side and the right hand side in (25). As $\delta \approx$ a few days, a coupon between t and t^δ is the exception rather than the rule. Moreover the resulting error (26) is not only exceptional but small and centered. As all XVA numbers are time and space averages over future scenario, we can and do neglect this feature in the sequel.

Lemma 5.1 *We have* $\beta_t IM^i_t = \mathrm{Nom}_i \times B_i(t) \times \widehat{S}_t$ *where*

$$B_i(t) = f(t) \times \begin{cases} e^{\sigma\sqrt{\delta}\Phi^{-1}(a_{im}) - \frac{\sigma^2}{2}\delta} - 1, & \nu_i \le 0 \\ 1 - e^{\sigma\sqrt{\delta}\Phi^{-1}(1-a_{im}) - \frac{\sigma^2}{2}\delta}, & \nu_i > 0. \end{cases}$$

Proof. This follows from (24)-(25) and from the Black property of $\widehat{S}$. ∎

[1] Instead of daily and monthly under typical market practice.

6 Sizing and Allocation of the Default Fund

Under the current regulation, the default fund of a CCP is sized according to the EMIR Cover 2 rule. The typical allocation of the total amount between the clearing members is proportional to their initial margins. Hence both the size and the allocation of the default fund are purely based on market risk, irrespective of the credit risk of the clearing members. The latter is only accounted for marginally and in a second step, by means of specific add-ons to the IM of the riskiest members.

As explained in Sect. 4.1, one may be interested in broader risk based alternatives for the sizing and/or allocation of the default fund. In the setup of our case study, we have the following explicit formulas.

Lemma 6.1 *We have*

$$\mathbb{E}_s\left[\left(\beta_{s^\delta}(\mathrm{MtM}^i_{s^\delta}+\Delta^i_{s^\delta})-\beta_s(\mathrm{MtM}^i_s+\mathrm{IM}^i_s)\right)^+\right]=\mathrm{Nom}_i\times A_i(s)\times\widehat{S}_s,$$

where

$$A_i(t)=(1-a_{im})\times f(t)\times e^{-\frac{\sigma^2\delta}{2}}\begin{cases}e^{\sigma\sqrt{\delta}\frac{\phi(\Phi^{-1}(a_{im}))}{1-a_{im}}}-e^{\sigma\sqrt{\delta}\Phi^{-1}(a_{im})}, & \nu_i\le 0\\ e^{\sigma\sqrt{\delta}\Phi^{-1}(1-a_{im})}-e^{-\sigma\sqrt{\delta}\frac{\phi(\Phi^{-1}(a_{im}))}{1-a_{im}}}, & \nu_i>0.\end{cases}$$

Proof. In view of (24)-(25), the conditional version of the identity $\mathbb{E}[X\mathbb{1}_{X\ge\mathbb{V}\mathrm{a}\mathbb{R}^a(X)}]=(1-a)\mathbb{ES}^a(X)$ yields

$$\begin{aligned}&\mathbb{E}_s\left[\left(\beta_{s^\delta}(\mathrm{MtM}^i_{s^\delta}+\Delta^i_{s^\delta})-\beta_s(\mathrm{MtM}^i_s+\mathrm{IM}^i_s)\right)^+\right]\\&=\mathrm{Nom}\times(1-a_{im})\times f(t)\left[\mathbb{ES}^{a_{im}}_s\left(\nu_i(\widehat{S}_t-\widehat{S}_{t^\delta})\right)-\mathbb{V}\mathrm{a}\mathbb{R}^{a_{im}}_s\left(\nu_i(\widehat{S}_t-\widehat{S}_{t^\delta})\right)\right].\end{aligned}$$

The desired result follows as the process $\widehat{S}$ is a Black martingale with volatility σ. ∎

Recalling (14):

Proposition 6.1 *We have*

$$\begin{aligned}\beta_t\mathrm{CVA}^{ccp}_t=\sum_i&\mathrm{Nom}_i\times\\&\left(\mathbb{1}_{t<\tau_i}\widehat{S}_t\int_t^T A_i(s)\gamma_i(s)e^{-\int_t^s\gamma_i(u)du}ds+\mathbb{1}_{\tau_i<t<\tau_i^\delta}E_i(\tau_i,\widehat{S}_{\tau_i},t,\widehat{S}_t)\right),\end{aligned}$$

where, setting $y_\pm=\frac{\ln(\widehat{S}_t/\widehat{S}_{\tau_i})}{\sigma\sqrt{\tau_i^\delta-t}}\pm\frac{1}{2}\sigma\sqrt{\tau_i^\delta-t}$,

$$E_i(\tau_i,\widehat{S}_{\tau_i},t,\widehat{S}_t)=f(\tau_i)\times\begin{cases}\widehat{S}_t\Phi(y_+)-\widehat{S}_{\tau_i}\Phi(y_-), & \nu_i\le 0\\ \widehat{S}_{\tau_i}\Phi(-y_-)-\widehat{S}_t\Phi(-y_+), & \nu_i>0.\end{cases}$$

