Master of Financial Engineering : Stochastic Calculus with applications to finance

Stéphane Menozzi,
Université d'Evry Val d'Essonne,
Laboratoire de Modélisation Mathématique d'Evry,
stephane.menozzi@univ-evry.fr,

The purpose of these lectures is to provide the mathematical background to apprehend a wide class of models appearing in finance. We focus on continuous processes through the study of Brownian motion, Itô's formula, Brownian driven Stochastic Differential Equations (SDEs), their correspondence with some appropriate Partial Differential Equations (PDEs). We will also investigate some associated strategies of dynamic/static pricing and hedging of options.

Here are the following key-points of the course.

- Preliminary results on Gaussian vectors
- Characterizations of Brownian motion (as a Gaussian process or as a process with stationary independent Gaussian increments)
- Construction of Brownian motion through the approach of Paul Lévy.
- Itô processes and Itô's formula.
- SDEs and some associated Parabolic PDEs : the Feynman-Kac representation formula and Applications to finance
- The Girsanov theorem.

The prerequisites are undergraduate probability (Markov chains, discrete time martingales, different convergence notions in probability). Some possible companion books to the lectures are the following :

[LG12] Le Gall, J.F., *Brownian Motion, Martingales, and Stochastic Calculus.* Springer.

[LL07] Lamberton, D. and Lapeyre, B. *Introduction to Stochastic Calculus Applied to Finance, Second Edition.* Chapman & Hall

[CM06] Comets, F. and Meyre, T. *Calcul stochastique et modèles de diffusions : Cours et exercices corrigés.* Dunod.

Some more advanced references are :

[KS98] Karatzas, I. and Shreve, S. *Brownian Motion and Stochastic calculus.* Springer.

[RW07a] Rogers, L.C.G. and Williams, D. *Diffusions, Markov Processes, and Martingales : Volume 1, Foundations.* Cambridge University press.

[RW07b] Rogers, L.C.G. and Williams, D. *Diffusions, Markov Processes and Martingales : Volume 2, Itô Calculus.* Cambridge University press

[IW81] Ikeda, N. and Watanabe, N. *Stochastic differential equations and diffusion processes.* North Holland.

Assessment : written examination.