Proof. We have

$$\beta_t \mathrm{CVA}_t^{ccp} = \sum_i \mathbb{1}_{t<\tau_i^\delta} \mathbb{E}_t \left[\left(\beta_{\tau_i^\delta} (\mathrm{MtM}^i_{\tau_i^\delta} + \Delta^i_{\tau_i^\delta}) - \beta_{\tau_i} (\mathrm{MtM}^i_{\tau_i} + \mathrm{IM}^i_{\tau_i}) \right)^+ \right]$$

$$= \sum_i \mathbb{1}_{t<\tau_i} \mathbb{E}_t \left[\mathbb{E}_{\tau_i -} \left(\left(\beta_{\tau_i^\delta} (\mathrm{MtM}^i_{\tau_i^\delta} + \Delta^i_{\tau_i^\delta}) - \beta_{\tau_i} (\mathrm{MtM}^i_{\tau_i} + \mathrm{IM}^i_{\tau_i}) \right)^+ \right) \right]$$

$$+ \sum_i \mathbb{1}_{\tau_i < t < \tau_i^\delta} \mathbb{E}_t \left[\left(\beta_{\tau_i^\delta} (\mathrm{MtM}^i_{\tau_i^\delta} + \Delta^i_{\tau_i^\delta}) - \beta_{\tau_i} (\mathrm{MtM}^i_{\tau_i} + \mathrm{IM}^i_{\tau_i}) \right)^+ \right]$$

$$= \sum_i \mathbb{1}_{t<\tau_i} \mathbb{E}_t \int_t^T \mathbb{E}_s \left[\left(\beta_{s^\delta} (\mathrm{MtM}^i_{s^\delta} + \Delta^i_{s^\delta}) - \beta_s (\mathrm{MtM}^i_s + \mathrm{IM}^i_s) \right)^+ \right] \gamma_i(s) e^{-\int_t^s \gamma_i(u) du} ds$$

$$+ \mathrm{Nom} \sum_i \mathbb{1}_{\tau_i < t < \tau_i^\delta} f(\tau_i) \, \mathbb{E}_t \left[\left(\nu_i (\widehat{S}_{\tau_i} - \widehat{S}_{\tau_i^\delta}) \right)^+ \right], \quad (27)$$

by virtue of (25) and of the conditional distribution properties of the DMO model (see Crépey et al. (2014, Section 8.2.1)). We conclude the proof by an application of Lemma 6.1 to the first line in (27) and of the Black formula to the second line. ∎

6.1 Default Fund as Economic Capital of the CCP

In this section we consider a default fund set, for instance in the context of XVA computations, as the economic capital of the CCP, in the sense of the conditional expected shortfall of its one-year ahead loss and profit. Namely, at time t (assuming for simplicity in this case study that the default fund is updated in continuous time)

$$\mathrm{DF}_t = \mathrm{EC}_t(L^{ccp}) = \mathbb{ES}_t^{a_{df}} \Big(\int_t^{t+1} \beta_t^{-1} \beta_s dL_s^{ccp} \Big) \quad (28)$$

(cf. (15)–(16)), where the integral involves the counterparty default losses and the CVA^{ccp} process as detailed in (17).

In practice, for numerical tractability, we work with $\mathbb{ES}_0^{a_{df}}$ instead of $\mathbb{ES}_t^{a_{df}}$ in (28). In other terms we compute a default fund term structure as opposed to a whole process. Computing a full-flesh conditional expected shortfall process as of (28) would require nested Monte Carlo simulation (and even doubly nested Monte Carlo in more complex models where CVA^{ccp} is not known analytically).

We use $m = 10^5$ simulated paths of S and default scenarios. All the reported numbers are in basis points (bp). We recall that the nominal "Nom" of the swap was fixed so that each leg equals $1 = 10^4$ bp at time 0. Unless stated otherwise we use $a_{im} = 85\%$ and $a_{df} = 99\%$.

The solid blue curves in Figure 3 show the resulting default fund term structures for $a_{df} = 85\%, 95.5\%$ and 99% (top to bottom). The respective dotted red and dashed green curves represent the analog results using value at risk instead of expected shortfall in (28), respectively ignoring the CVA terms (the second line) in (17).

The broadly decreasing feature of all curves reflects the run-off feature of the modeling setup. The comparison between the solid blue and the dotted red curves

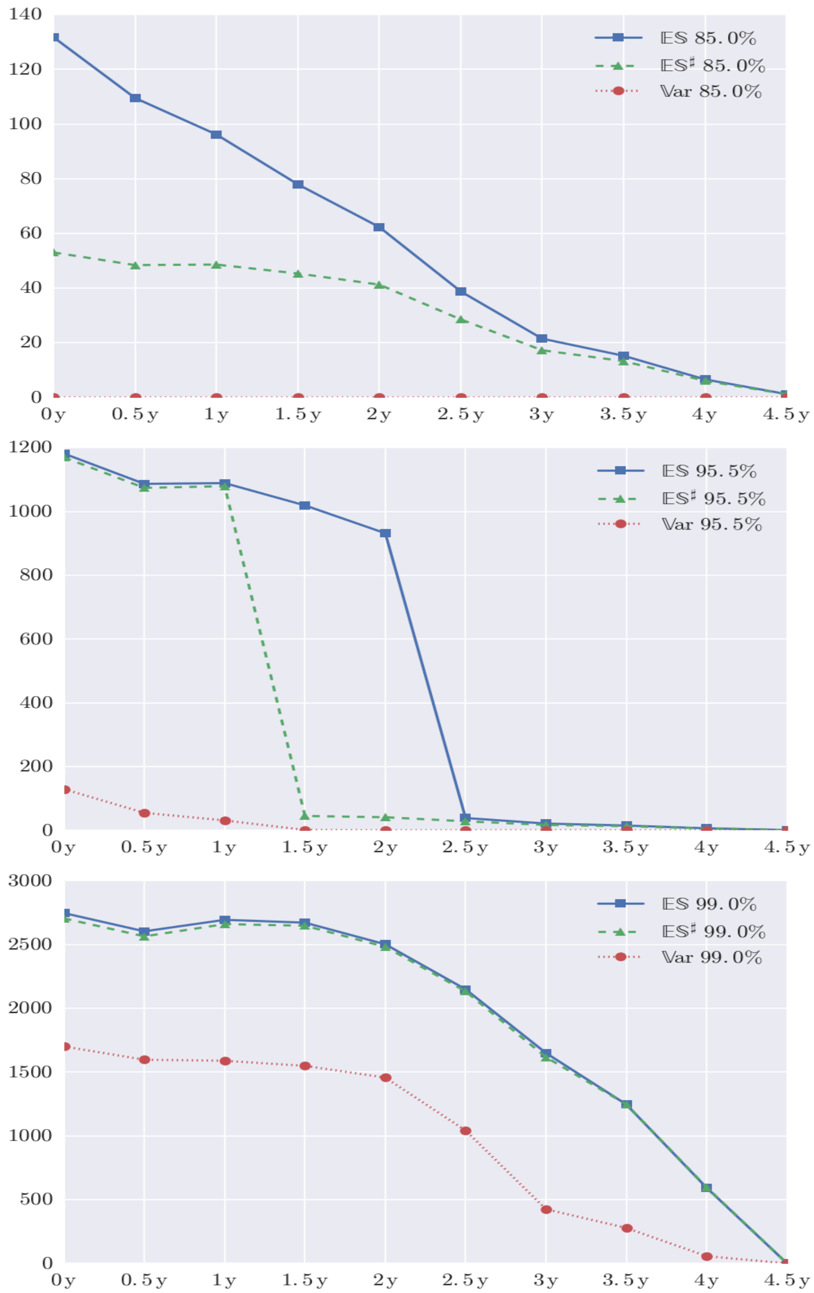

Figure 3: *Solid blue curves*: Economic capital based default fund of the CCP, as a function of time, for $a_{df} = 85\%, 95.5\%$ and 99% ($a_{im} = 85\%$). *Dotted red curves*: Analog results using value at risk instead of expected shortfall in (28). *Dashed green curves*: Analog results ignoring the CVA terms (the second line) in (17).

shows that for too low DF quantile levels a_{df}, the corresponding value-at-risk misses the right tail of the distribution of the losses: the 85% value at risk curve in the upper panel is visually indistinguishable from 0, so that the corresponding expected shortfall reduces to an expectation of the positive part of the losses. The comparison between the solid blue and the dashed green curves in Figure 3 reveals that when the DF quantile level a_{df} increases, the impact of the CVA terms in (28) decreases. It shows that the right tail of the distribution of the losses is driven by the counterparty default losses rather than by the volatile swings of CVA^{ccp}. This could be expected given the common shock intensity model that we use for the default times. Extreme swings of CVA^{ccp} could only arise in more structural credit models, where defaults are announced by volatile swings of CDS spreads.

This analysis is confirmed by Figure 4, which shows, for each time interval (with

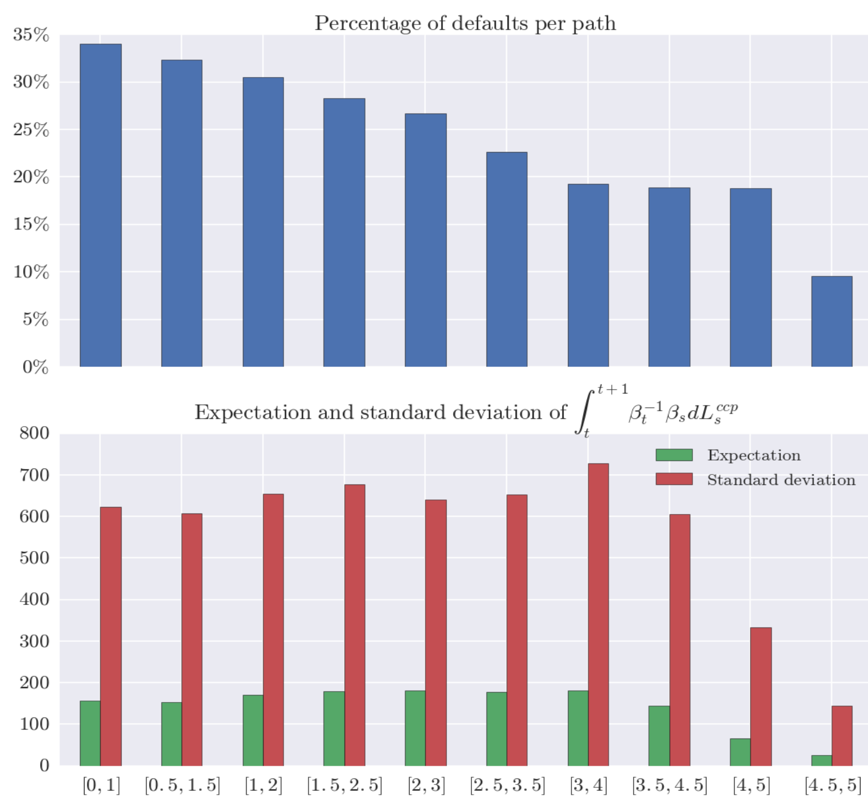

Figure 4: *Top*: Proportion of defaults per simulated path. *Bottom*: Expectation and standard deviation of the losses.

overlapping) $(0, 1), (0.5, 1.5), \ldots, (4.5, 5.5)$, the proportion of defaults per simulated path (upper panel) and the expectation and standard deviation of the corresponding losses (bottom panel). For instance, a proportion of 30% means that, over the 10^5 simulated paths, $30\% \times 10^5 = 3 \times 10^4$ defaults happened on the corresponding time interval. The run-off feature of the setup means that the clearing member portfolios purely amortize as time passes, but it also implies that defaulted clearing members are not replaced by new ones in the CCP. Hence, as time passes, there are less and less defaults on average (the mean and standard deviation of the losses take much more time to amortize, as the bottom panel of Figure 4 illustrates). Since the right tail of the losses is driven by the defaults, the EC based default fund exhibits the decreasing term structure shown by the solid blue curves in Figure 3.

Figure 5 represents, as a function of the IM quantile level a_{im}, the time-0 DF

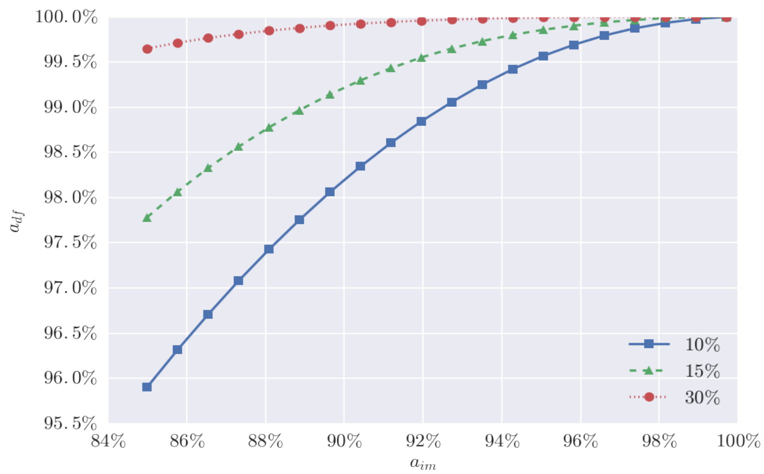

Figure 5: Time-0 DF quantile level a_{df} resulting in a default fund equal to 10% (solid blue curve), 15% (dashed green curve) or 30% (dotted red curve) of the total IM of the CCP, plotted as a function of the IM quantile level a_{im} of the clearing members.

quantile level a_{df} calibrated to the objective of a total default fund equal to 10% (solid blue curve), 15% (dashed green curve) or 30% (dotted red curve) of the total IM of all the clearing members—a range of values commonly encountered in the case of a CCP clearing interest rate derivatives. With $m = 10^5$ scenarios as we have, the a_{df} quantile level corresponding to a default fund equal to 50% or more of the total IM of the CCP, an order of magnitude not uncommon in the case of a CCP clearing CDS contracts, would be visually indistinguishable from 100% already for $a_{im} = 85\%$.

6.1.1 KVA of the CCP

The KVA of the CCP estimates how much it would cost the CCP to remunerate the clearing members at some hurdle rate h for their capital at risk in the default fund, namely (cf. (2) and (12))

$$\mathrm{KVA}_t^{ccp} = h\mathbb{E}_t\Big[\int_t^T e^{-(r+h)s}\mathrm{DF}_s ds\Big].$$

Of course this formula can be readily extended to a member dependent hurdle rate, taken for simplicity in our numerics as a common and exogenous constant $h = 10\%$.

Figure 6 shows the KVA term structures corresponding to a default fund sized by the

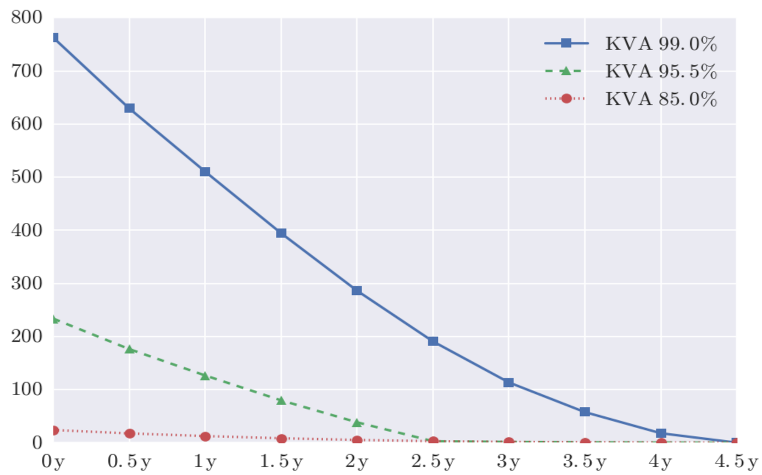

Figure 6: KVA term structures corresponding to the EC (solid blue) curves of Figure 3 ($h = 10\%$).

EC (solid blue) curves of Figure 3.

6.2 Default Fund Contributions

Let $\mathrm{EC}^{(-j)}$ denote the economic capital of the CCP deprived from its j^{th} member, i.e. with the j^{th} member replaced by the risk-free "buffer" in all its CCP transactions. Namely, at time t (cf. (16)-(17))

$$\begin{aligned}\mathrm{EC}_t^{(-j)} = \mathbb{ES}_t^{a_{df}}\Big(&\sum_{t<\tau_i^\delta\le t+1, i\neq j} \big(\beta_{\tau_i^\delta}(\mathrm{MtM}^i_{\tau_i^\delta} + \Delta^i_{\tau_i^\delta}) - \beta_{\tau_i}(\mathrm{MtM}^i_{\tau_i} + \mathrm{IM}^i_{\tau_i})\big)^+ \\ &- \big(\beta_t \mathrm{CVA}_t^{(-j)} - \beta_{t+1}\mathrm{CVA}_{t+1}^{(-j)}\big)\Big),\end{aligned}$$

where $\mathrm{CVA}_t^{(-j)}$ corresponds to the CVA of the CCP (cf. (14)) deprived from its j^{th} member.

In the line of Sect. 6.1, we can consider an allocation of the default fund between the clearing members proportional to the incremental change in economic capital attributable to each of them. Namely, as long as all the clearing members are alive (in particular at time 0)

$$\mu_t^{ec,i} = \frac{\Delta_i \mathrm{EC}_t}{\sum_j \Delta_j \mathrm{EC}_t}, \text{ where } \Delta_j \mathrm{EC}_t = \mathrm{EC}_t - \mathrm{EC}_t^{(-j)}.$$

A variant would be to allocate the default fund proportionally to the member incremental KVA^{ccp}. Let $\mathrm{KVA}_t^{(-j)} = h\mathbb{E}_t[\int_t^T e^{-(r+h)s}\mathrm{EC}_s^{(-j)}ds]$ denote the value of the KVA of the CCP deprived from its j^{th} member. The corresponding allocation is written as

$$\mu_t^{kva,i} = \frac{\Delta_i \mathrm{KVA}_t}{\sum_j \Delta_j \mathrm{KVA}_t}, \text{ where } \Delta_j \mathrm{KVA}_t = \mathrm{KVA}_t - \mathrm{KVA}_t^{(-j)}.$$

Figure 7 shows the time-0 default fund allocations based on member initial margin, member incremental economic capital and member incremental KVA, respectively represented by blue, red and green bars. In the upper panel the clearing members in the x axis are ordered by increasing position $|\nu_i|$, whereas in the lower panel they are ordered by increasing credit spread Σ_i (cf. Table 1). In the present setup where all portfolios are driven by a single Black-Scholes underlying, the initial margins, hence the blue bars in Figure 7, are simply proportional to the size $|\nu_i|$ (or nominal Nom_i) of the member positions. By contrast, the member incremental economic capital or KVA allocations (green and red bars) also take the credit risk of the members into account.

Figure 8 shows the term structures of the EC and KVA based allocation weights for each of the clearing members. We clearly see the impact of market but also credit risk on these term structures. At the beginning of the time period (and in particular at time 0), where defaults are, on average, still to come, with probabilities reflected by the time-0 credit spreads of the clearing members, the impact of credit risk is even predominant in the allocation weights.

7 Funding Strategies for Initial Margins

In the setup of our case study, the generic expressions (18) and (20) for the unsecured borrowing vs. specialist lender MVAs can be computed by deterministic time integration based on the following formulas.

Proposition 7.1 *The unsecured borrowing* MVA *of member i is given, at time 0, by*

$$\mathrm{MVA}_0^{ub,i} = \mathrm{Nom}_i\, S_0 \int_0^T B_i(s)\lambda_i(s)e^{-\int_0^s \gamma_i(u)du} ds.$$

Proof. By virtue of (18) and of the distributional properties of the DMO model, we have

$$\mathrm{MVA}_0^{ub,i} = \mathbb{E}\int_0^{T\wedge\tau_i} \beta_s\lambda_i(s)\mathrm{IM}_s^i ds = \mathbb{E}\int_0^T \beta_s\lambda_i(s)\, e^{-\int_0^s \gamma_i(u)du}\, \mathrm{IM}_s^i ds.$$

Hence the result follows from Lemma 5.1. ∎

Lemma 7.1 *We have*

$$\mathbb{E}_s\Big[\big(\beta_{s^\delta}(\mathrm{MtM}_{s^\delta}^i + \Delta_{s^\delta}^i) - \beta_s \mathrm{MtM}_s^i\big)^+\Big] = \mathrm{Nom}_i\, C(s)\, \widehat{S}_s, \tag{29}$$

where

$$C(s) = f(s)\left[\Phi\left(\frac{\sigma\sqrt{\delta}}{2}\right) - \Phi\left(-\frac{\sigma\sqrt{\delta}}{2}\right)\right]. \tag{30}$$

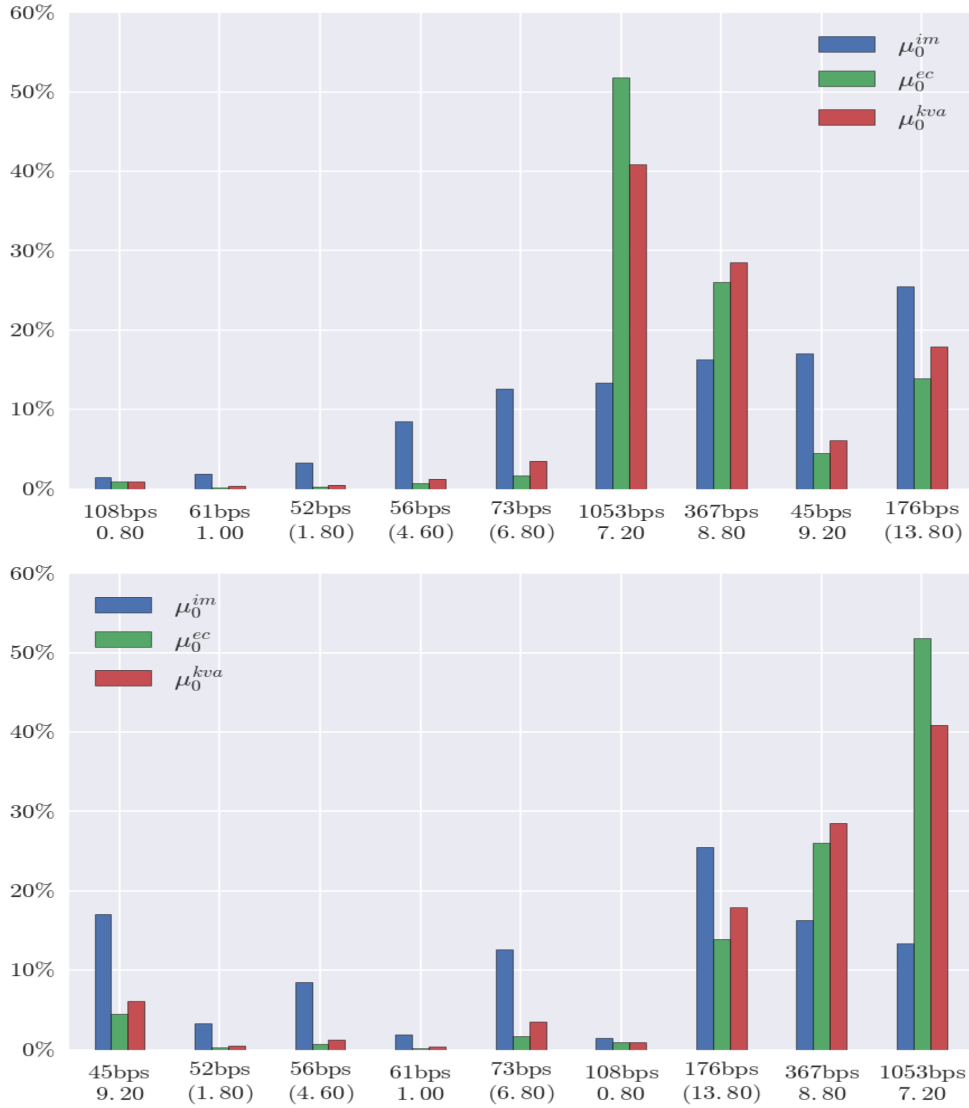

Figure 7: Time-0 default fund allocation based on member initial margin, member incremental EC and member incremental KVA. *Top*: Members ordered by increasing position $|\nu_i|$. *Bottom*: Members ordered by increasing credit spread Σ_i.

Figure 8: Default fund allocation weights term structures based on member incremental economic capital (in blue) or KVA (in green) for each member, ordered from left to right and top to bottom per increasing credit spread, as a function of time $t = 0, \ldots, 4.5$.

Proof. In view of (25), it comes:

$$\left(\beta_{s^\delta}(\mathrm{MtM}^i_{s^\delta}+\Delta^i_{s^\delta})-\beta_s\mathrm{MtM}^i_s\right)^+=\mathrm{Nom}\times f(s)\left(\nu_i(\widehat{S}_s-\widehat{S}_{s^\delta})\right)^+.$$

Hence the result follows from the Black formula. ∎

Proposition 7.2 *The specialist lender* MVA *of member i is given, at time 0, by*

$$\mathrm{MVA}^{sl,i}_0=\mathrm{Nom}_i\,S_0\int_0^T\Big(C(s)-A_i(s)\Big)\lambda_i(s)\,e^{-\int_0^s\gamma_i(u)du}ds.$$

Proof. Let

$$\begin{aligned}\xi^i_s&=\mathbb{E}_s\left[\left(\beta_{s^\delta}(\mathrm{MtM}^i_{s^\delta}+\Delta^i_{s^\delta})-\beta_s\mathrm{MtM}^i_s\right)^+\wedge\beta_s\mathrm{IM}^i_s\right]\\&=\mathbb{E}_s\left[\left(\beta_{s^\delta}(\mathrm{MtM}^i_{s^\delta}+\Delta^i_{s^\delta})-\beta_s\mathrm{MtM}^i_s\right)^+\right]\\&\qquad-\mathbb{E}_s\left[\left(\beta_{s^\delta}(\mathrm{MtM}^i_{s^\delta}+\Delta^i_{s^\delta})-\beta_s(\mathrm{MtM}^i_s+\mathrm{IM}^i_s)\right)^+\right]\\&=\mathrm{Nom}_i\,\widehat{S}_s\big(C(s)-A_i(s)\big),\end{aligned}$$

by Lemma 7.1 and Lemma 6.1. Note this is a predictable process. Hence

$$\begin{aligned}\mathrm{MVA}^{sl,i}_0&=\mathbb{E}\Big[\mathbb{1}_{\tau_i<T}(1-R_i)\Big(\big(\beta_{\tau_i^\delta}(\mathrm{MtM}^i_{\tau_i^\delta}+\Delta^i_{\tau_i^\delta})-\beta_{\tau_i}\mathrm{MtM}^i_{\tau_i}\big)^+\wedge\beta_\tau\mathrm{IM}^i_{\tau_i}\Big)\Big]\\&=\mathbb{E}\Big[\mathbb{1}_{\tau_i<T}(1-R_i)\mathbb{E}_{\tau_i}\Big(\big(\beta_{\tau_i^\delta}(\mathrm{MtM}^i_{\tau_i^\delta}+\Delta^i_{\tau_i^\delta})-\beta_{\tau_i}\mathrm{MtM}^i_{\tau_i}\big)^+\wedge\beta_\tau\mathrm{IM}^i_{\tau_i}\Big)\Big]\\&=\mathbb{E}\left[\mathbb{1}_{\tau_i<T}(1-R_i)\xi^i_{\tau_i}\right]=\mathbb{E}\left[\int_0^T\lambda_i(s)\,e^{-\int_0^s\gamma_i(u)du}\,\xi^i_s\,ds\right],\end{aligned}\tag{31}$$

where the conditional distribution properties of the DMO model were used in the last equality (see Crépey et al. (2014, Section 8.2.1)). ∎

Figure 9 shows the time-0 MVAs of the nine clearing members for unsecurely borrowed (top) vs. specialist lender (bottom) initial margin funding policies, for $a_{im}=$ 70% (blue), 80% (green), 90% (red) and 97.5% (purple). For each of the clearing members, its specialist lender MVA appears several times cheaper than its unsecured borrowing MVA (note the different scale of the y axis between the top and the lower panel in Figure 9).

As explained in Sect. 3.3, in a centrally cleared setup with daily remargining, the most important XVA numbers of a clearing member are its MVA and its KVA. Figure 10 compares the MVA and the KVA of each of the nine clearing members in our case study, under alternative specifications: unsecurely borrowed vs. specialist lender initial margin regarding the MVA, member incremental EC vs. member incremental KVA allocation of an EC based default fund regarding the KVA. The credit risk of the clearing members appears to be a more important driver of their MVA and KVA than their market risk: the bars of each given color are better ordered in the bottom panel, where they are ranked by increasing credit spread of the clearing members, than in the upper panel, where they are ranked by increasing position of the clearing members.

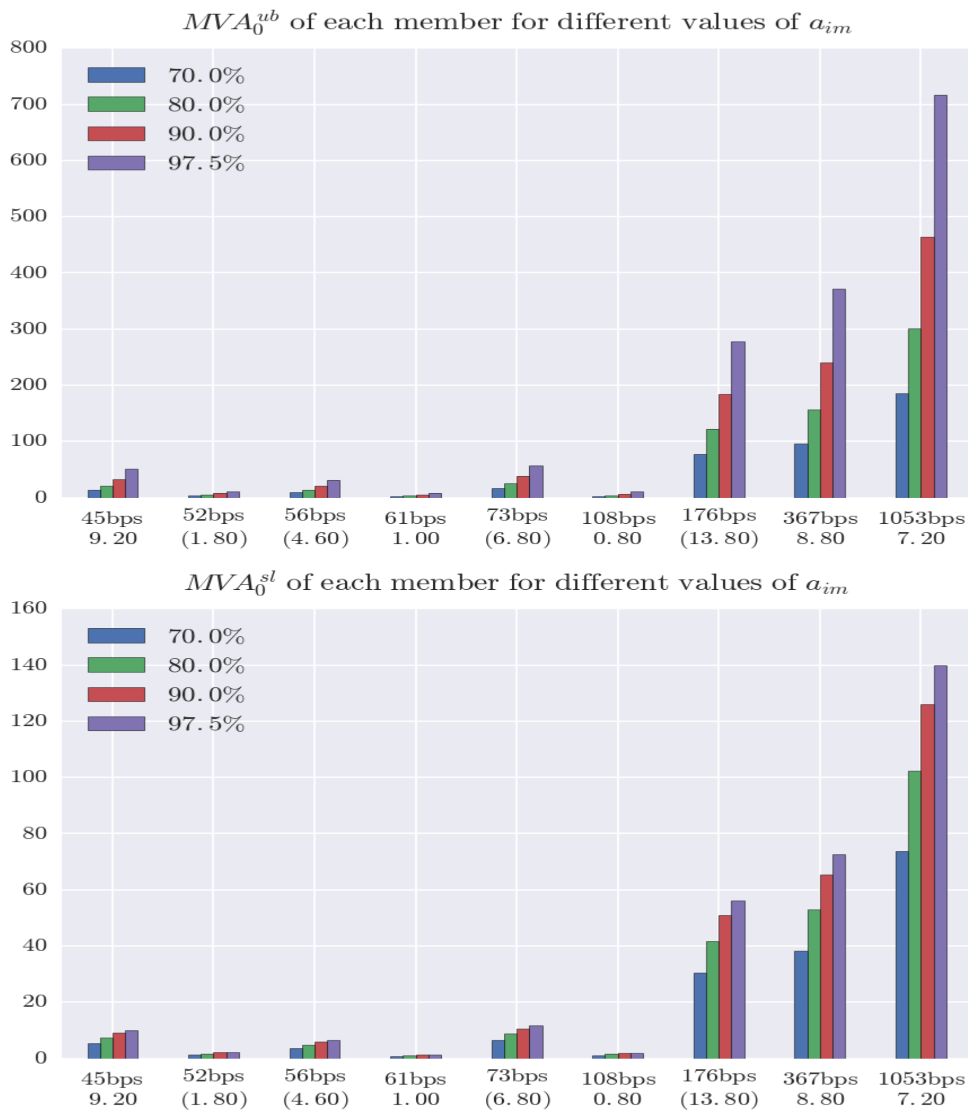

Figure 9: MVAs of the nine clearing members for unsecurely borrowed (top) vs. specialist lender (bottom) initial margin funding policies, for $a_{im} = 70\%$ (blue), 80% (green), 90% (red) and 97.5% (purple).

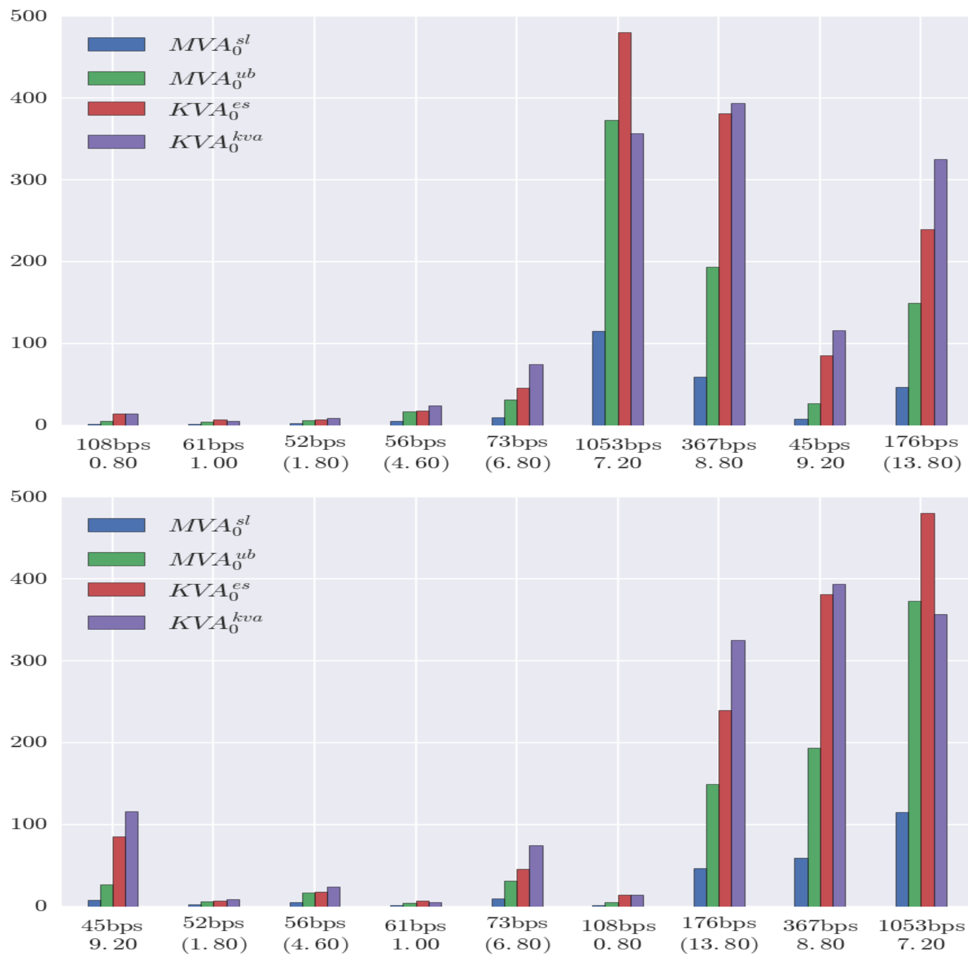

Figure 10: MVA and KVA for each of the clearing members ordered along the x axis by increasing position (top) or increasing credit spread (bottom).

8 Conclusion

In this work we consider two important capital and funding issues related to CCPs.

First, from the CCP perspective, we challenge the Cover 2 EMIR rule, for the sizing of the default fund, by an economic capital specification in the form of an expected shortfall of the one year ahead loss and profit of the CCP. We compare the usual IM based allocation of the default fund with an allocation proportional to the incremental impact of each clearing member on the economic capital of the CCP (or on the ensuing KVA). The EC based size and allocation of the default fund incorporate a mix of market and credit risk of the clearing members, by contrast with the purely market risk sensitive Cover 2 sizing rule and IM based allocation.

Second, from a clearing member perspective, we compare the MVAs resulting from two different strategies regarding the raising of their initial margin: the classical approach where the initial margin is unsecurely borrowed by the clearing member and a strategy where the clearing member delegates the posting of its initial margin to a specialist lender in exchange of a service fee. The alternative strategy yields a very significant MVA reduction.

We conclude that two major inefficiencies related to CCPs could be significantly compressed by resorting to alternative IM funding scheme and DF sizing, allocation and possibly remuneration policies. In the context of XVA computations, which entail projections over decades, it might be interesting for a bank to compute the MVA and KVA corresponding to these alternative IM and DF specifications even under the current regulatory environment, as a counterpart to the corresponding regulatory based XVA metrics.

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